# 1 Introduction - l1 to l4

1. From your life experience, give one example each of heat transfer by conduction, radiation and convection. Similarly, give one example each of diffusion mass transfer and convective mass transfer with and/or without heat transfer and chemical reaction.

2. Consider flow in a duct of square cross-section. The duct is subjected to axially uniform heat flux. At a particular cross-section, its three sides are held at uniform temperatures $T_{w1}$, $T_{w2}$ and, $T_{w3}$ and the fourth side is insulated. Define the *length-averaged heat transfer coefficients* at each of the four sides and the *overall heat transfer coefficient* for the cross-section. Assume that the bulk fluid temperature at the cross-section is $T_b$.

3. Consider practical flow situations depicted in figure 1. Identify the relevant flow domains ( including the planes of symmetry ). As discussed in lecture 2, classify each situation in terms of type of convective heat and/or mass transfer, incomp/compressible, wall/free flow, paraboloc or elliptic, single or two/multi-phase and dimensionality.

   (a) Air at room temperature at low and moderate speeds flows normal to a heated cylinder.

   (b) Air flow normal to a bank of tubes arranged in a square array.

   (c) Flow of high temperature and high velocity gas flowing over a gas turbine blade. Assume that flow separation has been prevented on either side of the blade.

   (d) Issuing of a particle-laden hot smoke from a tall chimney

   (e) Issuing of a hot water discharge into a stagnant lake

   (f) Decay of axi-symmetric swirling flow in a circular tube with a low ratio of swirl to axial velocity.

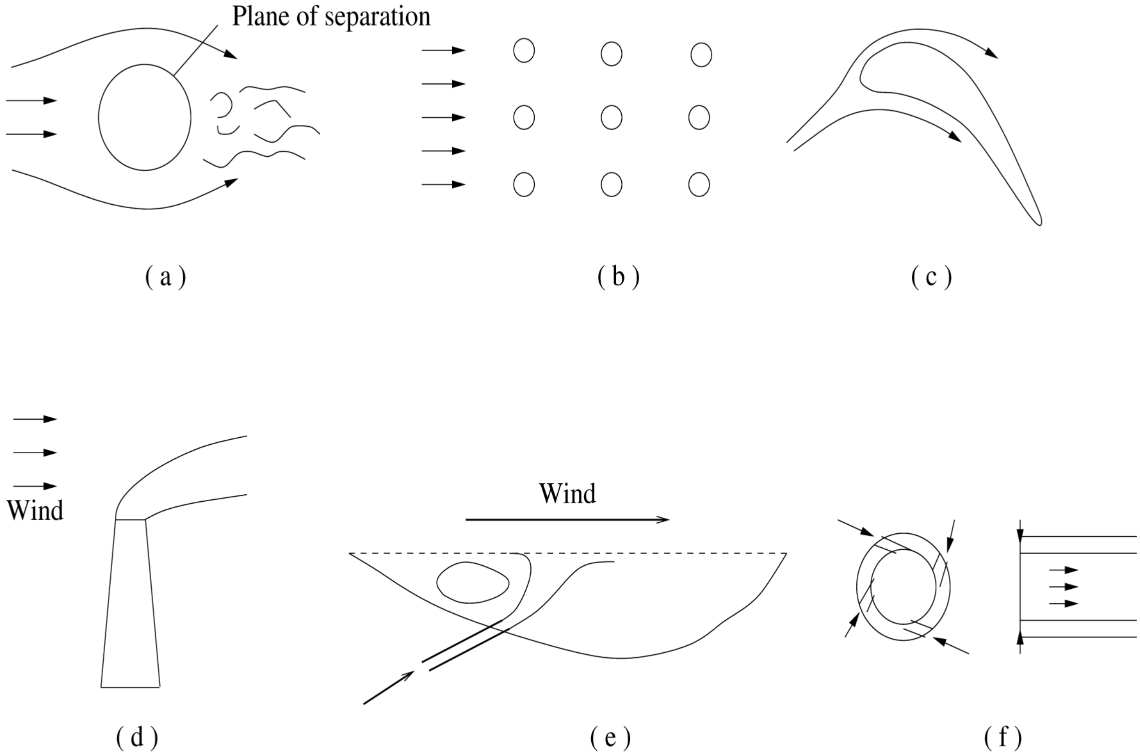



Figure 1: Examples of Different Flows

4. In lecture l3,

   (a) Consider slide 13. Analogous to equation ( 3 ), Derive momentum equations in directions 2 and 3. What are the new stresses identified ?

   (b) Consider slide 15. Assuming uniform fluid properties, rewrite equation ( 10 ) in non-conservative form. Now, multiply each equation in direction $x_i$ with velocity $u_i$. Addition of the 3 equations for i = 1,2 and 3 will yield an equation for kinetic energy

$$\rho \frac{D\,(V^2/2)}{Dt} = F \quad \text{where } V^2 = u_1^2 + u_2^2 + u_3^2$$

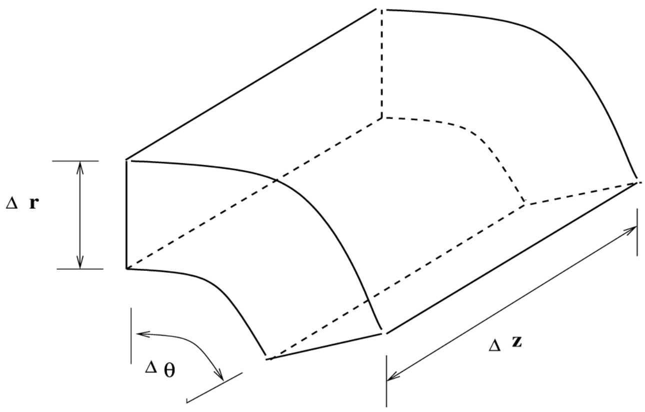
   Develop a compact expression for the right hand side F.

5. Consider lecture l4 - slide 3: Show that addition of equation ( 6 ) for all species k of the mixture implies that $\sum_k R_k = \sum_k \dot{m}_k'' = 0$.

6. In lecture 4 - slide 6 - consider eqn 9 in which the definition of $e^0$ includes contribution of kinetic energy $V^2/2$. However, in many heat transfer situations, the velocity is very small ( say, less tahn 100 m/s ) so that contribution of $V^2/2$ may be neglected. Show that this neglect will not alter the energy eqn ( 35 ) ( slide 17 ) written in terms of the enthalpy variable.

7. Show that in cylindrical polar coordinate system ( see figure 2 ), the bulk mass and the momentum equations are given by

$$\begin{aligned}
\frac{\partial \rho}{\partial t} &+ \frac{1}{r}\frac{\partial \rho\, r\, V_r}{\partial r} + \frac{1}{r}\frac{\partial \rho\, V_\theta}{\partial \theta} + \frac{\partial \rho\, V_z}{\partial z} = 0 \\
\frac{\partial \rho\, V_r}{\partial t} &+ \frac{1}{r}\frac{\partial \rho\, r\, V_r\, V_r}{\partial r} + \frac{1}{r}\frac{\partial \rho\, V_\theta\, V_r}{\partial \theta} + \frac{\partial \rho\, V_z\, V_r}{\partial z} - \rho\,\frac{v_\theta^2}{r} \\
&= -\frac{\partial p}{\partial r} + \frac{1}{r}\frac{\partial r\, \tau_{r,r}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r,\theta}}{\partial \theta} + \frac{\partial \tau_{r,z}}{\partial z} - \frac{\tau_{\theta,\theta}}{r} + \rho\, B_r \\
\frac{\partial \rho\, V_\theta}{\partial t} &+ \frac{1}{r}\frac{\partial \rho\, r\, V_r\, V_\theta}{\partial r} + \frac{1}{r}\frac{\partial \rho\, V_\theta\, V_\theta}{\partial \theta} + \frac{\partial \rho\, V_z\, V_\theta}{\partial z} + \rho\,\frac{v_\theta\, V_r}{r} \\
&= -\frac{1}{r}\frac{\partial p}{\partial \theta} + \frac{1}{r^2}\frac{\partial r^2\, \tau_{r,\theta}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta,\theta}}{\partial \theta} + \frac{\partial \tau_{\theta,z}}{\partial z} + \rho\, B_\theta
\end{aligned}$$




**CONTROL VOLUME OF A CYLINDRICAL POLAR COORDINATE SYSTEM**

Figure 2: Polar Coordiante System

$$\frac{\partial \rho\, V_z}{\partial t} + \frac{1}{r}\,\frac{\partial \rho\, r\, V_r\, V_z}{\partial r} + \frac{1}{r}\,\frac{\partial \rho\, V_\theta\, V_z}{\partial \theta} + \frac{\partial \rho\, V_z\, V_z}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial r\, \tau_{r,z}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta,z}}{\partial \theta} + \frac{\partial \tau_{z,z}}{\partial z} + \rho\, B_z$$

where, Stokes's stress-rate of strain relations are

$$\tau_{r,r} = 2\,\mu\,\frac{\partial V_r}{\partial r}, \quad \tau_{\theta,\theta} = 2\,\mu\,(\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}), \quad \tau_{z,z} = 2\,\mu\,\frac{\partial V_z}{\partial z}$$

$$\tau_{r,z} = \mu(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}) \quad \tau_{r,\theta} = \mu\left\{ r\,\frac{\partial}{\partial r}\,(\frac{V_\theta}{r}) + \frac{1}{r}\,\frac{\partial V_r}{\partial \theta}\right\} \quad \tau_{\theta,z} = \mu(\frac{\partial V_\theta}{\partial z} + \frac{1}{r}\frac{\partial V_z}{\partial \theta})$$

Similarly, show that energy equation will derive to

$$\frac{\partial \rho\, h}{\partial t} + \frac{1}{r}\,\frac{\partial \rho\, r\, V_r\, h}{\partial r} + \frac{1}{r}\,\frac{\partial \rho\, V_\theta\, h}{\partial \theta} + \frac{\partial \rho\, V_z\, h}{\partial z} = -\left[\frac{1}{r}\frac{\partial r\, q_r}{\partial r} + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta} + \frac{\partial q_z}{\partial z}\right] + \mu\,\Phi_{visc} + Q_{others}$$

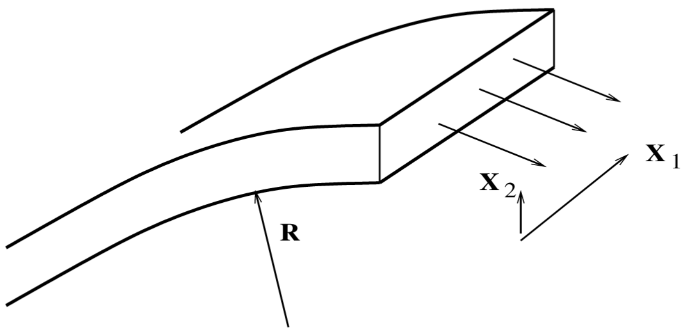



Figure 3: Curved passage of rectangular cross-section

where,

$$h = C_p\,(T - T_{ref}), \quad q_r = -k\,\frac{\partial T}{\partial r} \quad q_\theta = -\frac{k}{r}\,\frac{\partial T}{\partial \theta} \quad q_z = -k\,\frac{\partial T}{\partial z}$$

and the viscous dissipation is given by

$$\begin{aligned} \mu\,\Phi_{visc} &= \tau_{r,r}\,\frac{\partial V_r}{\partial r} + \tau_{\theta,\theta}\,(\frac{\partial V_\theta}{\partial \theta} + \frac{V_r}{r}) + \tau_{z,z}\,\frac{\partial V_z}{\partial z} \\ &+ \tau_{r,z}\,(\frac{\partial V_r}{\partial z} + \frac{\partial V_z}{\partial r}) + \tau_{r,\theta}\left\{r\,\frac{\partial}{\partial r}\,(\frac{V_\theta}{r}) + \frac{1}{r}\,\frac{\partial V_r}{\partial \theta}\right\} \\ &+ \tau_{\theta,z}\,(\frac{\partial V_\theta}{\partial z} + \frac{1}{r}\,\frac{\partial V_z}{\partial \theta}) \end{aligned}$$

8. Consider steady flow convection in a spiral plate heat exchanger ( figure 3 ) The flow passage is a curved one whose radius of curvature R ( $\theta$ ) changes cotinuously in the flow direction. The duct cross-section is rectangular. The equations in the fixed coordinate system $r$, $\theta$, $z$ ( see Bird, Stewart & Lightfoot, lecture 1, slide 15 ) must be transformed to ( $x_1$, $x_2$, $\Phi$ ) using transformations $x_2 = r - R(\theta), x_1 = z$ and $\Phi = \theta$.

   Develop the transport equations in the transformed coordinate system and identify the emergence of the centrifugal and coriolis force terms.

9. A large body of characteristic dimension 2d looses heat by natural convection to air under $(T_w - T_\infty)_{body} = 15^0$C. In order to estimate this heat

loss, a 1:2 reduced scaled model of the body is experimented with under $(T_w - T_\infty)_{model} = 20^0\mathrm{C}$. The measured heat loss is $Q_{model} = 200$ W. Assuming surface areas $A_{\text{body,model}} \propto$ (characteristic dimension)$^2$, estimate $Q_{body}$