

Hydrostatics and Stability

Dr. Hari Warrior

Model Questions & Answers

Problem set

1. A buoy is to be used to float a mooring chain. The buoy is cylindrical, of length 4 m, radius 0.75 m and is to float half submerged.

The mooring chain is made of steel of RD 7.95 and has a mass of 0.15 tonnes/m. The chain will be suspended in 10m of salt water.

Calculate:

- (a) The mass of the buoy;
(b) The thickness of the plate which should be used to construct the buoy if the buoy is to be made of metal of RD 7.5.

Ans: a) Mass of buoy + mass of chain = mass displaced by buoy

+ Mass displaced by chain

$$M_b + 10 \times 0.15t = \frac{1}{2} \times \Pi \times 0.75^2 \times 4 \times 1.025$$

$$M_b + 1.5t = 3.623t + 0.258t$$

$$M_b = 2.380 \text{ tones}$$

- (b) Let the thickness of the buoy metal be t meters:

$$M_b = v \times \rho$$

Where v is the volume of the metal and ρ is the density of the metal

$$\begin{aligned} \therefore 2.380 &= (2\Pi r t L + 2\Pi r^2 t) 7.15 \\ &= 7.15 \times 2\Pi r t (L + r) \\ &= 7.15 \times 2\Pi \times 0.75 \times 4.75t \\ &= 177.952t \end{aligned}$$

$$\therefore t = \frac{2.380}{177.052} = 0.0134 \text{ m}$$

i.e. Thickness of metal = 13.4 mm

2. A vessel has displacement 6200 tonnes KG, 8.0 m. Distribute 9108 tonnes of cargo between spaces Kg, 0.59 m and 11.45 m so that the vessel completes loading with a KG of 7.57 m

Ans:

Load 'w' tones at Kg 11.45 m.

Weight (tonnes)	KG	Moment
6200.0	8.00	49600.0
9108.0 – w	0.59	5373.7 – 0.59w
w	11.45	+ 11.45w
15308.0		54937.7 – 10.86w

$$KG = \frac{\text{moment of weight}}{\text{displacement}}$$

$$7.57 = \frac{54973.7 - 10.86w}{15308.0}$$

$$115881.6 = 54973.7 - 10.86w$$

$$W = 5612 \text{ tonnes}$$

Load 5612 tonnes at Kg 11.45 m.

Load 3496 tonnes at Kg 0.59 m.

3. A box shaped vessel length, 200 m, breadth 20 m and depth 10 m is loaded so that KG of the vessel is always equal to the draft. Find the maximum draft at which the vessel will be stable, and the GM at minimum KM.

Ans:

If the vessel had draft 'd'

$$KG = d$$

$$KM = KB + BM$$

For a box shape

$$KM = \frac{d}{2} + \frac{B^2}{12d}$$

$$KM = \frac{d}{2} + \frac{400}{12d}$$

$$KM = \frac{d}{2} + \frac{33.33}{d}$$

For

$$GM = 0$$

$$KG = KM$$

$$d = \frac{d}{2} + \frac{33.33}{d}$$

$$2d^2 = d^2 + 66.67$$

$$d^2 = 66.67 \text{ m}$$

$$d = 8.165 \text{ m}$$

By inspection of typical KM curve, vessel will be stable up to 8.165 m draft and unstable at greater drafts.

4. A vessel displacement 22600 tonnes, KG 8.2 m discharges 3000 tonnes of ballast from a mean Kg of 2.0 m. She loads 400 tonnes of cargo at a mean Kg of 7.8 m. A further parcel of 1200 tonnes of cargo remains to be loaded. Determine the mean Kg at which to load this cargo so that the final GM is at least 0.5 m.

Ans:

KM at displacement 32200 tonnes is 9.0 m.

Let x be the mean Kg at which to load cargo

Weight	Kg	Moment
22600	8.2	185300
11400	7.8	88900
1200	x	1200x
-3000	2	-6000
32200		268200 + 1200x

$$\text{Max . Kg} = KM - GM$$

$$= (9.0 - 0.5) \text{ m} = 8.5 \text{ m}$$

$$KG = \frac{\text{moment of weight}}{\text{displacement}}$$

$$8.5 = \frac{268200 + 1200x}{32200}$$

$$x = 4.67 \text{ m}$$

5. MV Nonesuch has draft 8m and KG, 10 m. Compare values from KN curves with values from the wall sided formula for 5°, 12° and 15° heel.

Ans: From Hydrostatic data

KM	11.6 m	KM	11.6 m	Displacement 28200 tonnes
KG	10.0 m	KB	4.2 m	
GM	<u>1.6 m</u>	BM	<u>7.4 m</u>	

$$GZ = \sin \theta (GM + \frac{1}{2} BM \tan^2 \theta)$$

$$= \sin \theta (1.6 + 3.7 \tan^2 \theta)$$

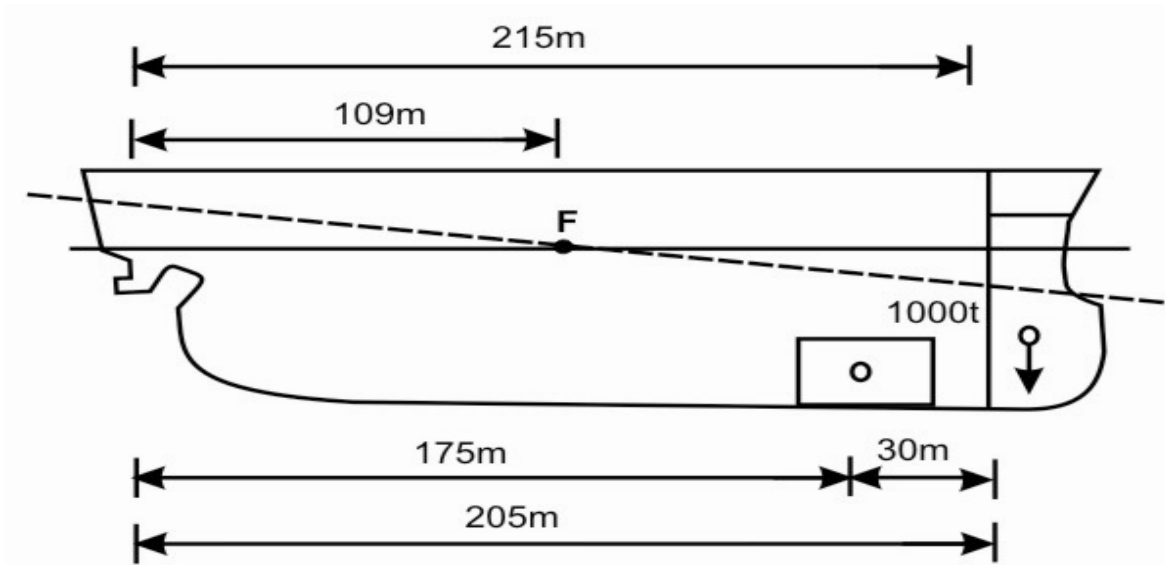
Heel	GZ
5°	0.140 m
12°	0.367 m
15°	0.483 m

From KN curves

	KN	KG sin θ	GZ m
5°	1.02	0.87	0.15
12°	2.49	2.08	0.41
15°	3.08 m	2.59	0.49

6. A vessel displacing 30000 tonnes is floating at drafts F 8.3 m, A 9.6 m. MCTC, 300 tonne m/cm. Centre of Floatation, 109 m forward of after perpendicular (AP), length, 210 m. Find the drafts fore and aft if 1000 tonnes of ballast are moved from a tank centre of gravity 175 m forward of AP to a tank 205 m forward of AP.

Ans:



$$\begin{aligned} \text{Change of trim} &= \frac{\text{moment changing trim}}{\text{MCTC}} \\ &= \frac{1000 \times 30}{300} = 100 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Change of aft} &= \frac{l}{L} \times \text{change of trim} \\ &= \frac{109}{210} \times 100 \text{ cm} = -51.9 \end{aligned}$$

$$\text{Change in trim forward} = +48.1 \text{ cm}$$

F	A
8.300 m	9.600 m
+ 0.481 m	- 0.519 m
8.781 m	9.081 m
8.781 m	9.081 m

Draft Forward 8.78 m

Aft 9.08 m

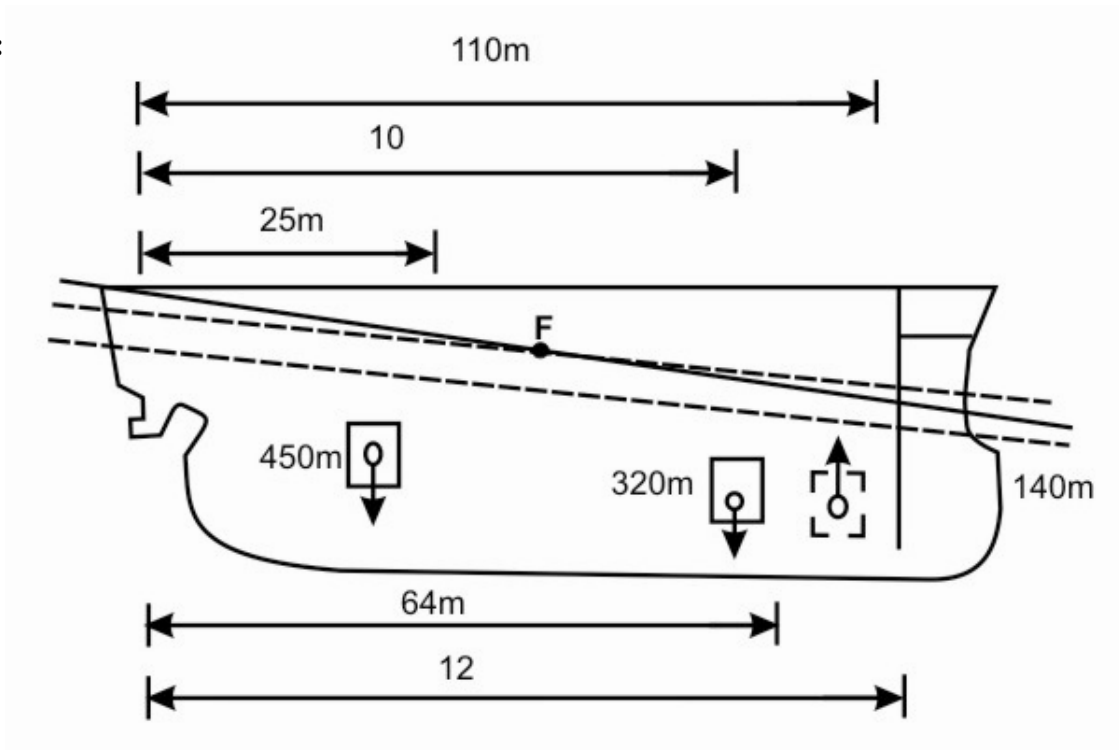
7. A vessel floating at draft Forward 9.84 m; Aft 10.62 m.

She loads

Weight (tonnes)	LCG from AP (m)
450	25
320	100
Discharges 140	110

TCP, 26 tonne/cm; MCTC, 148 tonne m/cm; LCF, 64 m forward of AP; length, 120m.

Ans :



Weight Tonne	LCG from F	Moments forward tonne m	Moments aft tonne m
450	39		17550
320	36	11520	
-140	46	-6440	
<hr/> 630		<hr/> 5080	<hr/> 17550
			<hr/> 5080
			<hr/> 12470 by stern

$$\text{Change of trim} = \frac{\text{moment changing trim}}{\text{MCTC}}$$

$$= \frac{12470}{148} = 84.3 \text{ cm}$$

$$\begin{aligned} \text{Change of trim after} &= \frac{1}{L} \times \text{change of trim} \\ &= \frac{64}{120} \times 84.3 = 45 \text{ cm} \end{aligned}$$

Change of trim forward = 39.3 cm

$$\begin{aligned} \text{Sinkage} &= \frac{w}{TPC} \\ &= \frac{693}{26} = 24.2 \end{aligned}$$

	F	A
Initial draft	9.840	10.620
Sinkage	0.393	+0.242
	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>
Trim	10.082 -0.393	10.862 +0.450
Final draft	<hr style="width: 100%;"/> 9.689 m	<hr style="width: 100%;"/> 11.312 m

8. **A vessel is about to enter a river port over a bar where the maximum depth at highwater is 9.2 m. She must have a minimum clearance of 0.5 m and is at present at draft. Forward 8.40 m, Aft 9.00 m tank. How much water must be discharged from an afterpeak tank LCG 7 m forward of AP?**

Ans:

TPC, 25 tonne/cm; MCTC, 125 tonne m/cm; LCF, midships; length, 220 m.

Maximum draft aft = 8.70 m

Present draft aft = 9.00 m

Change of draft aft = $\overline{0.30 \text{ m}}$

= 30 cm

d = 103 m

Let w be ballast to discharge

\pm change in draft aft = \pm sinkage \pm change in trim aft

$$-30 = -\frac{w}{TPC} - \frac{1}{L} \times \frac{w \times d}{MCTC}$$

$$30 = \frac{w}{25} + \frac{110 \times 103 \times w}{220 \times 125}$$

$$30 = 0.04w + 0.412w$$

$$30 = 0.452w$$

$$w = 66.4$$

Amount of ballast to discharge = 66.4 tonnes

$$\begin{aligned} \text{Change of trim} &= \frac{w \times d}{\text{MCTC}} \\ &= \frac{66.4 \times 103}{125} = 54.7 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change of trim after} &= \frac{l}{L} \text{ change of trim} \\ &= \frac{110}{220} \times 54.7 = 27.4 \text{ cm} \end{aligned}$$

Change of trim forward = 27.4 cm

$$\begin{aligned} \text{Rise} &= \frac{w}{\text{TPC}} \\ &= \frac{66.4}{25} = 2.7 \text{ cm} \end{aligned}$$

	F	A
Initial draft	8.40 m	9.00 m
Rise	0.03 m	0.03 m
	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>
	8.37 m	8.97 m
Trim	+0.27 m	-0.27 m
	<hr style="width: 50%; margin: 0 auto;"/>	<hr style="width: 50%; margin: 0 auto;"/>
Final draft	8.64 m	8.70 m

Note that it is always necessary to check the final draft forward as it is possible that it could exceed the maximum permissible draft.

In the special case of keeping the draft aft, or forward, constant (Figure) we have

$$0 = \pm \text{sinkage} \pm \text{change of trim aft}$$

Sinkage = change of trim aft

$$\frac{w}{\text{TPC}} = \frac{l}{L} \times \frac{w \times d}{\text{MCTC}}$$

From this equation, d represents the position to load a weight to keep the draft aft constant

$$d = \frac{L \times \text{MCTC}}{l \times \text{TPC}}$$

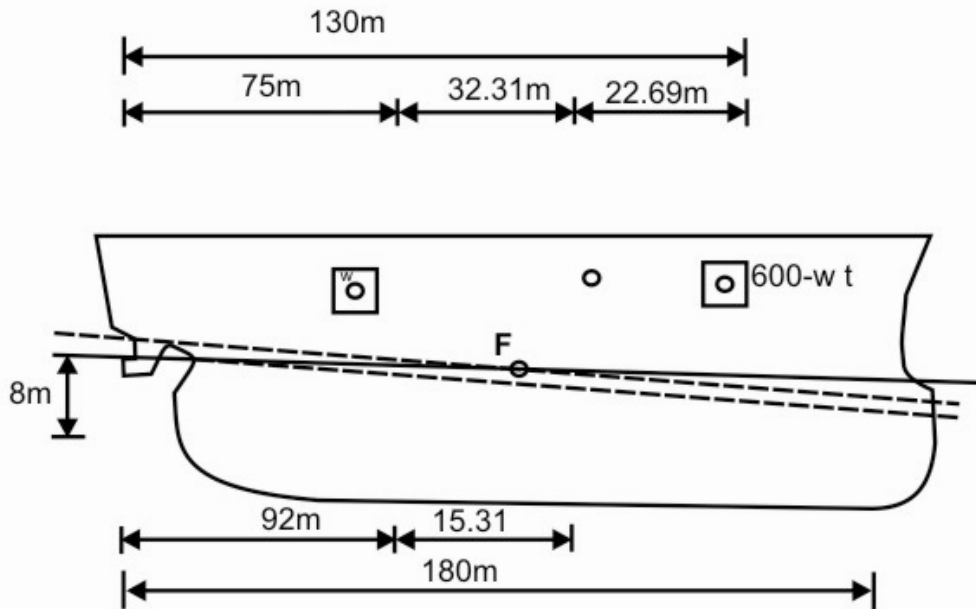
Note that the weight term does not appear in the equation. These are two limitations on the use of this equation:

1. The vessel could trim by the head until the forward draft is greater than the after draft.
2. The amount loaded could be greater than a moderate amount making the values of MCTC and TPC invalid.

In practice the position defined by d can be regarded as the centre of gravity of the weights loaded in compartments forward and aft of the position.

9. A vessel floating at draft: Forward, 7.00m; Aft, 8.00m. distribute 600 tonnes of cargo between compartment 1 LCG 75 m forward to AP, and compartment 2 LCG 130 m forward of AP so as to maintain draft aft constant. State the final draft forward.

Ans:



TCP, 23 tonne/cm; MCTC, 180 tonne/cm; LCF, 92m forward of AP; length 180m.

Distance of centre of gravity of weight from LCF

$$\begin{aligned}
 d &= \frac{L \times \text{MCTC}}{1 \times \text{TPC}} \\
 &= \frac{180 \times 180}{92 \times 23} \text{ m} \\
 &= 15.31 \text{ m forward of F}
 \end{aligned}$$

LCF from AP = 92.00 m

g from AP = 107.31 m 107.31 m

Compartment 1 = 75.00 m compartment 2 130.00m

32.31 m aft

22.69 m forward

Load 'w' tonne aft

$$32.31w = 22.69(600 - w)$$

$$55w = 22.69 \times 600$$

$$w = 248 \text{ tonne}$$

Load 248 tonne in Compartment 1

352 tonne in Compartment 2

$$\text{Sinkage} = \frac{w}{\text{TPC}} = \frac{600}{23} \text{ cm} = 26 \text{ cm}$$

$$\begin{aligned} \text{Change of trim} &= \frac{\text{moment changing trim}}{\text{MCTC}} \\ &= \frac{w \times d}{\text{MCTC}} \\ &= \frac{600 \times 15.31}{180} = 51 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change of trim aft} &= \frac{92}{180} \times 51 \text{ cm} \\ &= 26 \text{ cm} \end{aligned}$$

Change of trim forward = 25 cm

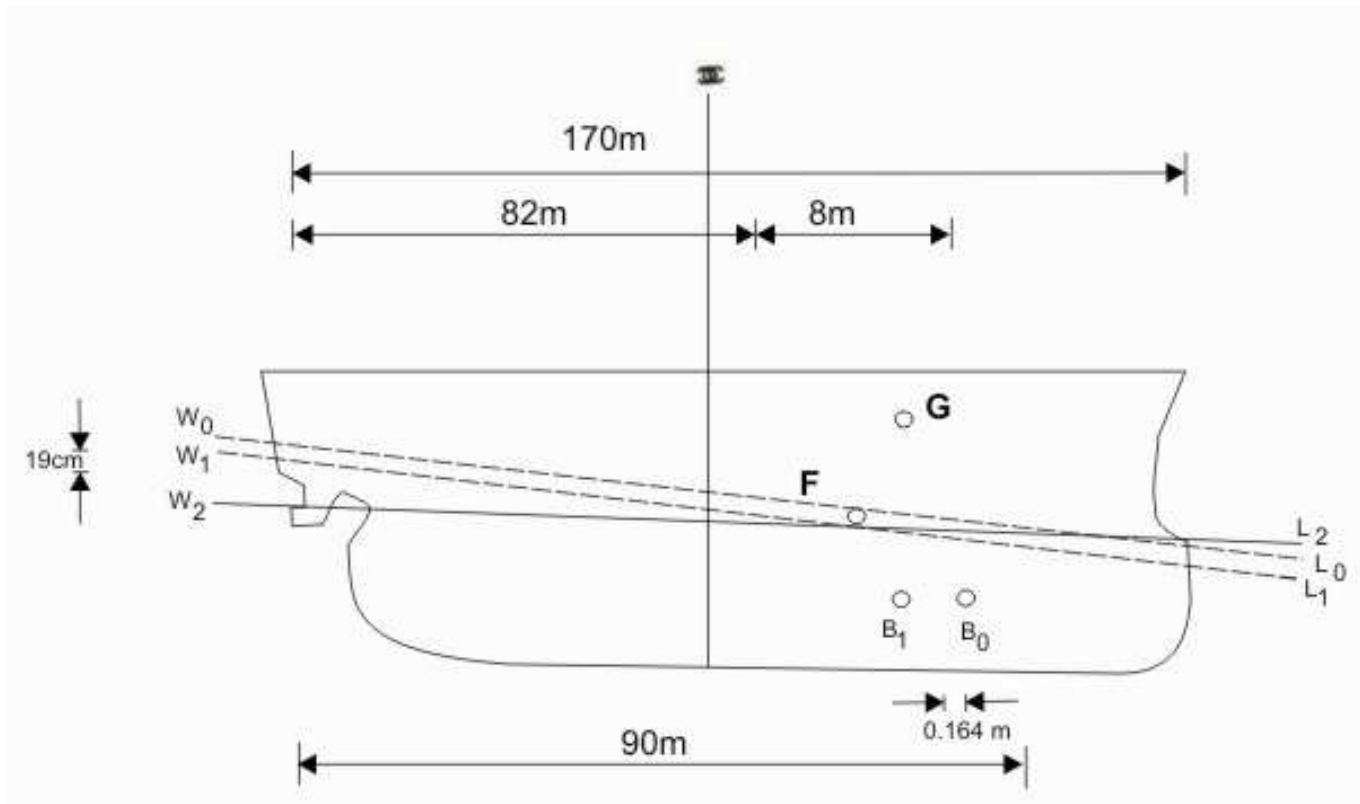
	F	A
Initial draft	7.00 m	8.00 m
	0.26 m	0.26 m
	<hr/>	<hr/>
Trim	7.26 m	8.26 m
	+0.25 m	-0.26 m
	<hr/>	<hr/>
Final drafts	7.51 m	8.00 m
	<hr/>	<hr/>

MEAN DRAFT

We were noted that the vessel trimmed to an even keel draft which was not the same as the average draft of the vessel before the weight was moved. Since a vessel trims about the center of flotation parallel sinkage will always be the change in draft at the centre of flotation.

Thus, in determining the amount of weight to be loaded or discharged when moving from one trimmed draft to a different trimmed draft, it is necessary to determine the change in draft at the centre of flotation. In Figure 8.8 (see page 198) the vessel length L is floating at a trimmed

water line W_0L_0 the centre of flotation is at F a distance 1 m from AP with the draft aft d_a the draft at F d_m and draft forward d_f . W_0C is a line drawn parallel to the deck of the vessel. Then L_0C the trim of the vessel



c = difference between d_a and d_m

Then using similar triangles

$$\frac{c}{I} = \frac{t}{L}$$

$$c = \frac{I}{L} \times t$$

$$d_m = d_a - \frac{I}{L} \times t$$

Note: in the unusual case of the vessel being trimmed by the head

$$d_m = d_a + \frac{I}{L} \times t$$

10. A vessel is floating at drafts: forward, 8.72 m, aft, 9.00 m in water density, 1.025 tonne/m³. She is to enter dock water density 1.004 tonne/m³. Find her drafts fore and aft in dock water, taking due account of the change of trim due to change of density.

Ans:

MCTC, 162 tonne m/cm. TPC, 29.8 tonne/cm; LCF, 82 m forward of AP, LCB, 90 m forward of AP. Length, 170 m. Displacement, 27 000 tonnes.

Initial mean draft

$$\begin{aligned}d_{ml} &= d_a - \frac{1}{L} \times t \\&= 9.00 - \frac{82}{170} \times 0.280 \\&= 9.000 - 0.135 \\&= 8.865 \text{ m}\end{aligned}$$

Final mean draft

$$\begin{aligned}d_m &= d_{ml} \times \frac{\rho_I}{\rho_F} \\&= 8.865 \times \frac{1.025}{1.004} \text{ m} \\&= 9.050 \text{ m}\end{aligned}$$

$$d_{mI} = 8.865 \text{ m}$$

$$\begin{aligned}s &= 0.185 \\&= 0.19 \text{ m}\end{aligned}$$

$$\begin{aligned}\nabla &= \frac{W}{\rho_0} \\&= \frac{27000}{1.025} \text{ m}^3 \\&= 26341.5 \text{ m}^3\end{aligned}$$

$$\begin{aligned}v &= \nabla \left(\frac{\rho_I}{\rho_F} - 1 \right) \\&= 26341.5 \times \left(\frac{1.025}{1.004} - 1 \right) \\&= 551 \text{ m}^3\end{aligned}$$

$$\begin{aligned}B_0 B_1 &= \frac{v \times d}{\nabla + v} \\&= \frac{551 \times 8}{26341.5 + 551} \text{ m} \\&= 0.164 \text{ m}\end{aligned}$$

Since B has moved aft trim is by head

$$\begin{aligned} \text{Trim} &= \frac{\text{moment changing trim}}{\text{MCTC}} \\ &= \frac{W \times B_0B_1}{\text{MCTC}} \\ &= \frac{27000 \times 0.164}{162} = 27.32 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change of trim aft} &= \frac{l}{L} \times \text{change of trim} \\ &= \frac{82}{170} \times 27.3 = 13.2 \text{ cm} \end{aligned}$$

Change of trim forward = 14.1 cm

	F	A
Initial draft	8.72	9.00
Sinkage	0.19	0.19
	<hr/>	<hr/>
	8.91	9.19
Trim	+0.14	-0.13
	<hr/>	<hr/>
Final draft	9.05 m	9.06 m
	<hr/>	<hr/>

11. A vessel about to dry dock is in the following condition.
- | | |
|-----------------------|--------------------------------------|
| Draft | forward, 6.10 m; aft, 6.70 m |
| KM₀ | 7.20 m; KG₀, 6.8 m |
| MCTC | 155 tonne/cm |
| TPC | 22 tonne/cm |
| LCF | 80 m forward of AP |
| Length | 180 m |
| Displacement | 11 000 tonnes |

Find

- The GM of the vessel at the critical instant.
- The righting moment at 1° heel.
- The drafts fore and aft at the critical instant.

$$\begin{aligned} \text{Ans : } p &= \frac{t \times \text{MCTC}}{l} \\ &= \frac{60 \times 155}{80} = 116.3 \text{ tonne} \end{aligned}$$

$$\begin{aligned} G_0G_1 &= \frac{p \times KG_0}{W - p} & M_0M_1 &= \frac{P \times KM_0}{W} \\ &= \frac{116.3 \times 6.80}{11000 - 116.3} & &= \frac{116.3 \times 7.20}{11000} \end{aligned}$$

$$= 0.0727 \text{ m} \qquad = 0.0761 \text{ m}$$

$$G_0M_0 = 0.6000 \text{ m} \qquad G_0M_0 = 0.6000 \text{ m}$$

$$G_1M_0 = 0.5273 \text{ m} \qquad G_0M_1 = 0.5239 \text{ m}$$

$$\begin{aligned} \text{Righting moment} &= (W - P) G_1M_0 \sin\theta \\ &= (11000 - 116.3) \times 0.5273 \times \sin 1^\circ \\ &= 100.16 \text{ tonne} \end{aligned}$$

$$\begin{aligned} \text{Righting moment} &= W \times W \times G_0M_1 \sin\theta \\ &= 11000 \times 0.5239 \times \sin 1^\circ \\ &= 100.56 \text{ tonne} \end{aligned}$$

Within the limits of reasonable rounding the righting moments are equal. The apparent difference in the measurement of stability given by G_1M_0 and G_0M_1 can be explained in terms of righting moment

$$\begin{aligned} \text{'Bodily rise'} &= \frac{P}{TPC} \\ &= \frac{116.3}{22} \text{ cm} \\ &= 5.3 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change in trim aft} &= \frac{l}{L} \times t \\ &= \frac{80}{180} \times 60 = 26.7 \text{ cm} \end{aligned}$$

$$\text{Change in trim forward} = 33.3 \text{ cm}$$

	Fm	Am
Initial draft	6.100	6.700
Rise	0.053	0.053
	6.47	6.647
Trim	+0.333	-0.267
	6.380	6.380

12. A vessel is to be drydocked and is in the following condition

Drafts

Forward	7.92 m; aft, 9.30 m
KM₀	11.43 m; KG₀, 10.90 m
MCTC	400.5 tonne m/cm
TPC	28.1 tonne/cm
LCF	88.5 m forward of AP
Length	174 m
Displacement	28 200 tonnes

The depth of water in the dock is initially 10.00 m. Find the effective GM of the vessel after the water level has fallen by 1.2 m in dock. What are the drafts of the vessel after the fall?

Ans :	Depth of water	10.00 m
	After draft	9.30 m
	Clearance	0.70 m
	Fall	1.20 m
	Change in draft aft	0.50 m = 50 cm

If P is the upthrust \pm change in draft = \pm bodily rise \pm change in trim aft

$$50 = \pm \frac{P}{\text{TPC}} \pm \frac{1 \times P \times 1}{L \times \text{MCTC}}$$

$$- 50 = - \frac{P}{38.1} - \frac{88.5^2 \times P}{174 \times 400.5}$$

$$50 = 0.2626P + 0.1123P$$

$$50 = 0.1396P$$

$$P = 358.2 \text{ tonnes}$$

$$G_0G_1 = \frac{P \times KG_0}{W - P}$$

$$= \frac{358.2 \times 10.90}{28200 - 358.2}$$

$$= 0.140 \text{ m}$$

$$G_0M_0 = 0.530 \text{ m}$$

$$G_1M_0 = 0.390 \text{ m}$$

$$\begin{aligned} \text{Bodily rise} &= \frac{P}{TPC} \\ &= \frac{358.2}{38.1} \text{ cm} \\ &= 9.40 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change in trim} &= \frac{P \times l}{MCTC} \\ &= \frac{358.2 \times 88.5}{400.5} \text{ cm} \\ &= 79.15 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change in trim aft} &= \frac{l}{L} \times t \\ &= \frac{88.5}{174} \times 79.2 = 40.6 \text{ cm} \end{aligned}$$

$$\text{Change of trim fwd} = 38.9$$

	F_m	A_m
Initial draft	7.920	9.300
Bodily rise	0.094	0.094
Trim	$\frac{7.826}{+0.389}$	$\frac{9.206}{-0.406}$
Drafts after fall	$\frac{8.215}{\text{---}}$	$\frac{8.800}{\text{---}}$

13. A vessel floating at drafts: forward, 8.70m; aft, 9.40 m, grounds at a point 30 m aft of the forward perpendicular. Estimate the drafts of the vessel and the GM after the tide has fallen by 70 cm

Ans :

MCTC	340 tonne m/cm
TPC	28 tonne/cm
KG_0	7.60 m, KM_0 8.40 m
Length	162 m
LCF	82 m forward of AP

Displacement

29 000 tonnes

$$y = \frac{P}{TPC} + \frac{X \times P \times X}{L \times MCTC}$$

$$70 = \frac{P}{28} + \frac{54 \times 54 \times P}{162 \times 340}$$

$$70 = 0.0357P + 0.0529P$$

$$P = 789.7 \text{ tonnes} \approx 790 \text{ tonne}$$

$$\begin{aligned} G_0G_1 &= \frac{P \times KG_0}{W - P} \\ &= \frac{790 \times 7.80}{29000 - 790} \\ &= 0.218 \text{ m} \end{aligned}$$

$$G_0M_0 = 0.700 \text{ m}$$

$$G_1M_0 = 0.482 \text{ m}$$

$$\begin{aligned} \text{Bodily rise} &= \frac{P}{TPC} \\ &= \frac{790}{28} \text{ cm} \\ &= 28 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change in trim} &= \frac{P \times X}{MCTC} \\ &= \frac{790 \times 54}{340} \text{ cm} \\ &= 125.5 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Change in trim aft} &= \frac{l}{L} \times t \\ &= \frac{78}{162} \times 125.5 \text{ cm} \\ &= 60 \text{ cm} \end{aligned}$$

$$\text{Change of trim frd} = 65.5 \text{ cm}$$

	Forward (m)	Aft (m)
Initial draft	8.70	9.40
Rise	- 0.28	- 0.28
Trim	8.42	9.12
	- 0.66	+0.60
Final draft	7.76	9.72

14. A box vessel: length, 110 m; breadth, 12 m; depth, 8 m is floating at draft 6 m. A midships compartment extending the full breadth of the vessel length 9 m is bilged. If the vessel has KG 4.8 m and is floating in salt water. Find

- (a) the bilged draft;
- (b) the GM of the vessel in the initial condition;
- (c) the GM of the vessel in the bilged condition
- (d) the GM of the vessel at the bilged draft;
- (e) the righting moment of the vessel at 1° heel in conditions (b), (c) and (d)

Ans : Intact volume before bilging = intact volume after bilging

$$LBD_i = LBd_b - lBd_b$$

$$110 \times 12 \times 6 \text{ m}^3 = 110 \times 12 \times d_b - 9 \times 12 \times d_b$$

$$7920 \text{ m}^3 = 1320d_b - 108d_b$$

$$7920 \text{ m}^3 = 1212d_b$$

$$d_b = 6.535 \text{ m}$$

$$\begin{aligned} \text{Displacement} &= LBd_i \rho_{sw} \\ &= 7920 \times 1.025 \text{ tonne} \\ &= 8118 \text{ tonne} \end{aligned}$$

KM intact

$$KM = \frac{d_b}{2} + \frac{B^2}{12d_b}$$

$$= \frac{6}{2} + \frac{12^2}{12 \times 6}$$

$$= 3 + 2 = 5 \text{ m}$$

$$GM = 5 - 4.8 = 0.2 \text{ m}$$

$$\begin{aligned} \text{Righting moment} &= W \times GM \times \sin\theta \\ &= 7920 \times 0.2 \times \sin 1^\circ \text{ tonne m} \\ &= 27.64 \text{ tonne m} \end{aligned}$$

KM bilged

$$\begin{aligned}
KM &= KB + \frac{1}{V} \\
&= \frac{d_b}{2} + \frac{(L - 1) \times B^2}{12(L - 1) \times B \times d_b} \\
&= \frac{d_b}{2} + \frac{B^2}{12d_b} \\
&= \frac{6.535}{2} + \frac{144}{12 \times 6.535} \\
&= 3.268 + 1.836 \\
&= 5.104 \text{ m}
\end{aligned}$$

$$KG = 4.800 \text{ m}$$

$$GM = 0.304 \text{ m}$$

$$\begin{aligned}
\text{Righting moment} &= W \times GM \times \sin\theta \\
&= 7920 \times 0.304 \times \sin 1^\circ \\
&= 52.02 \text{ tonne m}
\end{aligned}$$

KM intact at bilged draft

$$\begin{aligned}
KM &= \frac{d_b}{2} + \frac{B^2}{12d_b} \\
&= \frac{6.535}{2} + \frac{144}{12 \times 6.535} \\
&= 3.268 + 1.836 \\
&= 5.104 \text{ m}
\end{aligned}$$

$$KG = 4.800 \text{ m}$$

$$GM = 0.304 \text{ m}$$

$$\begin{aligned}
\text{Intact displacement at } 6.535 &= L \times B \times d_b \times \rho \\
&= 110 \times 12 \times 6.535 \times 1.025 \\
&= 8841.9 \text{ tonne}
\end{aligned}$$

$$\begin{aligned}
\text{Righting moment} &= W \times GM \times \sin 1^\circ \\
&= 8841.9 \times 0.304 \times \sin 1^\circ \\
&= 49.91 \text{ tonne m}
\end{aligned}$$

In this problem the bilged GM is greater than the initial GM.

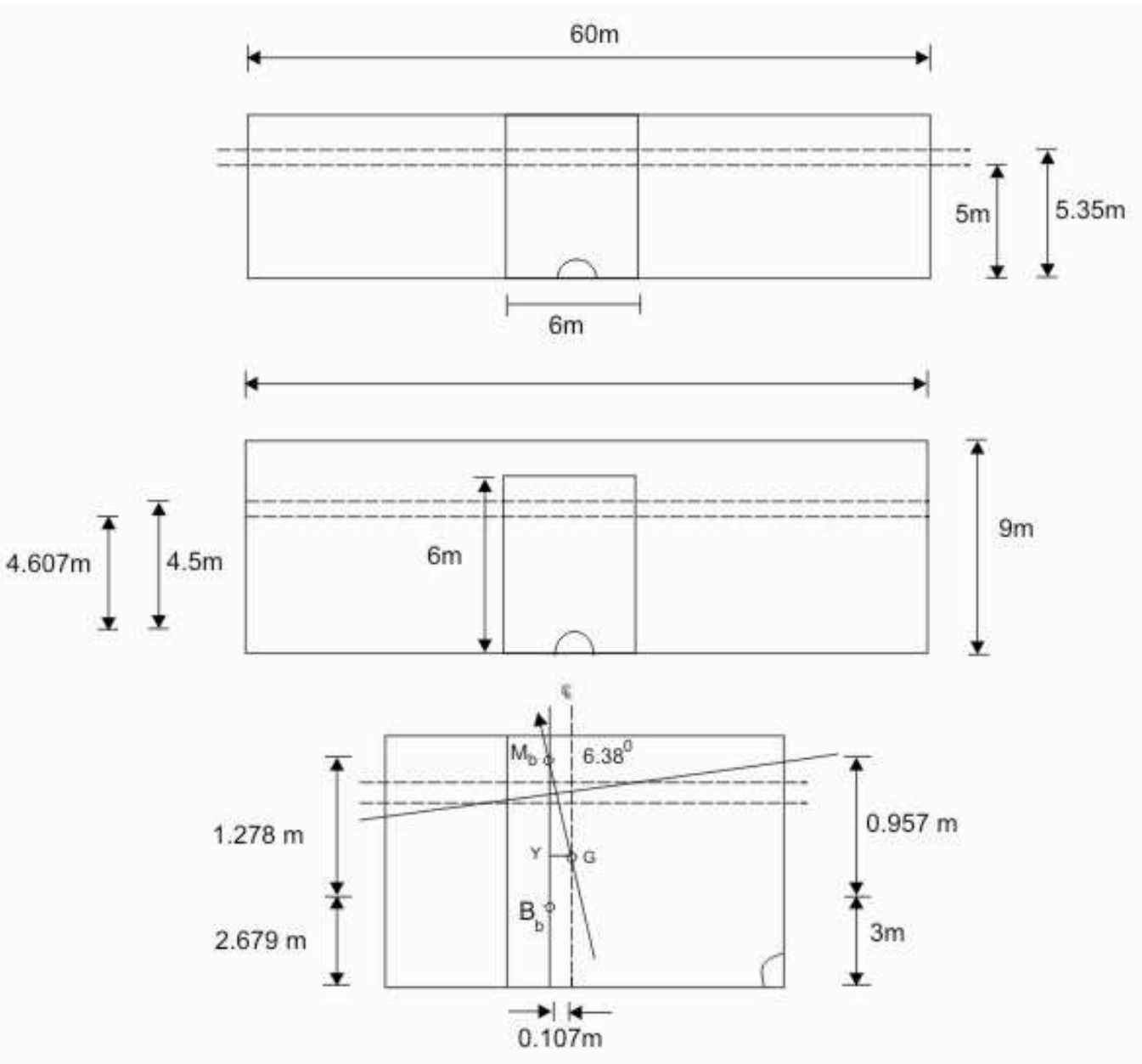
This is not a general result, a change in the dimensions of the vessel or the intact draft could have produced a reduction in GM. In the case of a full breadth compartment being bilged, the bilged GM of a box vessel is the same as GM for the intact vessel at the bilged draft.

However, since the bilged vessel has less displacement than the intact vessel, the righting moment is reduced and the vessel is less stable than it would have been had it been intact. Also

the freeboard is reduced with the consequent effect on the GZ curve as described in Chapter 6. If the compartment does not extend the full breadth and/or depth of the vessel the situation is modified.

15. A box vessel length, 60 m; breadth, 9 m is floating at a draft of 5 m and has KG, 3.0 m. find the list if empty midships side compartments length 6 m, breadth 6 m is bilged.

Ans :



Intact volume before bilging = intact volume after bilging

$$LBd_i = LBd_b - lbd_b$$

$$60 \times 9 \times 5 = 60 \times 9 \times d_b - 6 \times 6 \times d_b$$

$$d_b = 5.357 \text{ m}$$

$$KB_1 = \frac{d_b}{2}$$

$$= \frac{5.357}{2} \text{ m} = 2.679 \text{ m}$$

h from side

	Area (m^2)	Centroid (m)	Moment (m^3)
$L \times B$	540	4.5	2430
$l \times b$	- 36	3.0	- 108
	<hr/> 504		<hr/> 2322
	<hr/>		<hr/>

$$h = \frac{2322}{504} = 4.607 \text{ m}$$

$$G\gamma = h - \frac{B}{2} = 4.607 - 4.5 = 0.107 \text{ m}$$

$$\begin{aligned} I_{\text{side}} &= \frac{LB^3}{3} - \frac{lb^3}{3} \\ &= \frac{60 \times 9^3}{3} - \frac{6 \times 6^3}{3} \\ &= 14580 \text{ m}^4 - 432 \text{ m}^4 = 14148 \text{ m}^4 \end{aligned}$$

$$\begin{aligned} I_b &= I_{\text{side}} - Ah^2 \\ &= 14148 \text{ m}^4 - 504 \times 4.607^2 \text{ m}^4 \\ &= 3451 \text{ m}^4 \end{aligned}$$

$$\begin{aligned} B_1M_b &= \frac{I_b}{\tilde{N}} \\ &= \frac{3451}{2700} \text{ m} \end{aligned}$$

$$B_1M_b = 1.278 \text{ m}$$

$$KB = 2.679 \text{ m}$$

$$KM_b = 3.957 \text{ m}$$

$$KG = 3.000 \text{ m}$$

$$\gamma M_b = 0.957 \text{ m}$$

$$\tan\theta = \frac{G\gamma}{\gamma M_b}$$

$$= \frac{0.107}{0.957}$$

$$\theta = 6.38^\circ$$

16. A box vessel has length, 180 m; breadth, 20 m is floating at an even keel draft of 12 m. KG, 8 m.

Find the drafts of the vessel fore and aft if an empty full breadth end compartment length, 12 m is bilged. Vessel floating in fresh water.

Ans : Intact volume before bilging = Intact volume after bilging

$$\begin{aligned}
L \times B \times d_i &= (L - 1) \times B \times d_b \\
180 \times 20 \times 12 &= 168 \times 20 \times d_b \\
d_b &= 12.857 \text{ m} \\
G\gamma &= \frac{1}{2} \\
&= \frac{12}{2} = 6 \text{ m} \\
W &= L \times B \times d_b \times \rho \\
&= 180 \times 20 \times 12 \times 1.000 \text{ tonne} \\
&= 43200 \text{ tonnes} \\
B_1M_L &= \frac{I_b}{\tilde{N}} \\
&= \frac{(L - 1)^3 B}{12 (L - 1) B d_b} \\
&= \frac{(L - 1)^2}{12 d_b} \\
&= \frac{168^2}{12 \times 12.857} = 182.9 \text{ m} \\
KB_1 &= 6.4 \text{ m} \\
KM_L &= 189.3 \text{ m} \\
KG &= 8.0 \text{ m} \\
GM_L &= 181.3 \text{ m} \\
MCTC &= \frac{W \times GM_L}{100L} \\
&= \frac{43200 \times 181.3}{100 \times 180} \\
&= 435.1 \text{ tonne m/cm}
\end{aligned}$$

Note that L is used because in the derivation of MCTC, L is the length of the structure.

$$\begin{aligned}
\text{Change of trim} &= \frac{W \times G\gamma}{MCTC} \\
&= \frac{43200 \times 6}{435.1} = 595.7 \text{ cm} \\
\text{Change of trim aft} &= \frac{(L - 1)}{2L} \times t \\
&= \frac{168}{2 \times 180} \times 595.7 = 278 \text{ cm} \\
\text{Change of trim fwd} &= 318 \text{ cm}
\end{aligned}$$

		Forward (m)	Aft (m)	
Bilged draft		12.86	12.86	
Trim		3.18 m	- 2.78 m	
Heel	GZ	Displacement	Righting moment	
0	0	32000	0	—
5	0.04	32000	1280	—
10	0.09	32000	2880	—
20	0.19	32000	6080	—
30	0.29	32000	9280	—

Final drafts 16.04 m 10.08 m

17. A vessel has the following values GZ at the angles of heel indicated

0	5	10	25	30	40
0	0.04	0.09	0.09	0.29	0.32

The vessel is displacing 32000 tonnes, has KG, 10.3 m and KM, 10.8 m.

She is floating at draft 11m.

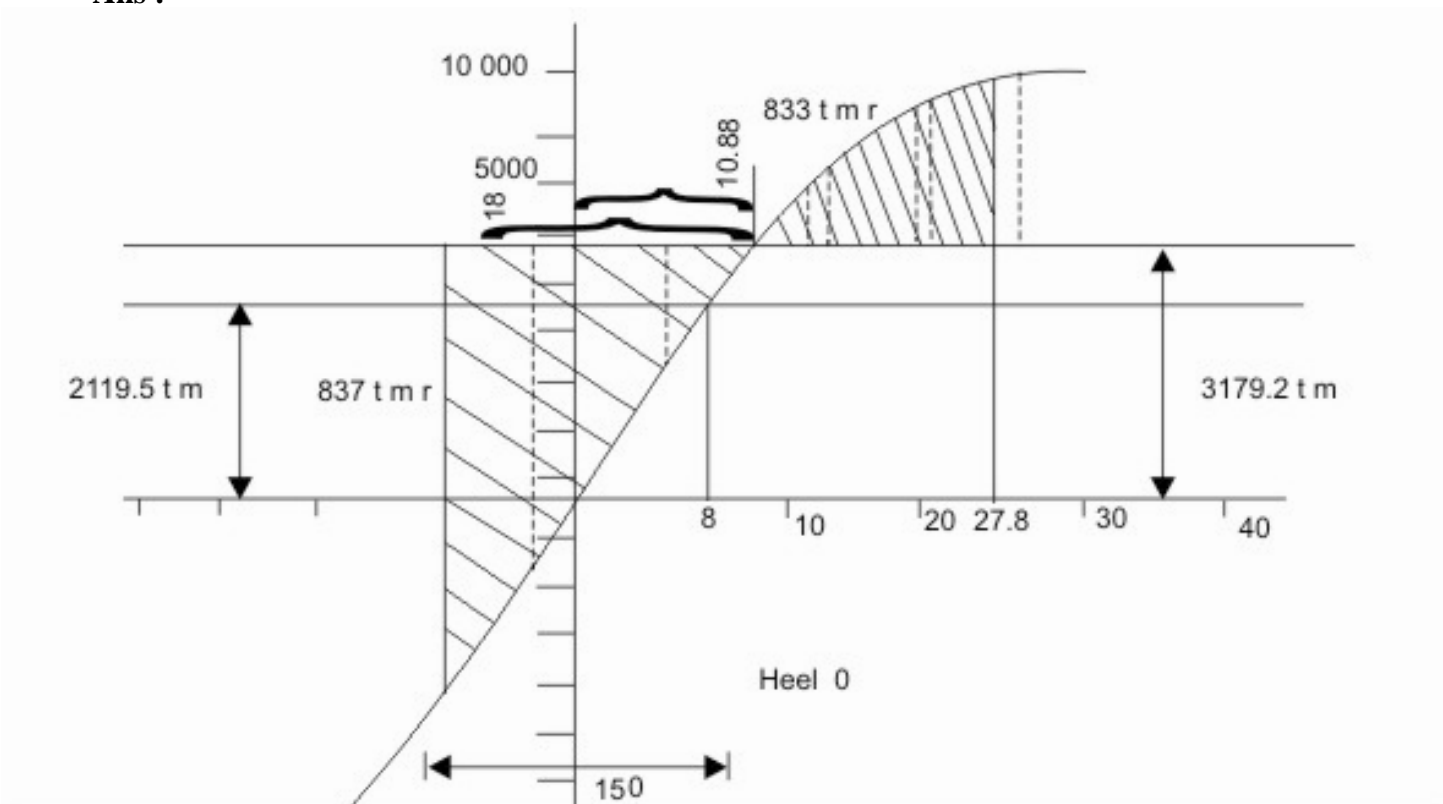
The windage area is 3800 m² and the centroid of the area is 6 m above the water line.

The angle of deck edge immersion is 23°.

The angle of flooding is 34°.

Assess the ability of the vessel to withstand heeling due to wind

Ans :



40	0.32	32000	10240
----	------	-------	-------

$$\begin{aligned} \text{wind lever} &= \frac{d}{2} + \text{centroid windage area above WL} \\ &= 5.5 + 6 \\ &= 11.5 \text{ m} \end{aligned}$$

$$\begin{aligned} \lambda_0 &= \frac{\text{windage area} \times \text{wind force} \times \text{wind level terms}}{1000} \\ &= \frac{3800 \times 48.5 \times 11.5}{1000} \\ &= 2119.45 \end{aligned}$$

$$1.5\lambda_0 = 3179.18$$

From curve area S_1 extend of 18° . Interval 6°

Heel	Ord	SM	F(Area)
0	5300	1	5300
1	3450	3	10350
2	1890	3	5670
3	00	1	0
			21320

$$s_1 = \frac{3}{8} \times h \times \Sigma F(\text{Area})$$

$$= \frac{3}{8} \times \frac{6}{57.3} \times 21320 \text{ tonne m radians}$$

$$s_2 = 837.1 \text{ tonne m radians}$$

Taking 18° for S_2 as first approximation

Station	Ord	SM	F(Area)
0	0	1	0
1	2100	3	6300
2	4050	3	12150
3	5690	1	5690
			24140

$$s_2 = \frac{3}{8} \times h \times \Sigma F(\text{Area})$$

$$= \frac{3}{8} \times \frac{6}{57.3} \times 24140 \text{ tonne m radians}$$

$$= 647.9$$

$$s_1 \text{ Difference} = \frac{837.1}{110.8}$$

Reduction in θ assuming area approximately rectangular

$$5690 \theta_c = 110.8$$

$$\theta_c = \frac{110.8}{5690}$$

$$= \frac{110.8 \times 57.3}{5690}$$

$$= 1.12^\circ$$

$$\text{Range} = 18^\circ - 1.12^\circ$$

$$= 16.88^\circ$$

$$\text{Interval} = 5.66^\circ$$

Station	Ord	SM	F(Area)
0	0	1	0
1	1900	3	5700
2	3800	3	11400
3	5400	1	5400
			22500

$$s_1 = \frac{3}{8} \times \frac{5.66}{57.3} \times 22500 \text{ tonne m radians}$$

$$= 833.4 \text{ tonne m radians}$$

which is sufficiently close

$$\therefore \theta_{dy} = 10.8^\circ + 16.88^\circ$$

$$= 27.68^\circ$$

$$\theta_f = 34^\circ \quad \theta_{dy} \text{ satisfactory}$$

$$\theta_1 = 8^\circ$$

$$0.65 \theta_{dc} = 0.65 \times 23^\circ$$

$$= 14.95^\circ$$

$\therefore \theta_1$ satisfactory