ME-662 CONVECTIVE HEAT AND MASS TRANSFER

A. W. Date
Mechanical Engineering Department
Indian Institute of Technology, Bombay
Mumbai - 400076
India

LECTURE-33 COUETTE FLOW MODEL

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1. Momentum Transfer with wall suction/blowing
2. General convective mass transfer
3. Interpretation of g^*
4. Estimate of evaporation/burning time

Reminder of Gov Eqns - L32($\frac{1}{12}$)

In the Couette flow model, u = const $\times$ y , (d/dx) = 0 and A = const. Hence, under steady state

$$\frac{d}{dy}\ [N_{\Psi,y}] = \frac{d}{dy}\ \left[\rho_m\, v\, \Psi - (\Gamma + \Gamma_t)_\Psi \frac{d\ \Psi}{dy}\right] = S_\Psi$$

Ψ	Γ_Ψ	S_Ψ
1	0	0
u	$\mu + \mu_t$	0
ω_k	$\rho_m\,(D + D_t)$	R_k
η_α	$\rho_m\,(D + D_t)$	0
h_m	$(k_m + k_{m,t})/cp_m$	- $d(\sum m''_{y,k}\, h_k)/dy$

where $\dot{Q}_{rad}$ is neglected and $m''_{y,k} = -\rho_m\, D\,(d\omega_k/dy)$. Also, $\rho_m\, V\, A = \dot{m}_w$ = const.

Momentum Transfer - 1 - L33($\frac{2}{12}$)

1. For Ψ = u,

$$\frac{d}{dy}\left[N_w\, u - (\mu + \mu_t)\,\frac{d\,u}{dy}\right] = 0$$

Integrating once and noting that u = 0 at y = 0 and $(\mu + \mu_t)\,(d\,u/dy)|_{y=0} = \tau_w$, the constant of integration C = - τ_w. Hence,

$$\int_0^{\infty} \frac{du}{N_w\, u + \tau_w} = \int_0^{\delta} \frac{dy}{\mu + \mu_t} = C_1 \quad \text{say}$$

$$= \frac{1}{N_w} \ln\left[1 + \frac{N_w\, U_\infty}{\tau_w}\right]$$

2. But,

$$\frac{N_w\, U_\infty}{\tau_w} = \frac{\rho\, V_w\, U_\infty}{\tau_w} = \frac{V_w/U_\infty}{C_{f,x}/2} = B_f \quad \text{(Blowing Parameter)}$$

Momentum Transfer - 2 - L33($\frac{3}{12}$)

1. Therefore, $\ln(1 + B_f) = C_1\, N_w = C_1\, \rho\, U_\infty\, B_f\, (C_{f,x}/2)$
2. As $B_f \rightarrow 0$, let $C_{f,x} = C_{f,x,V_w=0}$.
3. Then, assuming C_1 remains independent of whether V_w is finite or zero
$$\frac{C_{f,x,V_w}}{C_{f,x,V_w=0}} = \frac{\ln(1 + B_f)}{B_f}$$
4. This eqn is applicable to both laminar and turbulent flow . It is derived for dp/dx = 0 but can be taken to be valid for mild Pr gr.

General Conv. Mass Transfer - 1 - L33($\frac{4}{12}$)

1. For all 4 types of mass transfer, and an appropriately defined conserved property Ψ , $N_w = N_{\Psi,y}$ = const. Hence, for conserved property ($\Psi - \Psi_w$)

$$\begin{aligned} \frac{d}{dy}\left[N_w\left(\Psi-\Psi_w\right)-\left(\Gamma+\Gamma_t\right)\frac{d\left(\Psi-\Psi_w\right)}{dy}\right] &= 0 \text{ or} \\ N_w\left(\Psi-\Psi_w\right)-\left(\Gamma+\Gamma_t\right)\frac{d\,\Psi}{dy} &= C_1 \quad \text{(say)} \end{aligned}$$

2. Then, writing this eqn in w- and T-states,

$$C_1 = N_w\left(\Psi_T-\Psi_w\right) = -\,\Gamma\,\frac{d\,\Psi}{dy}|_w$$

3. Recall that T-state is deep inside the neighbouring phase where Ψ is uniform and hence $(d\,\Psi/dy)_T$ = 0. Also, at the w-state, Γ_t = 0.

General Conv. Mass Transfer - 2 - L33($\frac{5}{12}$)

1. Hence, replacing $C_1 = N_w\,(\Psi_T - \Psi_w)$, we have

$$N_w\,(\Psi - \Psi_T) - (\Gamma + \Gamma_t)\,\frac{d\,\Psi}{dy} = 0$$

2. Integrating this Eqn from w-state (y = 0) to ∞-state (y = δ)

$$\frac{1}{N_w}\int_0^\infty \frac{d\,\Psi}{(\Psi - \Psi_T)} = \int_0^\delta \frac{dy}{\Gamma + \Gamma_t} = C_2 \quad (\text{ say })$$

3. Or, integration of LHS gives

$$N_w = \frac{1}{C_2}\ln\,(1 + B_\Psi) \quad \rightarrow \quad B_\Psi = \frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T} \quad \text{(and)}$$

$$N_w = \frac{C_1}{\Psi_T - \Psi_w} = \frac{-\,\Gamma\,(d\,\Psi/dy)_w}{\Psi_T - \Psi_w}$$

General Conv. Mass Transfer - 3 - L33($\frac{6}{12}$)

1. Now, consistent with the theory of heat transfer, we may write

$$-\Gamma \frac{d\,\Psi}{dy}|_w = g \times (\Psi_w - \Psi_\infty)$$

where g is the mass transfer coefficient (kg/m^2-s)

2. Then,

$$N_w = g \times \left(\frac{\Psi_\infty - \Psi_w}{\Psi_w - \Psi_T}\right) = g \times B_\Psi \quad \text{and}$$

$$g = \frac{1}{C_2} \frac{\ln(1 + B_\Psi)}{B_\Psi}$$

3. Let $g \rightarrow g^*$ as $B_\Psi \rightarrow 0$. Further, let C_2 remain same for with and without mass transfer. Then

$$\frac{g}{g^*} = \frac{\ln(1 + B_\Psi)}{B_\Psi}$$

Comments - 1 - L33($\frac{7}{12}$)

1. Thus, the ficticious g^* flux is given by

$$N_w = g^* \ln\left(1 + B_\Psi\right) \quad \text{where} \quad \frac{1}{g^*} = C_2 = \int_0^\delta \frac{dy}{\Gamma + \Gamma_t}$$

2. Thus, g^* may be viewed as the sum of layer-by-layer resistances to mass transfer in the considered phase over the width δ
3. This interpretation of g^* enables its evaluation from known $\Gamma\left(y\right) = \Gamma\left(\Psi\right)$ in a laminar BL and from known $\Gamma_t\left(y\right)$ from a turbulence model (mixing length, for example) in a turbulent BL. Thus, the Couette flow model permits study of property variations.
4. In fact, if Γ = const and $\Gamma_t = 0$ then $g^* = \Gamma/\delta$ which is same as the Stefan flow model with $g^* = \Gamma/L$.

Comments - 2 - L33($\frac{8}{12}$)

1. Further, if we consider case of pure heat transfer in the presence of suction or blowing , with $\Psi = h_m = c_p\, T$

$$-k\,\frac{d\,T}{dy}|_w = g\,c_p\,(T_w - T_\infty) = h_{cof,V_w}\,(T_w - T_\infty)$$

   where h_{cof,V_w} is heat transfer coefficient.

2. Then, $h_{cof,V_w} = g/c_p$ and $h_{cof,V_w=0} = g^*/c_p$
3. Hence,

$$\frac{g}{g^*} = \frac{h_{cof,V_w}}{h_{cof,V_w=0}} = \frac{St_{x,V_w}}{St_{x,V_w=0}} = \frac{\ln\,(1+B_h)}{B_h} \quad \rightarrow \quad B_h = \frac{T_\infty - T_w}{T_w - T_T}$$

4. This relationship was found to be applicable in a real boundary layer in lecture 30. Thus, the Couette flow model captures most features of a real boundary layer.

Evaporation/Burning times - 1 - L33($\frac{9}{12}$)

1. The previous expressions can be used instantaneously , to estimate evaporation/burning times . Thus

$$\rho_l \, \frac{dV}{dt} = - \, \dot{m}_w = - \, A_w \, N_w = - \, A_w \, g^* \, \ln \left(1 + B_\Psi\right)$$

   Integrating from t = 0 (V = V_i) to t = $t_{evpa,burn}$ (V = 0) gives

$$t_{evap,burn} = - \, \frac{\rho_l}{\ln \left(1 + B_\Psi\right)} \int_{V_i}^{0} \frac{dV}{A_w \, g^*}$$

2. For a liquid drop and diffusion mass transfer , $A_w = 4 \, \pi \, r_w^2$, V = $(4/3) \, \pi \, r_w^3$, and $g^* = \Gamma_{mh}/r_w$. Hence,

$$t_{evap,burn} = - \, \frac{\rho_l}{\ln \left(1 + B_\Psi\right)} \int_{r_{w,i}}^{0} \frac{r_w}{\Gamma_{mh}} \, dr_w = \frac{\rho_l \, D_{w,i}^2}{8 \, \Gamma_{mh} \, \ln \left(1 + B_\Psi\right)}$$

Evaporation/Burning times - 2 - L33($\frac{10}{12}$)

1. For a liquid drop and Convective mass transfer , g^* can be determined by a short-cut method . Thus,

$$\frac{\dot{m}_{w,conv}}{\dot{m}_{w,diff}} = \frac{g^* \, 4\,\pi\, r_w^2 \, \ln\,[1+B]}{\rho_m \, D \, 4\,\pi\, r_w \, \ln\,[1+B]} = \frac{1}{2}\left[\frac{g^* \, D_w}{\rho_m \, D}\right] = \frac{Sh}{2}$$

   where Sh $\equiv$ Sherwood Number .

2. Using analogy between HT & MT (Le = 1)

$$Sh = \frac{g^* \, D_w}{\Gamma_{mh}} = 2 + 0.6\, Re^{0.5}\, Sc^{1/3} \quad \rightarrow \quad Re = \frac{|\, u_g - u_p \,|\, D_w}{\nu_m}$$

   where $|\, u_g - u_p \,| \;\equiv$ relative vel between drop and gas.

3. Then,

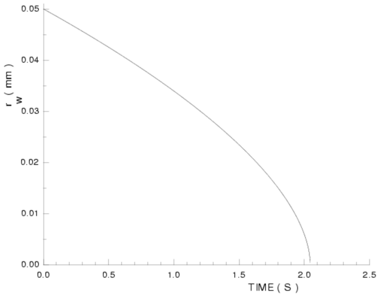
$$t_{evap,burn} = -\frac{2\,\rho_l}{\ln\,(1+B_\Psi)} \int_{r_{w,i}}^{0} \frac{r_w \, dr_w}{\Gamma_{mh}\,(2 + 0.6\, Re_{D_w}^{0.5}\, Sc^{1/3})}$$

   This evaluation requires numerical integration

Problem - L33($\frac{11}{12}$)

Prob: A water droplet ($D_{w,i}$ = 1 mm) at 25^{0}C evaporates in air (RH - 25 %, T = 25^{0}C) with u_{rel} = 5 m/s. Estimate evaporation time. Take Sc = 0.6 .

Soln: This is inert MT without HT. The mass fractions are: $\omega_{v,\infty}$ = 0.0078, $\omega_{v,w}$ = 0.02. Therefore $B_m = 0.0124$. ρ_m = 1.177 kg/m^3, ρ_l = 1000 kg/m^3, $D_m = 2.376 \times 10^{-5}$ and $\nu_m = D_m \times Sc = 1.42 \times 10^{-5}$.



Num. Int. - Δt = 0.01 sec.

Ans: Evaporation time at $r_w = 0$ is 2.045 sec . If $u_{rel} = 0$, then Evaporation time = 4.66 sec.

Summary - L33($\frac{12}{12}$)

1. In the Couette flow model with A = const, u = const $\times$ y and $d\Psi$ / dx = 0, we have shown that

$$N_w = g^* \ln\left(1 + B_\Psi\right) \quad \text{where} \quad \frac{g}{g^*} = \frac{\ln\left(1 + B_\Psi\right)}{B_\Psi}$$

2. The ficticios g^* flux is interpreted as the sum of layer-by-layer resistances to mass transfer in the considered phase over boundary layer width
3. In pure momentum and heat transfer in the presence of suction/blowing

$$\frac{C_{f,x,V_w}}{C_{f,x,V_w=0}} = \frac{\ln\left(1 + B_f\right)}{B_f} \qquad \frac{St_{x,V_w}}{St_{x,V_w=0}} = \frac{\ln\left(1 + B_h\right)}{B_h}$$

4. In the following lectures, we shall develop similar results using algebraic Reynolds flow model .