

Quizzes and short questions QUANTUM ELECTRONICS
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Module 5: Quizzes and short questions:

1. A linear harmonic oscillator is prepared in a state such that the probability of finding it with an energy of $5/2 \hbar\omega$ is 0.2 and with an energy of $11/2 \hbar\omega$ is 0.8. Write down two possible kets describing the oscillator at $t = 0$.

2. Q: Consider a single mode radiation in a state given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

Obtain the expectation value of the electric field in the state.

3. Q: Consider a two mode state given by

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0_1, 0_2\rangle + \frac{1}{\sqrt{2}}|1_1, 1_2\rangle$$

Obtain the expectation value of the number of photons in this state.

4. Q: What is the expectation value of energy in a coherent state $|\alpha\rangle$ with $\alpha = 5$?

5. Q: Consider the following single mode state

$$|\psi\rangle = \frac{1}{\sqrt{0.2}}|n\rangle + \frac{1}{\sqrt{0.2}}|n+1\rangle$$

Obtain the average number of photons in this state.

Answers of module 5 Quizzes and short questions:

A1: $|\psi\rangle_1 = \sqrt{0.2}|2\rangle + \sqrt{0.8}|5\rangle$ and $|\psi\rangle_2 = \sqrt{0.2}|2\rangle - \sqrt{0.8}|5\rangle$

Hence the efficiency for SHG will become zero when $\Delta k = 2\pi/L$ which for the values given gives us $\Delta k = 1.26 \text{ cm}^{-1}$.

A2 : The electric field operator is given by

$$\hat{E} = i \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} [\hat{a}e^{-i(\omega t - kz)} - \hat{a}^\dagger e^{+i(\omega t - kz)}]$$

Thus the expectation value of the electric field in the given state will be

$$\langle\psi|\hat{E}|\psi\rangle = \sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}} \sin(\omega t - kz)$$

A3: The expectation value of the number of photons is given by

$$\langle\psi|(\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2)|\psi\rangle = \frac{1}{2} \langle 1_1, 1_2 | (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2) | 1_1, 1_2 \rangle = 1$$

A4 : $\langle\alpha|\hat{a}^\dagger \hat{a}|\alpha\rangle = 5$

Hence $\langle E \rangle = 5.5 \hbar\omega$

A5: A: $\langle\psi|\hat{a}^\dagger \hat{a}|\psi\rangle = \frac{1}{2} [\langle n | + \langle n + 1 |] \hat{a}^\dagger \hat{a} [|n\rangle + |n + 1\rangle] = \left(n + \frac{1}{2}\right)$