Module 2: Simple Solutions of the one-dimensional Schrodinger Equation

2.1 For a free particle, the most general solution of the 1-dimensional Schrodinger equation is given by $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{+\infty} a(p)\, exp\left[\frac{i}{\hbar}\left(p\,x - \frac{p^2}{2\mu}t\right)\right] dp$. Assume $\Psi(x.0) = \frac{1}{\left(\pi\sigma_0^2\right)^{1/4}} \exp\left[-\frac{x^2}{2\sigma_0^2}\right] \exp\left[\frac{i}{\hbar} p_0 x\right]$. The value of $a(p)$ is given by

(a) $a(p) = \left(\frac{\sigma_0^2}{\pi\hbar^2}\right)^{1/4} \exp\left[-\frac{2\left(p-p_0\right)^2 \sigma_0^2}{\hbar^2}\right]$

(b) $a(p) = \left(\frac{\sigma_0^2}{\pi\hbar^2}\right)^{1/4} \exp\left[-\frac{\left(p-p_0\right)^2 \sigma_0^2}{2\hbar^2}\right]$

(c) $a(p) = \left(\frac{\sigma_0^2}{\pi\hbar^2}\right)^{1/4} \exp\left[-\frac{\left(p-p_0\right)^2 \sigma_0^2}{4\hbar^2}\right]$

(d) $a(p) = \left(\frac{\sigma_0^2}{\pi\hbar^2}\right)^{1/4} \exp\left[-\frac{4\left(p-p_0\right)^2 \sigma_0^2}{\hbar^2}\right]$

[Answer (b)]

2.2 Consider a wave packet given by $\psi(x,y,0) = \left[\left(\frac{1}{\pi\sigma_0^2}\right)^{\frac{1}{4}} \exp\left[-\frac{x^2}{2\sigma_0^2}\right] e^{\frac{i}{\hbar}p_0 x}\right] \psi_b(y)$

where

$$\left.\begin{aligned} \psi_b(y) &= \frac{1}{\sqrt{b}} \qquad |y| < b/2 \\ &= 0 \qquad\quad |y| > b/2 \end{aligned}\right\}$$

$P(p_y)\, dp_y$ represents the probability of the *y*-component of the momentum between p_y and $p_y + dp_y$; it will be given by

(a) $\frac{b}{2\pi\hbar}\frac{\sin^2\left(p_y b/2\hbar\right)}{\left(p_y b/2\hbar\right)^2}dp_y$

(b) $\frac{b}{2\pi\hbar}\frac{\sin^2\left(p_y b/\hbar\right)}{\left(p_y b/\hbar\right)^2}dp_y$

(c) $\sqrt{\frac{b}{2\pi\hbar}}\frac{\sin\left(p_y b/2\hbar\right)}{p_y b/2\hbar}dp_y$

(d) $\sqrt{\frac{b}{2\pi\hbar}}\frac{\sin\left(p_y b/\hbar\right)}{p_y b/\hbar}dp_y$

[Answer (a)]

2.3 For a particle in a one dimensional box, the wave function is given by

$$\begin{aligned}\psi(x) &= N\sin\left(\frac{3\pi x}{L}\right) \qquad 0<x<L\\ &= 0 \qquad\qquad x<0 \text{ and } x>L\end{aligned}$$

The normalization constant N is given by:

(a) $N=\sqrt{\frac{1}{L}}$

(b) $N=\sqrt{\frac{2}{L}}$

(c) $N=\sqrt{\frac{3}{L}}$

(d) $N=\sqrt{\frac{4}{L}}$

[Answer (b)]

2.4 Find the potential function $V(x)$ for which the wavefunction is $\psi(x)=N\exp\left(-\frac{\mu S}{\hbar^2}|x|\right)$

(a) $V(x)=-S\delta(x)$

(b) $V(x)=+S\delta(x)$

(c) $V(x)=0$

(d) $V(x)=-S$

[Answer (a)]

2.5 Consider the wavefunction $\psi(x) = N\exp\left(-\frac{\mu S}{\hbar^2}|x|\right); -\infty < x < +\infty$. Calculate the normalization constant N.

(a) $N = \frac{\mu S}{\hbar^2}$

(b) $N = \sqrt{\frac{\mu S}{\hbar^2}}$

(c) $N = \sqrt{\frac{\hbar^2}{\mu S}}$

(d) $N = \frac{\hbar^2}{\mu S}$

[Answer (b)]