4 Turbulent Flows - l21 to l30

4.1 Formal Aspects

1. Lecture 21 → Applying Reynolds's averaging rules, derive time-averaged continuity and momentum equations shown on slide 9.

2. Lecture 21 → Applying Reynolds's averaging rules, derive time-averaged energy equation shown on slide 10. Write out fully expanded form of turbulent kinetic energy dissipation rate $\rho_m\,\epsilon$.

3. Lecture 21 → Starting from N-S equations for instantaneous variables, derive Turbulent Kinetic Energy equation (Hint: Follow through slides 3 to 6)

4. By making appropriate boundary layer approximations, show that the turbulent KE equation for a 2-dimensional boundary layer is given by

$$\rho\left[u\,\frac{\partial k}{\partial x}+v\,\frac{\partial k}{\partial y}\right] = -\,\rho\left[\overline{u'v'}\,\frac{\partial u}{\partial y}+(\overline{u'^2}-\overline{v'^2})\frac{\partial u}{\partial x}\right] - \frac{\partial}{\partial y}\left\{\overline{(p'+k)v'}\right\}+\mu\,\frac{\partial^2 k}{\partial y^2}-\rho\,\epsilon$$

   Interpret meaning of each term. Explain why $(\overline{u'^2}-\overline{v'^2})\,\partial u/\partial x\to 0$ in moderate pressure gradient boundary layer. Hence show that in regions where convection and diffusion of k are negligible, the reduced equation implies a *mixing length* type formulation of the eddy viscosity.

5. Lecture 22 → Using the properties of isotropic turbulence, show that the estimates of the magnitude of ϵ shown in slide 16 are correct.

6. Lecture 22 → On slide 18, it is assumed that $u_2^{'}=A\,V^{'}$ where $A>1$. Justify this assumption on the basis of Vorticity Dynamics (slide 13 - lecture 23)

7. Lecture 23 → Derive transport equation for a two-point correlation tensor B_{ij} for a non-homogeneous anisotropic turbulent flow (slides 4,5 and 6). Hence, derive simplified equation for homogeneous turbulent flow.

8. Lecture 27 → From the result of the previous problem, derive transport equation for a one-point correlation stress tensor $\overline{u_i^{'}u_j^{'}}$ (slide 3)

9. Lecture 24 $\rightarrow$ Discuss the phenomenological arguments followed by dimensional analysis to show that the near-wall velocity profile can be taken as $u^+ = F\,(y^+)$ (see slides 2 to 4)

10. Lecture 24 $\rightarrow$ Discuss the main arguments leading to the 3-layer ' law of the wall ' shown on slide 9.

11. Compare the above prescription with

$$y^+ = u^+ + \exp\left(-5.4\,\kappa\right)\left[\exp\left(\kappa\,u^+\right) - 1 - \kappa\,u^+ - \frac{(\kappa\,u^+)^2}{2} - \frac{(\kappa\,u^+)^3}{6}\right]$$

and the ' continuous law of the wall ' shown on slides 14 & 15 (lecture 24) for (dp/dx) = 0.

12. Lecture 25 $\rightarrow$ Consider the dimensionless mean kinetic energy equation for a zero pressure gradient boundary layer shown in slide 5. Using the 3-layer law, evaluate each term on the right hand side of this equation. The values of $\overline{u'v'}/u_\tau^2$ may be read from fig (a) in slide 3. Hence, verify the plots shown on slide 6.

13. Lecture 26 $\rightarrow$ Interpret meaning of each term of the e-ϵ equation set shown in slide 17. Follow the arguments on slides 18 and 19 to estimate C_1 and C_2.

14. Lecture 27 $\rightarrow$ Using Low Re_t ASM (slides 7 and 8) for a 2D boundary layer flow, derive expressions for $\overline{u'v'}$, $\overline{u'^2}$ and $\overline{v'^2}$

15. Lecture 27 $\rightarrow$ (slide 12) Derive transport equation for turbulent heat flux correlation $\overline{u_i'T'}$. Using the modelling assumptions(slides 13 and 14) , write the transport equation for $\overline{v'T'}$ for a 2D boundary layer flow.

4.2 Predictive Aspects

1. Using similarity solution for a laminar boundary layer, estimate values of the onset ($Re_{x,ts}$) and the end of transition ($Re_{x,te}$) Reynolds numbers using (i) Cebeci model and (ii) Fraser & Milne model

2. Repeat the previous problem for an adverse pressure gradient boundary layer with $U_\infty = C\,x^{-0.04}$

3. Using Power law profile, show that for $U_\infty = C\,x^m$,

$$p^+ = (\frac{\nu}{\rho\,u_\tau^3})\,\frac{dp}{dx} = \frac{-204.14\,m\,Re_x^{-0.7}}{(3.86\,m+1)^{0.3}}$$

4. Write a computer program to calculate u^+ as a function of y^+ from mixing length model with Van-Driest damping. Use

$$A^+ = \frac{25}{a\left[v_w^+ + b\left\{\frac{p^+}{1+c\,v_w^+}\right\}\right]+1}$$

with a = 7.1, b = 4.25, c = 10 and if $p^+ > 0,\ \ b = 2.9,\ \ c = 0$ or, if $v_w^+ < 0,\ \ a = 9$

For $v_w = 0$, carry out computations for ($U_\infty = C\,x^m$) and $Re_x = 10^6$ with m = -0.2, 0.0 and 0.6. Compare your results with the 3-layer law. Estimate p^+ using power-law velocity profile in each case.

5. Evauate PF(u^+) using 3-layer law as well as the continuous law for Pr = 0.001, 0.7, 10, 100, 500. Plot the variation of PF with u^+ in each case on a single graph. Note values of PF as $u^+ \to \infty$. Compare this value with that obtaied using

$$PF_\infty = 9.24\left[(\frac{Pr}{\mathrm{Pr}_T})^{0.75}-1\right]\left[1+0.28\ \exp((-\,0.007\,\frac{Pr}{Pr_T)}\right]$$

where $Pr_T = 0.85 + 0.0309\,(Pr+1)/Pr$.

6. Consider a 2-D converging nozzle of inlet width W_i, exit width W_e and length L in which fluid enters with a velocity U_0. Calculate the discharge coefficient of this nozzle as a function of Re $= U_0\,W_i/\nu$ assuming that turbulent boundary layer develops on both walls right from the inlet. Take (L/W_i) = 3 and (W_i/W_e) = 1.5.

7. Consider fully developed turbulent flow between two parallel plates separated by distance 2b. Assume that in the turbulent core, the velocity distribution is given by

$$u^+ = 5.5 + 2.5 \ln \left[y^+ \left\{ \frac{1.5(1 + y/b)}{1 + 2\,(y/b)^2} \right\} \right]$$

where y is measured from the duct centerline. Determine $f_{D_h} = F(Re_{D_h})$ and compare with $f = 0.046\, Re^{-0.2}$ (Hint: Integrate the above expression to determine $\overline{u}^+$).

Also determine $u_{cl}/\overline{u}$ and compare with the values estimated from log-law and power-law.

8. In the above problem, let there be heat transfer with axially constant wall heat flux q_w at both the plates. Using analogy method, show that

$$T_{cl} - T_w = -\frac{q_w}{\rho\, Cp\, u_\tau} \left[5\, Pr + 2.5 \ln \left(\frac{b^+}{5} \right) \right]$$

when following assumptions are made.

   (a) The flow is divided in two layers only; that is (i) $0 < y^+ < 11.6$ and $11.6 < y^+ < b^+$
   (b) $Pr_T = 1$ and $(\nu_t/\nu) >> 1$ in the second layer.
   (c) $\tau = \tau_w\,(1 - y/b)$ where y is measured from bottom plate
   (d) $Pr >> 1$

Further, assuming that $(T_w - T_{cl})/(T_w - T_b) = (y/b)^{1/7}$, develop expression for Nusselt number Nu_{D_h} and evaluate its value for for Pr = 10. 100 and 1000 and $Re_{D_h} = 10^5$. Compare these values with those evaluated from Dittu-Boelter and Gnielinski correlations.

9. Using the Integral Equation, develop a computer program to calculate laminar, transitional through to turbulent boundary layer.

10. Lecture 29 $\rightarrow$ (slide 14) Using similarity variables mentioned on this slide, write a computer program to calculate laminar, transitional through to turbulent boundary layer. Adapt this program to calculate development of laminar, transitional through to turbulent boundary layer on an ellispse (slide 15). Compare your results with those predicted by the integral method (slides 10 t0 13)

11. Lecture 30 $\rightarrow$ (slide 10) Extend the computer program of the previous problem to include heat transfer

12. Lecture 30 $\rightarrow$ (slide 12) Assuming 1/7th power-law profile for temperature and axial velocity, show that $(T_w - T_{cl})/(T_w - T_b) = (6/5)$

13. Lecture 30 $\rightarrow$ (slide 15) Liquid Hg flows through a long tube (2.5 cm id) with a velocity of 1 m/s. Assuming axially constant wall heat flux, evaluate Nu using Analogy method and the correlation by Sliecher and Rouse. Given: ρ = 13264 kg / m^3, Cp = 136.5 J / kg-K, k = 115 W/m-K and $\mu = 90 \times 10^{-5}$ N-s/m^2.

14. Lecture 30 $\rightarrow$ (slide 15) Lubricating oil is to be cooled from 75^{0}C to 40^{0}C while flowing through 1.25 cm dia tube with a velocity of 3 m/s. The tube surface temperature is 25^{0}C. Calculate the length required assuming correlation due to Sliecher and Rouse and the pumping power. Given: ρ = 865 kg / m^3, Cp = 1780 J / kg-K, k = 0.14 W/m-K and $\nu = 9 \times 10^{-6}$ m^2/s.

15. Air at 5 bar and 800 C flows over a surface (0.3 m long) such that $U_\infty = 20 - 33.3\,x$ m/s where x is measured in meters. The surface temperature is 600 C. Assuming that turbulent boundary layer originates from the leading edge of the surface, estimate variation of C_{fx} and heat transfer coefficient h_x with x. Hence, estimate the average heat transfer coefficient. (Hint: Determine C_{fx} using integral method and h_x using $T^+ = Pr_T\,(u^+ + PF)$ at 5 - 6 axial locations and prepare a table. Then determine average h by numerical integration.)