Vibrations of Structures

Module I: Vibrations of Strings and Bars

Exercises

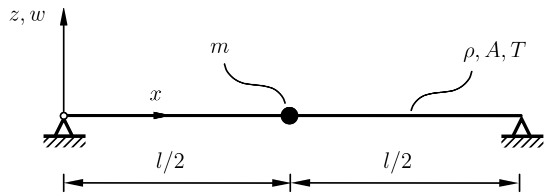

Figure 1: Exercise 1

1. A taut string carries a point mass m at the center, as shown in Fig. 1. Determine the eigenfrequencies and mode shapes of transverse vibration of the string What happens when $m/\rho Al \to \infty$, and $m/\rho Al \to 0$.

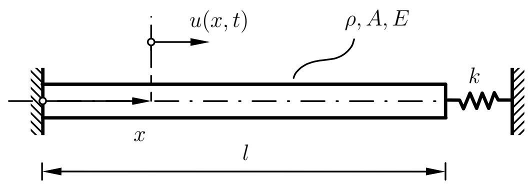

Figure 2: Exercise 2

2. A uniform homogeneous bar is fixed at the left end, and flexibly connected at the right end with a spring of stiffness k, as shown in Fig. 2. Using the variational formulation, derive the equation of motion, and the boundary conditions of the system. For $k = EA/l$, determine the first two eigenfrequencies, and the corresponding modes of vibration.

3. A uniform homogeneous circular bar of length l and area of cross-section A, carries a thin disc, as shown in Fig. 3. Assuming that the moment of inertia of the disc is $I_D = \rho A^2$ and $a = l/2$, determine the characteristic equation for torsional vibrations of the system. Determine the eigenfrequencies and modes of torsional vibrations of the bar.

4. The cross-sectional area of the tapered bar shown in Fig. 4 varies as $A(x) = A_0(1 - x/2l)$. The bar is forced at the center by a concentrated harmonic force $F(t) = F_0 \cos \Omega t$, as shown in the figure. Determine the exact solution of forced vibration of the bar. Also, determine the location of maximum normal stress in the bar.

5. Using Galerkin's method, discretize the equation of motion of a hanging string. Use the comparison functions as $P_i(x) = x^i$, $i = 1, 2, \ldots, N$. For $N = 2$ determine the eigenfrequencies from the discretized system and compare with the exact solutions.

6. A homogeneous tapered bar of circular cross-section is shown in Fig. 5. Using Rayleigh's quotient, estimate the fundamental circular frequency of the bar in longitudinal vibration for

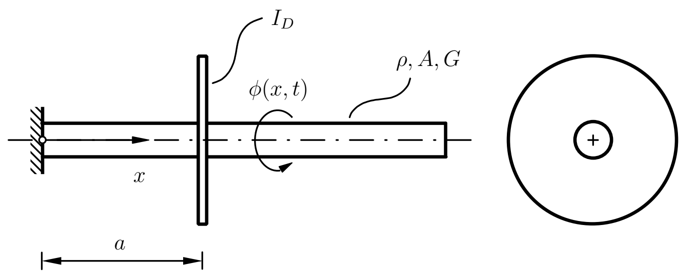

Figure 3: A circular bar with a disc

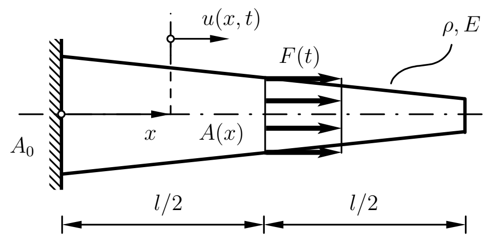

Figure 4: Exercise 4

the following choices of admissible functions: (a) First eigenfunction for longitudinal vibration of a bar with constant cross-section. (b) Admissible functions of the form $H_k(x) = (x/l)^k$, where k is an integer. Determine the value of k that yields the lowest value of the fundamental frequency? (c) The static deflection function of a vertically hanging bar.

7. Show that the initial value problem

$$\mu(x)w_{,tt} + \mathcal{K}[w] = 0, \qquad w(x,0) = w_0(x), \text{ and } w_{,t}(x,0) = v_0(x),$$

can be converted to the problem with forcing and homogeneous boundary conditions

$$\mu(x)w_{,tt} + \mathcal{K}[w] = w_0(x)\dot{\delta}(t) + v_0(x)\delta(t), \qquad w(x,0) = 0, \text{ and } w_{,t}(x,0) = 0.$$

8. Determine the Green's function of a sliding-fixed taut string.

9. A sliding-fixed taut string of length l is excited by a uniformly force $q(x,t) = Q_0 \cos \Omega t$, as shown in Fig. 6. Determine the steady-state response of the string using: (a) Eigenfunction expansion method, and (b) Green's function method.

10. An axially translating string excited by an impulsive transverse point force at $x = \bar{x}$ is described by the equation of motion

$$\rho A[w_{,tt} + 2vw_{,xt} + v^2 w_{,xx}] - Tw_{,xx} = \delta(t-\tau)\delta(x-\bar{x}).$$

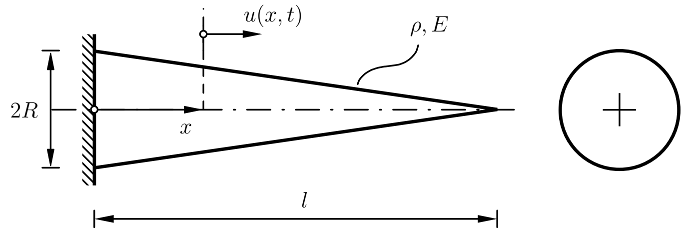

Figure 5: Exercise 6

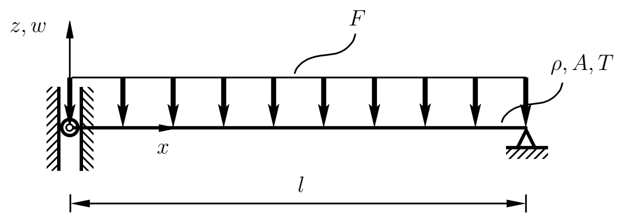

Figure 6: Exercise 9

Show that the response (Green's function) of the string is given by

$$w(x,\bar{x},t,\tau) = \mathcal{H}(t-\tau)\sum_{n=1}^{\infty}\frac{2}{n\pi\rho A c}\sin\left[\frac{n\pi}{cl}\left\{(c^2-v^2)(t-\tau)+v(x-\bar{x})\right\}\right]\sin\frac{n\pi\bar{x}}{l}\sin\frac{n\pi x}{l},$$

where $\mathcal{H}(\cdot)$ is the Heaviside step function, and $c = \sqrt{T/\rho A}$.