ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-42 DIFFUSION JET FLAMES

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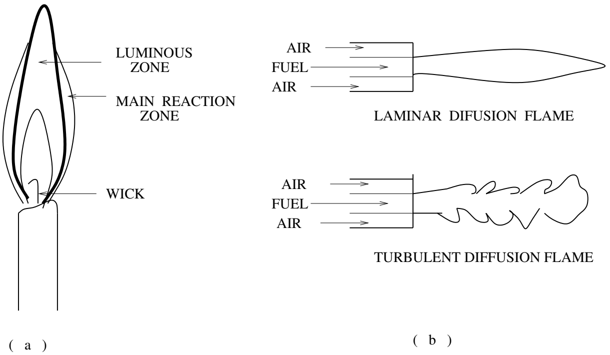
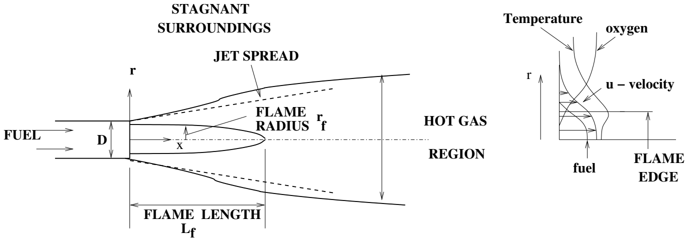
Definition L42($\frac{1}{19}$)



In diffusion flames, the source of fuel and oxidiser are physically separated . The candle flame is an example. In combustion of gaseous hydrocarbon fuels, fuel flows through the inner pipe of the burner whereas the air flows through the concentric outer pipe .

Main Objective L42($\frac{2}{19}$)



(a) LAMINAR BURNING JET (b) PROFILES

The main objective is to predict

1. Flame Length L_f
2. Flame shape or $r_f\ (x)$

in stagnant surroundings assuming SCR:

1 kg of fuel $+\ R_{st}$ kg of oxidant air $= (1 + R_{st})$ kg of product.

Governing Eqns L42($\frac{3}{19}$)

$$
\begin{aligned}
\frac{\partial}{\partial x}(\rho_m\, u\, r) + \frac{\partial}{\partial r}(\rho_m\, v\, r) &= 0 \quad \left(\frac{dp}{dx} = 0\right) \\
\frac{\partial}{\partial x}(\rho_m\, u\, r\, u) + \frac{\partial}{\partial r}(\rho_m\, v\, r\, u) &= \frac{\partial}{\partial r}\left[\mu_{eff}\, r\, \frac{\partial u}{\partial r}\right] \\
\frac{\partial}{\partial x}(\rho_m\, u\, r\, \omega_{fu}) + \frac{\partial}{\partial r}(\rho_m\, v\, r\, \omega_{fu}) &= \frac{\partial}{\partial r}\left[\rho_m\, D_{eff}\, r\, \frac{\partial \omega_{fu}}{\partial r}\right] \\
&- r \mid R_{fu} \mid \\
\frac{\partial}{\partial x}(\rho_m\, u\, r\, \omega_{ox}) + \frac{\partial}{\partial r}(\rho_m\, v\, r\, \omega_{ox}) &= \frac{\partial}{\partial r}\left[\rho_m\, D_{eff}\, r\, \frac{\partial \omega_{ox}}{\partial r}\right] \\
&- r \mid R_{ox} \mid \\
\frac{\partial}{\partial x}(\rho_m\, u\, r\, h_m) + \frac{\partial}{\partial r}(\rho_m\, v\, r\, h_m) &= \frac{\partial}{\partial r}\left[\frac{k_{eff}}{c_{pm}}\, r\, \frac{\partial h_m}{\partial r}\right] \\
&+ r \mid R_{fu} \mid \Delta H_c
\end{aligned}
$$

Laminar Vel Prediction - 1 - L42($\frac{4}{19}$)

Main assumption: Properties are uniform

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial r} + \frac{v}{r} &= 0 \quad \text{and} \\
u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial r} &= \frac{\nu_m}{r}\frac{\partial}{\partial r}\left[r\,\frac{\partial u}{\partial r}\right]
\end{aligned}$$

$$\psi\left(x,\eta\right) \equiv \nu_m\, x\, f\left(\eta\right) \quad \text{and} \quad \eta \equiv C\,\frac{r}{x}$$

$$\begin{aligned}
u &\equiv \frac{1}{r}\,\frac{\partial\psi}{\partial r} = \frac{C^2\,\nu_m}{x}\left[\frac{f'}{\eta}\right] \\
v &\equiv -\frac{1}{r}\,\frac{\partial\psi}{\partial x} = \frac{C\,\nu_m}{x}\left[f' - \frac{f}{\eta}\right]
\end{aligned}$$

Boundary conditions are: f(0) = $f'(0)$ = 0 and $f'\left(\infty\right)$ = 0.

Laminar Vel Prediction - 2 - L42($\frac{5}{19}$)

Substitutions give similarity Eqn

$$\frac{f\,f'}{\eta^2} - \frac{f\,f''}{\eta} - \frac{f'^{\,2}}{\eta} = \frac{d}{d\eta}\left[f'' - \frac{f}{\eta}\right] \quad \text{or}$$

$$\frac{d}{d\eta}\left[f'' - \frac{f}{\eta} + \frac{f\,f'}{\eta}\right] = 0$$

Integrating from $\eta = 0$ to η and noting BCs f (0) = f' $(0) = 0$, we get $f\,f' = f' - \eta\,f''$. The soln is

$$f = \frac{\eta^2}{1+\eta^2/4}, \quad f' = \frac{2\,\eta}{(1+\eta^2/4)^2}, \quad f'' = \frac{2\,(1-3\,\eta^2/4)}{(1+\eta^2/4)^3} \quad \text{or}$$

$$u = \frac{C^2\,\nu_m}{x}\left[\frac{2}{(1+\eta^2/4)^2}\right], \quad v = \frac{C\,\nu_m}{x}\left[\frac{\eta-\eta^3/4}{(1+\eta^2/4)^2}\right]$$

Determination of C - L42($\frac{6}{19}$)

Multiply momentum eqn by r and integrate from r = 0 to r = ∞.
Then

$$\frac{\partial}{\partial x}\left[\int_0^\infty \rho_m\, u^2\, r\, dr\right] + \{\rho_m\, v\, u\, r\,|_\infty - \rho_m\, v\, u\, r\,|_0\} = \left\{\mu_m\, r\,\frac{\partial u}{\partial r}\,|_\infty - \mu_m\, r\,\frac{\partial u}{\partial r}\,|_0\right\}$$

From BCs, terms in curly brackets are zero.
Hence, substituting for u, we have

$$J_{mom} = 2\,\pi \int_0^\infty \rho_m\, u^2\, r\, dr = \frac{16}{3}\,\pi\,\rho_m\,\nu_m^2\,C^2 = \rho_0\,U_0^2\,(\frac{\pi}{4}\,D^2) = \text{constant}$$

$$C = \frac{\sqrt{3}}{8}\,Re\,(\frac{\rho_0}{\rho_m})^{0.5} = \frac{1}{4\,\nu_m}\sqrt{\frac{3\,J_{mom}}{\pi\,\rho_m}} \rightarrow Re = \frac{U_0\,D}{\nu_m}$$

Final Soln - L42($\frac{7}{19}$)

$$
\begin{aligned}
\eta &= \frac{\sqrt{3}}{8}\,(\frac{\rho_0}{\rho_m})^{0.5}\,Re\,\frac{r}{x} \\
u^* &= \frac{u\,x}{\nu_m} = \frac{3}{32}\,Re^2\,\left[1+\eta^2/4\right]^{-2}\,(\frac{\rho_0}{\rho_m}) \\
\frac{u}{U_0} &= \frac{3}{32}\,(\frac{D}{x})\,Re\,\left[1+\eta^2/4\right]^{-2}\,(\frac{\rho_0}{\rho_m}) \\
\frac{u}{u_{max}} &= \frac{1}{(1+\eta^2/4)^2} \;\rightarrow\; \frac{u}{u_{max}} = \frac{1}{2}\ \text{at}\ \eta_{1/2} = 1.287
\end{aligned}
$$

Because at any x, u $\rightarrow$ 0 as $y \rightarrow \infty$, it is difficult to identify the jet-width exactly. Hence, by convention, $\eta_{1/2}$ characterises the *jet half-width* ($r_{1/2}$). Thus,

$\frac{r_{1/2}}{x} = \frac{\eta_{1/2}}{C} = 1.287 \times \frac{8}{\sqrt{3}\,Re}\,(\frac{\rho_0}{\rho_m})^{-0.5} = \frac{5.945}{Re}\,(\frac{\rho_0}{\rho_m})^{-0.5} = \tan\alpha$

where α is the jet-spread angle.

Prediction of L_f and r_f(x) - 1 - L42($\frac{8}{19}$)

Assuming Le = 1 and SCR, all eqns can be rendered in conserved-property form

$$\frac{\partial}{\partial x}(\rho_m\, u\, r\, \Phi) + \frac{\partial}{\partial r}(\rho_m\, v\, r\, \Phi) \;=\; \frac{\partial}{\partial r}\left[\Gamma_m\, r\, \frac{\partial \Phi}{\partial r}\right]$$

$$\Phi = \omega_{fu} - \frac{\omega_{ox}}{R_{st}} = h_m + \Delta H_c\, \omega_{fu} \;=\; \frac{u}{U_0} \;\rightarrow\; R_{st} = \frac{A}{F}$$

To locate the flame, we define conserved scalar Φ = f

$$f \equiv \frac{\Phi - \Phi_A}{\Phi_F - \Phi_A} = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) - (\omega_{fu} - \omega_{ox}/R_{st})_A}{(\omega_{fu} - \omega_{ox}/R_{st})_F - (\omega_{fu} - \omega_{ox}/R_{st})_A}$$

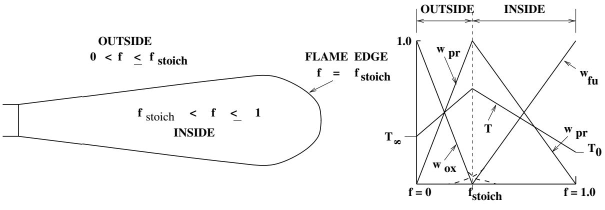
where subscripts A and F refer to Air and Fuel Streams . Hence,

$$f = \frac{(\omega_{fu} - \omega_{ox}/R_{st}) + 1/R_{st}}{1 + 1/R_{st}} \quad \text{because} \quad \omega_{ox,\infty} = \omega_{fu,F} = 1$$

Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{9}{19}$)

The flame is located where $(\omega_{fu} - \omega_{ox}/R_{st}) = 0$. Hence,

$$
\begin{aligned}
f &= f_{stoich} = 1/(1 + R_{st}) \quad \text{(flame edge)} \\
f &= f_{stoich} + \omega_{fu}/(1 + 1/R_{st}) \quad \text{(inside)} \\
f &= f_{stoich} - \ (\omega_{ox}/R_{st})/(1 + 1/R_{st}) \quad \text{(outside)}
\end{aligned}
$$



(a) STATES OF MIXTURE FRACTION f

(b) STATES OF MASS FRACTIONS

Prediction of L_f and r_f(x) - 3 - L42($\frac{10}{19}$)

If we take $\Phi = h^* = h_m + \Delta H_c \, \omega_{fu}$ then

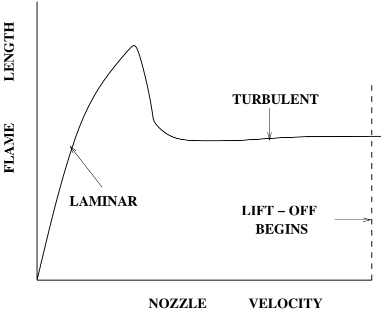
$$
\begin{aligned}
f = h^* \quad &= \quad \frac{(h_m + \Delta H_c \, \omega_{fu}) - (h_m + \Delta H_c \, \omega_{fu})_A}{(h_m + \Delta H_c \, \omega_{fu})_F - (h_m + \Delta H_c \, \omega_{fu})_A} \\
&= \quad \frac{cp_m \, (T - T_\infty) + \Delta H_c \, \omega_{fu}}{cp_m \, (T_0 - T_\infty) + \Delta H_c}
\end{aligned}
$$

Thus, noting that r_f corresponds to $\eta_f = C \, (r_f / x)$ and f_{stoich}, and r_f = 0 at x = L_f

$$
\begin{aligned}
\Phi \quad &= \quad \frac{u}{U_0} = f = h^* = \frac{3}{32} \, (\frac{D}{x}) \, (\frac{\rho_0}{\rho_m}) \, Re \, (1 + \eta^2/4)^{-2} \\
\frac{r_f}{x} \quad &= \quad \frac{16}{3^{0.5} \, Re} \, (\frac{\rho_m}{\rho_0})^{0.5} \left[\left\{ \frac{3}{32} \, (\frac{D}{x}) \, (\frac{\rho_0}{\rho_m}) \, \frac{Re}{f_{stoich}} \right\}^{0.5} - 1 \right]^{0.5} \\
\frac{L_f}{D} \quad &= \quad \frac{3}{32} \, (\frac{\rho_0}{\rho_m}) \, \frac{Re}{f_{stoich}} = \frac{3}{32} \, Re \, (1 + R_{st}) \, (\frac{\rho_0}{\rho_m})
\end{aligned}
$$

Turbulent Jet Flame - L42($\frac{11}{19}$)

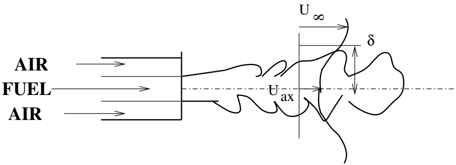
1. In Laminar flames, L_f increases with U_0 ,
2. In Turbulent flames, $L_f \simeq$ const.and
3. Radial distribution of u is nearly uniform over greater part of δ.



Eqns of slide 3 apply with

$$\rho_m D_{eff} \simeq \frac{\mu_{eff}}{Sc_t}$$

$$\rho_m \alpha_{eff} \simeq \frac{\mu_{eff}}{Pr_t}$$



with $Sc_t = Pr_t = 0.9$

Velocity Prediction - 1 - L42($\frac{12}{19}$)

1. The simplest formula[1] is $\mu_{eff} = 0.01 \times \rho_m \, |u_\infty - u_{ax}| \, \delta$
2. For stagnant surroundings $u_\infty = 0$ and $u_{ax} = u_{max}$. Also, from experiments, $(\delta / r_{1/2}) \simeq 2.5$. Hence, $\mu_{eff} = 0.0256 \, \rho_m \, u_{max} \, r_{1/2} \neq F\,(r)$
3. Thus, all laminar solns apply with μ changed to μ_{eff} . Hence

$$u^*_{max} = \frac{u_{max} \, x}{\nu_m} = (\frac{3}{32}) \, Re^2_{turb} \, (\frac{\rho_0}{\rho_m})$$

$$\frac{r_{1/2}}{x} = \frac{5.945}{Re_{turb}} \, (\frac{\rho_0}{\rho_m})^{-0.5}$$

$$= 5.945 \times \left[\frac{0.0256 \, \rho_m \, u_{max} \, r_{1/2}}{\rho_m \, U_0 \, D} \right] \, (\frac{\rho_0}{\rho_m})^{-0.5} \quad \text{Hence}$$

$$\frac{u_{max}}{U_0} = 6.57 \, (\frac{D}{x}) \, (\frac{\rho_0}{\rho_m})^{0.5} \quad \text{or, Combining with } u^*_{max}$$

[1] Spalding D. B. Combustion and Mass Transfer, Pergamon Press, Oxford (1979)

Velocity Prediction - 2 - L42($\frac{13}{19}$)

$$
\begin{aligned}
(\frac{u_{max}}{U_0})^2 &= 3.662\,(\frac{D^2}{r_{1/2}\,x})\,(\frac{\rho_0}{\rho_m}) \\
\frac{r_{1/2}}{x} &= \frac{3.662}{(6.57)^2} = 0.0848 \;\; \text{constant}
\end{aligned}
$$

This result agrees very well with Expt data for (x / D) $>$ 6.5. Replacing $r_{1/2}$ and u_{max}, we have

$$
\begin{aligned}
\mu_{eff} &= 0.0256\,\rho_m \times 6.57\,U_0\,(\frac{D}{x})\,(\frac{\rho_0}{\rho_m})^{0.5} \times (0.0848\,x) \\
&= 0.01426\,(\rho_0\,\rho_m)^{0.5}\,U_0\,D \simeq \;\; \text{constant}
\end{aligned}
$$

Also, since $\eta_{1/2}$ = 1.287,

$$
\eta = 1.287\,(\frac{r}{r_{1/2}}) \;\; \rightarrow \;\; \frac{u}{u_{max}} = \left[1 + 0.414\,(\frac{r}{r_{1/2}})^2\right]^{-2}
$$

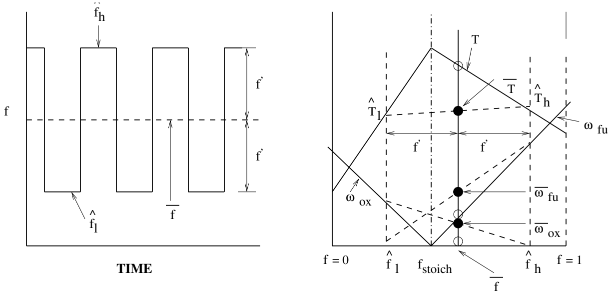
Prediction of L_f and r_f(x) - 1 - L42($\frac{14}{19}$)

1. A turbulent flame is essentially unsteady and its edges are jagged. Fragments of gas intermittently detach from the main body of the flame and flare, diminishing in size. Turbulence affects not only L_f but also the entire reaction zone near the edge of the flame. Compared with a laminar flame, this zone is also much thicker .
2. This implies that if the time-averaged values $\overline{\omega}_{fu}$ and $\overline{\omega}_{ox}$ are plotted with radius r, then the two profiles show considerable overlap around the crossover point f = f_{stoich}.
3. Unlike the overlap in a laminar flame , which is caused by finite chemical kinetic rates, however, in turbulent diffusion flames, the overlap is caused by turbulence.
4. In the presence of turbulence, R_{fu} actually experienced is not as high as that estimated from $\overline{R}_{fu} \propto \overline{\omega}_{fu}^{x}\, \overline{\omega}_{ox}^{y}$. This is because the fuel and oxidant at a point are present at *different times* - must allow for probability .

Prediction of L_f and $r_f(x)$ - 2 - L42($\frac{15}{19}$)

Since all laminar solutions are applicable to *time-averaged quantities* , we may write

$$\overline{\Phi} = \frac{\overline{u}}{U_0} = \overline{f} = \overline{h^*} = 6.57\,(\frac{D}{x})\,(\frac{\rho_0}{\rho_m})\,\left[1 + 57.6\,(\frac{r}{x})^2\right]^{-2}$$



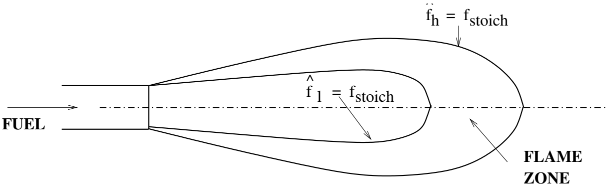
(a) TIME VARIATION OF f

(b) STATE DIAGRAM

Prediction of L_f and r_f(x) - 3 - L42($\frac{16}{19}$)

1. With reference to the figure, suppose that the value of $\hat{f}$ truly fluctuates between a low value $\hat{f}_l$ and a high value $\hat{f}_h$.
2. Let us assume that the fluid spends *equal* times at the two extremes and sharply moves from one extreme to the other
3. Then, $\overline{f} = \frac{1}{2}\,(\hat{f}_h + \hat{f}_l)$ and $f' = \frac{1}{2}\,(\hat{f}_h - \hat{f}_l)$
4. Thus, $\overline{\omega}_{fu} = 0.5\,(\hat{\omega}_{fu,l} + \hat{\omega}_{fu,h})$ (filled circle) $> \omega_{fu}$ (open circle) corresponding to $\overline{f} > f_{stoich}$.
5. Likewise, $\overline{\omega}_{ox} = 0.5\,(\hat{\omega}_{ox,l} + \hat{\omega}_{ox,h}) > \omega_{ox}$ (which is zero) corresponding to $\overline{f} > f_{stoich}$.
6. $\overline{T} = 0.5\,(\hat{T}_l + \hat{T}_h) <$ T corresponding to $\overline{f} > f_{stoich}$.
7. The above observations will also apply when $\overline{f} < f_{stoich}$. Thus, in general, finite amounts of fuel and oxidant are found when $\overline{f} = f_{stoich}$
8. If f_{stoich} does not lie between $\hat{f}_l$ and $\hat{f}_h$ then $\overline{T}$, $\overline{\omega}_{ox}$ and $\overline{\omega}_{fu}$ will of course correspond to $\overline{f}$.

Prediction of L_f and $r_f(x)$ - 4 - L42($\frac{17}{19}$)



The flame zone will be a finite volume enclosed by the $\hat{f}_l = \overline{f} - f' = f_{stoich}$ (inner) and $\hat{f}_h = \overline{f} + f' = f_{stoich}$ (outer) surfaces. The $\overline{f} = f_{stoich}$ surface will lie between the two surfaces.

Prediction of L_f and $r_f(x)$ - 5 - L42($\frac{18}{19}$)

Thus

$$
\begin{aligned}
\frac{r_{f,out}}{x} &= \left[\frac{1}{57.6}\left\{\sqrt{(\frac{6.57}{f_{stoich}-f'})\,(\frac{D}{x})\,(\frac{\rho_0}{\rho_m})}-1\right\}\right]^{0.5} \\
\frac{r_{f,in}}{x} &= \left[\frac{1}{57.6}\left\{(\sqrt{\frac{6.57}{f_{stoich}+f'}})\,(\frac{D}{x})\,(\frac{\rho_0}{\rho_m})-1\right\}\right]^{0.5} \\
\frac{r_{f,stoich}}{x} &= \left[\frac{1}{57.6}\left\{\sqrt{(\frac{6.57}{f_{stoich}})\,(\frac{D}{x})\,(\frac{\rho_0}{\rho_m})}-1\right\}\right]^{0.5} .
\end{aligned}
$$

where f' is estimated from

$$
\begin{aligned}
f' &\simeq l_m \times |\frac{\partial \bar{f}}{\partial r}|_{stoich} = (0.1875 \times r_{1/2}) \times |\frac{\partial \bar{f}}{\partial r}|_{stoich} \\
&= 24.07\,(\frac{\rho_0}{\rho_m})\,(\frac{D}{x})\,(\frac{r_{f,stoich}}{x})\,\left[1+57.6\,(\frac{r_{f,stoich}}{x})^2\right]^{-3}
\end{aligned}
$$

Prediction of L_f and r_f(x) - 6 - L42($\frac{19}{19}$)

Setting $r_{f,in,out,stoich} = 0$, $x = L_f$ can be estimated from

$$\frac{L_{f,out}}{D} = \frac{6.57}{f_{stoich} - f'}\left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{L_{f,in}}{D} = \frac{6.57}{f_{stoich} + f'}\left(\frac{\rho_0}{\rho_m}\right)$$

$$\frac{L_{f,stoich}}{D} = \frac{6.57}{f_{stoich}}\left(\frac{\rho_0}{\rho_m}\right) = 6.57\,(1 + R_{st})\left(\frac{\rho_0}{\rho_m}\right)$$

Thus, if $L_{f,stoich}$ is regarded as the mean flame length then, knowing $f_{stoich} = (1 + R_{st})^{-1}$, the flame length can be estimated for any fuel. Although the above relations are only approximate, they do embody the form of the experimentally determined empirical correlations

$$L_{f,exp} = F\left(D, R_{st}, \frac{\rho_0}{\rho_\infty}, \frac{\rho_0}{\rho_m}\right) \quad \rightarrow \quad \frac{\rho_0}{\rho_\infty} \simeq \frac{\rho_{fu}}{\rho_\infty} \quad \text{in most cases}$$