Vibrations of Structures

Module II: Wave Propagation and Scattering

Exercises

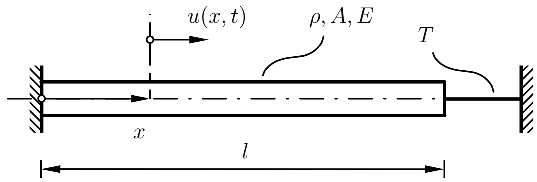

Figure 1: Exercise 1

1. A uniform homogeneous bar is under tension due to a string, as shown in Fig. 1. If the string suddenly snaps, determine the transient motion and stress waves set-up in the bar.

2. An infinte string at rest is excited by a force $q(x,t) = F(t)\delta(x)$. Determine the subsequent motion of the string when (a) $F(t) = F_0\delta(t)$ (impulse), and (b) $F(t) = F_0\mathcal{H}(t)$ (Heaviside step). (Use Fourier transform for space and Laplace transform for time.)

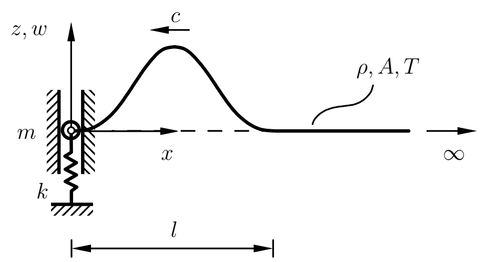

Figure 2: Exercise 3

3. A semi-infinite string is connected at $x = 0$ to a spring-mass system as shown in Fig. 2. At $t = 0$, a waveform given by

$$f(\xi) = \begin{cases} A\left(1 - \cos\frac{2\pi\xi}{l}\right), & \xi \in [0, l] \\ 0, & \xi \geq l \end{cases}$$

is incident at $x = 0$ from the right. Analyze the wave reflection process when (a) $m = 0$ and $k \neq 0$, (b) $m \neq 0$ and $k = 0$, and (c) $m \neq 0$ and $k \neq 0$. In case (c), what happens if the incident wave is resonant, or non-resonant?

4. Two semi-infinite bars of different materials and diameters are joined, as shown in Fig. 3. A positive traveling longitudinal wave $f_I(x - c_1 t)$ in the left bar is incident on the junction at $x = 0$. Determine the reflected and transmitted waves $f_R(x + c_1 t)$ and $f_T(x - c_2 t)$, respectively.

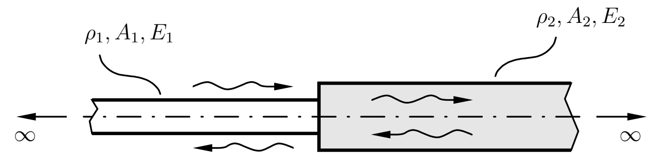

Figure 3: Exercise 4

5. Analyze the wave scattering process in Exercise 4 if a thin damping material (modeled as a discrete dashpot with viscous damping coefficient d) is introduced at the junction between the two bars in Fig. 3. Define $\eta = |C_R|^2 + |C_T|^2$ as the fraction of the average incident power after the scattering process. For what value of d will η be minimized.

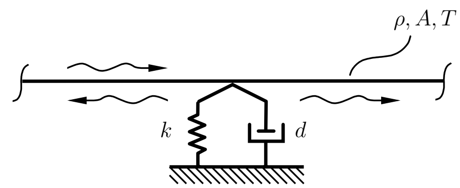

Figure 4: Exercise 6

6. An infinite string is provided a support with internal damping, as shown in the Fig. 4. A positive traveling harmonic wave is incident from the left. Determine d for maximizing the absorption of the incident power by the support. (Minimize the function η defined in Exercise 5.)

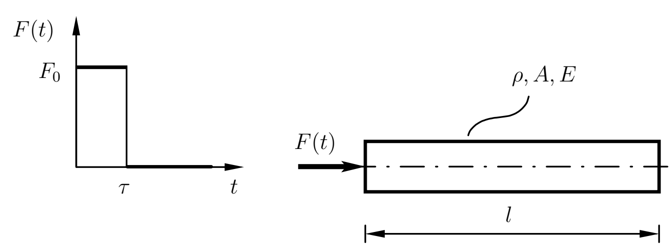

Figure 5: Exercise 7

7. A uniform homogeneous bar of length l, density ρ, and section-modulus EA is subjected to an axial force of the form $F(t) = F_0[\mathcal{H}(t) - \mathcal{H}(t-\tau)]$, where τ is a constant, as shown in Fig. 5. Determine the motion of the bar in terms of the traveling elastic waves set-up inside it when $0 < \tau < 2l/c$ and $2l/c < \tau < 4l/c$.