ME-662 CONVECTIVE HEAT AND MASS TRANSFER

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LECTURE-38 CONV M T - COUETTE FLOW MODEL

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1. Gas Injection - Effect of property variation and ω_T - LBL
2. Gas Injection - Effect of property variation and ω_T - TBL
3. Benzene evaporation in convective environment

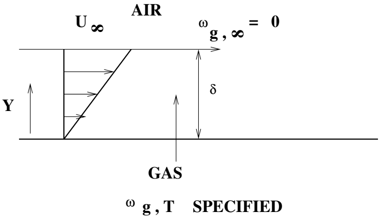
Couette flow model permits effects of fluid property variations to be studied.

Gas Injection ($\omega_T = 1$) - L38($\frac{1}{14}$)

Prob: Consider laminar Couette flow of air in which a gas with a specified $\omega_{g,T}$ is injected . Develop relationship $(g/g^*) \sim B$ when the gas is CO_2, He and H_2 . and study the effect of $\omega_{g,T}$.

Soln: In the Couette flow model, $\partial(\rho\, u)/\partial x = 0 = \partial(\rho\, v)/\partial y$. Hence, $N_w = \rho_w\, v_w = \rho\, v$ = const. The species transfer Eqn

$$N_w\, \frac{\partial \omega_g}{\partial y} = \frac{\partial}{\partial y}\, (\rho_m\, D \frac{\partial \omega_g}{\partial y})$$



Integrating once
$N_w\, (\omega_{g,y} - \omega_{g,w})$ =

$$\rho_m\, D\, \frac{\partial \omega_g}{\partial y}|_y - \rho_m\, D\, \frac{\partial \omega_g}{\partial y}|_w$$

Now, boundary condition gives (next slide)

Soln (Contd) - 1 - L38($\frac{2}{14}$)

$$N_w = \frac{\rho_m \, D \, \partial \omega_g / \partial y|_w}{\omega_{g,w} - \omega_{g,T}}$$

Hence

$$N_w \, (\omega_{g,y} - \omega_{g,w}) \;=\; \rho_m \, D \, \frac{\partial \omega_g}{\partial y}|_y - N_w \, (\omega_{g,w} - \omega_{g,T}) \quad \text{or}$$

$$\rho_m \, D \, \frac{\partial \omega_g}{\partial y}|_y \;=\; N_w \, (\omega_{g,y} - \omega_{g,T})$$

where D = const $\neq F(\omega_g)$ because p & T are const, but

$$\rho_m \;=\; \frac{p}{R_u \, T} \, M_{mix} = \frac{p}{R_u \, T} \, (\sum \, \frac{\omega_j}{M_j})^{-1}$$

$$=\; \frac{p}{R_u \, T} \, \left[\frac{M_g \, M_a}{M_a \, \omega_g + M_g \, (1 - \omega_g)} \right]$$

Soln (Contd) - 2 - L38($\frac{3}{14}$)

Substitution and intgration from y = 0 to δ gives

$$\int_{\omega_{g,w}}^{0} \frac{d\,\omega_g}{a\,\omega_g^2 + b\,\omega_g + c} = \frac{N_w\,R_u\,T\,\delta}{p\,M_g\,M_a\,D} \text{ with}$$

$$a = (M_a - M_g), \quad b = M_g - \omega_{g,T}\,(M_a - M_g), \quad c = -M_g\,\omega_{g,T}$$

where the LHS is given by

$$\text{LHS} = \frac{1}{\sqrt{b^2 - 4\,a\,c}} \ln\left[\frac{2\,a\,\omega_g + b - \sqrt{b^2 - 4\,a\,c}}{2\,a\,\omega_g + b + \sqrt{b^2 - 4\,a\,c}}\right]_{\omega_{g,w}}^{0}$$

$$= \frac{1}{M_g + \omega_{g,T}\,(M_a - M_g)} \ln\left[1 + B + \omega_{g,T}\,B\,(\frac{M_a}{M_g} - 1)\right]$$

where

$$B = \frac{0 - \omega_{g,w}}{\omega_{g,w} - \omega_{g,T}} = \frac{\omega_{g,w}}{\omega_{g,T} - \omega_{g,w}} \quad \text{and} \quad \omega_{g,w} = \omega_{g,T} \times \frac{B}{1 + B}$$

Soln (Contd) - 3 - L38($\frac{4}{14}$)

Now, for the Couette flow model

$$N_w = g\, B, \quad \text{and} \quad \frac{R_u\, T}{p\, M_g} = \frac{1}{\rho_g}$$

Therefore

$$\text{RHS} = \frac{N_w\, R_u\, T\, \delta}{p\, M_g\, M_a\, D} = \frac{g\, B\, \delta}{\rho_g\, M_a\, D}$$

Equating LHS = RHS and rearranging

$$\left(\frac{g\, \delta}{\rho_g\, D}\right) = \left(\frac{M_a}{M_g}\right) \left[\frac{\ln\,(1 + B^*)}{B^*}\right] \quad \text{where}$$

$$B^* = B\, \left\{1 + \omega_{g,T}\, \left(\frac{M_a}{M_g} - 1\right)\right\}. \quad \text{Hence}$$

$$\left(\frac{g}{g^*}\right)_{vp} = \frac{\ln\,(1 + B^*)}{B^*} \quad \text{(Ans)} \quad \rightarrow \quad \left(\frac{g}{g^*}\right)_{cp} = \frac{\ln\,(1 + B)}{B}$$

where subscript 'vp' for variable and 'cp' for const property.

Soln - $(\frac{g}{g^*}) \sim B$ for $\omega_{g,T} = 1$ - L38($\frac{5}{14}$)

B	cp	vp_{CO_2}	vp_{He}	vp_{H_2}	$\omega_{g,w}$
0	1.0	1.0	1.0	1.0	0.0
.25	.893	.926	.571	.422	.200
.50	.811	.864	.422	.291	.333
1.0	.693	.768	.291	.189	.500
1.5	.611	.695	.228	.144	.600
2.0	.549	.638	.189	.117	.667
2.5	.501	.591	.163	.0998	.714
3.0	.462	.552	.144	.0873	.750

1. $\omega_{g,T} = 1$ implies that the gas is the only transferred substance . Also, $B^* = B\, M_a/M_g$.
2. $(g/g^*)_{vp,CO_2} > (g/g^*)_{cp}$ because $M_{CO_2} > M_{air}$
3. For He and H_2, this trend reverses.
4. $\omega_{g,w}$ increases with B

Soln - $(\frac{g}{g^*}) \sim B$ for $\omega_{g,T} = 0.01$ - L38($\frac{6}{14}$)

B	cp	vp_{CO_2}	vp_{He}	vp_{H_2}	$\omega_{g,w}$
0	1.0	1.0	1.0	1.0	0.0
.25	.893	.893	.887	.888	.002
.50	.811	.811	.802	.792	.0033
1.0	.693	.694	.681	.668	.005
1.5	.611	.612	.598	.584	.006
2.0	.549	.550	.536	.522	.0067
2.5	.501	.502	.488	.474	.0071
3.0	.462	.463	.449	.435	.0075

1. $\omega_{g,T} = .01$ implies that the gas in the transferred substance is a small fraction - rest is air.
2. $(g/g^*)_{vp,CO_2} \simeq (g/g^*)_{cp}$
3. For He and H_2, $(g/g^*)_{vp} < (g/g^*)_{cp}$
4. $\omega_{g,w}$, though small, increases with B

Correlation with $(\frac{M_{mix,\infty}}{M_{mix,w}})$ - L38$(\frac{7}{14})$

Here, $M_{mix,w} = M_a\, M_g/(M_a\, \omega_{g,w} + M_g\, (1 - \omega_{g,w}))$ and $M_{mix,\infty} = M_a$ (because $\omega_{g,\infty} = 0$). Hence, from slide 4, and using $\omega_{g,w} = \omega_{g,T} \times B/(1+B)$

$$B^* = B\left\{1 + \omega_{g,T}\,(\frac{M_a}{M_g} - 1)\right\}.$$

$$\frac{B^*}{B} = 1 + (\frac{1+B}{B})\,(\frac{M_{mix,\infty}}{M_{mix,w}} - 1)$$

$$\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln(1+B^*)}{B^*} \times \frac{B}{\ln(1+B)}$$

This shows dependence on $M_{mix,w}/M_{mix,\infty}$ and B as recommended correction from boundary layer flow model. If $\omega_{g,T} = 0$, $B^* = B$. If $\omega_{g,T} = 1$, $B^* = B\,(M_a/M_g)$

Turbulent Couette Flow - 1 - L38($\frac{8}{14}$)

Here, the governing Eqn will be

$$N_w\,(\omega_g - \omega_{g,T}) = \rho_m\,(D + D_t)\,\frac{d\,\omega_g}{dy}$$

where

$$\begin{aligned}
\rho_m\,D_t &= \rho_m\,\frac{\nu_{t,ref}}{Sc_t} \quad \text{But, from Van-Driest model} \\
\nu_{t,ref} &= \frac{\mu_t}{\rho_{ref}} = l_m^2\,\frac{\partial u}{\partial y} \;\rightarrow\; \frac{\partial u}{\partial y} = C \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(\kappa\,y^+)^2\,\left\{1 - \exp(-\frac{y^+}{A^+})\right\}^2 \quad \text{and} \\
&= C\,(\frac{\nu_{ref}}{u_\tau})^2\,(0.08\,\delta^+)^2 \;\text{ for }\; y^+ > 26 \;\text{ where} \\
C\,(\frac{\nu_{ref}}{u_\tau})^2 &= C\,\frac{\nu_{ref}^2\,\rho_{ref}}{\tau_w} = C \times \frac{\mu_{ref}\,\nu_{ref}}{\mu_{ref}\,C} = \nu_{ref}
\end{aligned}$$

Turbulent Couette Flow - 2 - L38($\frac{9}{14}$)

Substituting for D_t and ρ_m, we have

$$
\begin{aligned}
N_w\,(\omega_g - \omega_{g,T}) &= \rho_m\, D\,(1 + \frac{\nu_{t,ref}}{Sc_t\, D})\,\frac{d\,\omega_g}{dy} \\
&= (\frac{D\, p\, M_a\, M_g}{R_u\, T}) \times \frac{u_\tau / \nu_{ref}}{M_a\,\omega_g + M_g\,(1 - \omega_g)} \\
&\times \quad F \times \frac{d\,\omega_g}{dy^+} \quad \text{where}
\end{aligned}
$$

$$
\begin{aligned}
F &= 1 + (\frac{Sc}{Sc_t})\,(\kappa\, y^+)^2\, \left\{ 1 - \exp(-\frac{y^+}{A^+}) \right\}^2 \quad y^+ < 26 \\
&= 1 + (\frac{Sc}{Sc_t})\,(0.08\,\delta^+)^2 \quad y^+ > 26
\end{aligned}
$$

Turbulent Couette Flow - 3 - L38($\frac{10}{14}$)

Taking $N_w = g\,B$, $(p\,M_g)/(R_u\,T) = \rho_g$ and $u_\tau = U_\infty\,\sqrt{C_{f,x}/2}$,

$$\text{LHS} = \left(\frac{g}{\rho_g\,U_\infty}\sqrt{\frac{2}{C_{f,x}}}\,Sc\right) \times \text{ INT where INT} = \int_0^{\delta^+} \frac{dy^+}{F}$$

$$\text{RHS} = \frac{M_a}{B}\int_{\omega_{g,w}}^{0} \frac{d\,\omega_g}{(\omega_g - \omega_{g,T})\ \{M_a\,\omega_g + M_g\,(1-\omega_g)\}}$$

$$= \frac{\ln(1+B^*)}{B^*} \quad \rightarrow \quad B^* = B\left\{1 + \omega_{g,T}\left(\frac{M_a}{M_g} - 1\right)\right\}$$

Taking $A^+ = 26$ and $Sc_t = 0.9$, we have

INT = 9.62 for $CO_2 - Air$, Sc = 0.96

INT = 14.57 for $H_2 - Air$ and He-Air, Sc = 0.22

Turbulent Couette Flow - 4 - L38($\frac{11}{14}$)

Therefore

$$\frac{g_{vp}}{\rho_g\, U_\infty} \times \sqrt{\frac{2}{C_{f,x}}} \times Sc = \frac{1}{\text{INT}} \times \frac{\ln\left(1+B^*\right)}{B^*}$$

and

$$\frac{(g/g^*)_{vp}}{(g/g^*)_{cp}} = \frac{\ln\left(1+B^*\right)}{B^*} \times \frac{B}{\ln\left(1+B\right)}$$

This result is same as that for a Laminar boundary layer. This is because it is assumed that the value of INT is same for 'cp' and 'vp' conditions.

Note that g_{vp} is significantly influenced by INT (Sc) .

Evaporation of C_6H_6 - L38($\frac{12}{14}$)

Prob: C_6H_6 evaporates from the outer surface of a circular cylinder in air flowing at 6 m/s normal to the cylinder. From expts, $h_{cof,v_w=0}$ = 85 W/m^2-K and B = 0.9 . Allowing for property variations, estimate N_w and ω_w . Given: Sc = 1.71, Pr = 0.71, $cp_{C_6H_6}$ = 1.69 kJ/kg-K and cp_a = 1.01 kJ/kg-K.

Soln: Here,

$$B = \frac{\omega_{v,\infty} - \omega_{v,w}}{\omega_{v,w} - 1} = 0.9 \quad \rightarrow \omega_{v,w} = 0.4737 \quad \text{(Ans)}$$

Therefore, $\omega_{v,m} = 0.5\,(\omega_{v,\infty} + \omega_{v,w})$ = 0.2368.
$c_{pm} = 1.69 \times 0.2368 + 1.01 \times 0.7632$ = 1.171 kJ/kg-K.
Hence, $g^* = (h_{cof,v_w=0}/c_{pm})$ = 0.0726 kg/m^2-s .
Also, $M_{mix,\infty}$ = 29 and
$M_{mix,w} = (0.4737/78 + 0.5263/29)^{-1}$ = 41.286.

Soln (Contd.) - L38($\frac{13}{14}$)

For Flow over a cylinder[1], $Nu_{cp} \propto Pr^{0.37}$.
Therefore, using the short-cut empirical formula

$$\frac{g_{vp}}{g^*_{cp}} = \frac{\ln(1+B)}{B} \times (\frac{Pr}{Sc})^{0.37} \times (\frac{M_{mix,\infty}}{M_{mix,w}})^{-0.67}$$

$$= \frac{\ln(1+0.9)}{0.9} \times (\frac{0.71}{1.71})^{0.37} \times (\frac{29}{41.286})^{-0.67} = 0.6525$$

Therfore, $g = 0.0726 \times 0.6525$ = 0.0474 kg/m^2-s (Ans) .
Thus, the effect of property variations is to reduce g_{vp} compared to g_{cp}.

[1] Zhukauskas A *Heat Transfer from Tubes in Crossflow*, Eds: Hartnett J P and Irvine T F, Adv H T, vol 8, Academic Press, (1972)

Soln (Contd.) - L38($\frac{14}{14}$)

If we followed the Couette flow theory, then in this case,

$$B^* = B \left\{ 1 + \omega_{g,T} \, (\frac{M_a}{M_g} - 1) \right\} = 0.3346$$

Hence

$$(\frac{g}{g^*})_{vp} = \frac{\ln{(1 + 0.3346)}}{0.3346} = 0.8626$$

But, for variable properties, $h_{cof,vp} = h_{cof,cp} \times Pr^{.25}$.
Therefore, $g_{vp} = g^*_{cp} \times (0.71)^{0.25} \times 0.8626$ = 0.0575 kg/m^2-s.
This value is greater than that obtained from the empirical formula. Thus, Couette flow theory provides an approximate answer due to linear velocity profile assumption.