Module 9: The JWKB Approximation & Applications

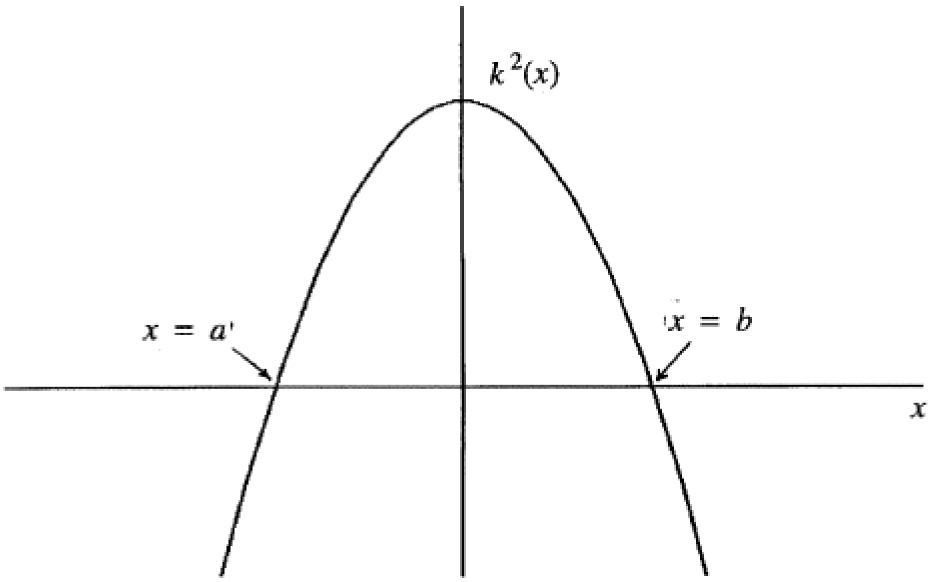

Fig. 9.1

9.1 Consider a typical $k^2(x)$ variation as shown in Fig. 9.1. We consider an exponentially decaying solution in the region $x > b$: $\psi(x) = \frac{1}{\sqrt{\kappa(x)}} \exp\left[-\int_b^x \kappa(x)\, dx\right]$; $\kappa^2(x) = -k^2(x)$.

The solution in the region $a < x < b$ will be

(a) $\psi(x) = \frac{2}{\sqrt{k(x)}} \sin\left[\int_x^b k(x)\, dx + \frac{\pi}{4}\right]$

(b) $\psi(x) = \frac{2}{\sqrt{k(x)}} \cos\left[\int_x^b k(x)\, dx + \frac{\pi}{4}\right]$

(c) $\psi(x) = \frac{1}{\sqrt{k(x)}} \sin\left[\int_x^b k(x)\, dx + \frac{\pi}{4}\right]$

(d) $\psi(x) = \frac{1}{\sqrt{k(x)}} \cos\left[\int_x^b k(x)\, dx + \frac{\pi}{4}\right]$

[Answer (a)]

9.2 We assume that the variation of $k^2 \;\; x$ as shown in Fig. 9.1 is symmetric in $x \;\; a = -b$. We consider the antisymmetric solution in the region $-b < x < b$; i.e., in the region $-b < x < b$, the JWKB wave function is given by $\psi \;\; x = \frac{1}{\sqrt{k \;\; x}} \sin\left[\int_0^x k \;\; x \;\; dx\right]$. The JWKB solution in the region $x > b$ will be $\left(\alpha = \int_0^b k \;\; x \;\; dx + \frac{\pi}{4}\right)$

(a) $\psi \;\; x = \frac{1}{\sqrt{\kappa \;\; x}} \exp\left[-\int_b^x \kappa \;\; x \;\; dx\right]$

(b) $\psi \;\; x = \frac{1}{\sqrt{\kappa \;\; x}} \exp\left[+\int_b^x \kappa \;\; x \;\; dx\right]$

(c) $\psi \;\; x = \frac{\sin\alpha}{\sqrt{\kappa \;\; x}} \exp\left[-\int_b^x \kappa \;\; x \;\; dx\right] - \frac{\cos\alpha}{\sqrt{\kappa \;\; x}} \exp\left[+\int_b^x \kappa \;\; x \;\; dx\right]$

(d) $\psi \;\; x = \frac{\sin\alpha}{\sqrt{\kappa \;\; x}} \exp\left[+\int_b^x \kappa \;\; x \;\; dx\right] - \frac{\cos\alpha}{2\sqrt{\kappa \;\; x}} \exp\left[-\int_b^x \kappa \;\; x \;\; dx\right]$

[Answer (d)]