

Linear Algebra

$$x + 3y = 6$$

$$x - y = 2$$

$$x = 3 \quad y = 1 \quad \text{sol}$$

THE ONLY SOL.

$$a_1x + a_2y = b_1'$$

$$b_1x + b_2y = b_2'$$

- 1) Lines intersect at one pt Unique
- 2) Lines coincide infinite sol
- 3) Lines are parallel No Sol

m equations n unknown

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

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$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

Matrix Notation

$$A = (a_{ij})_{m \times n}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$Ax = b$$

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Problem Given $m \times n$ m —

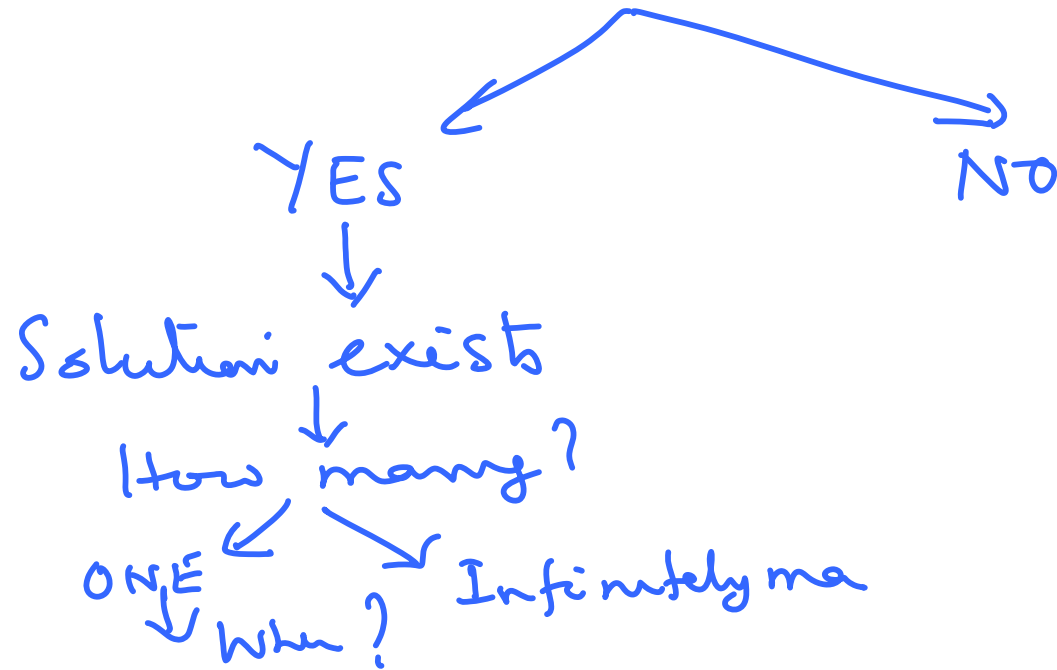
For different b find sol.

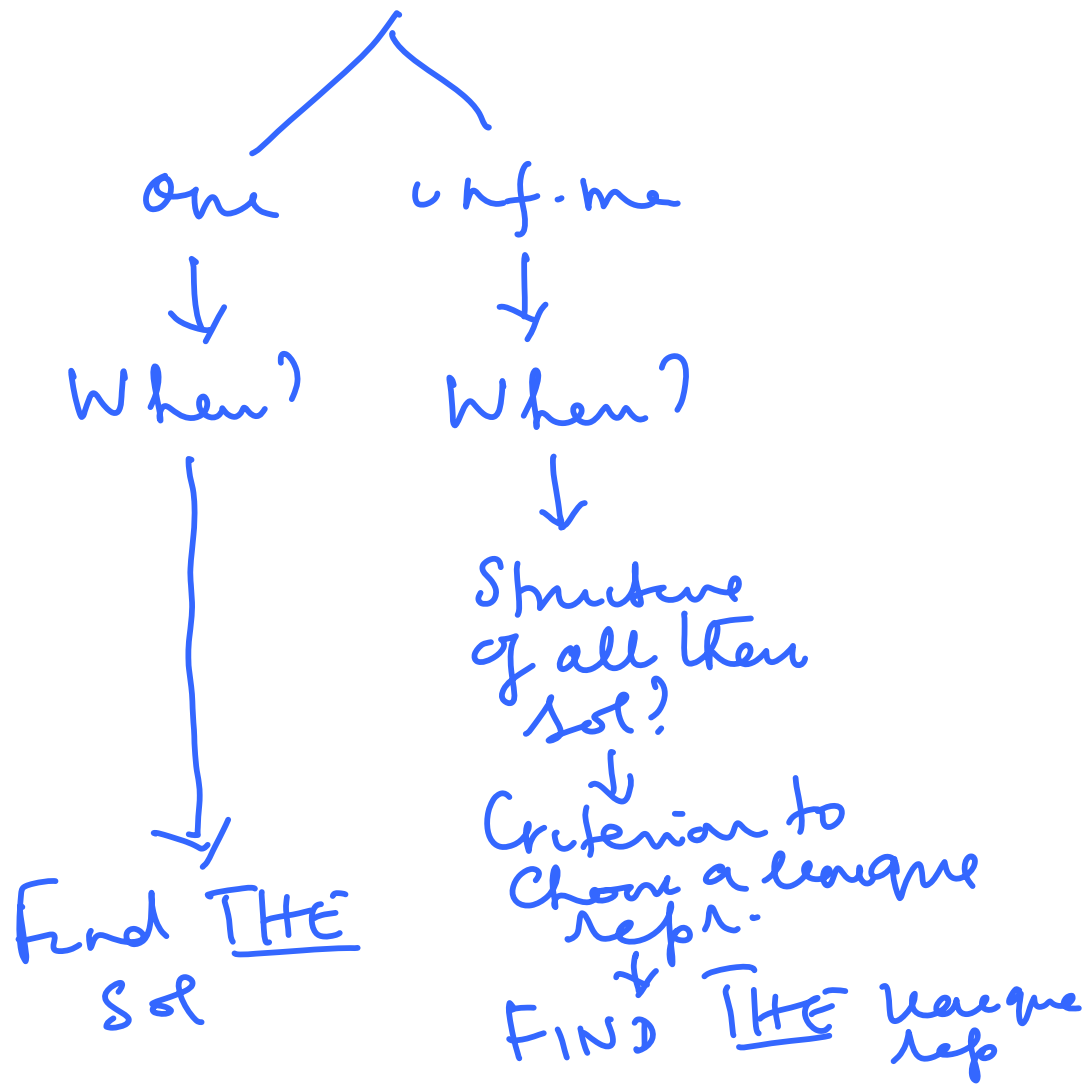
1st Question : What is the condition
that b should satisfy for a sol
to exist.

Suppose we have this condition $[C]$

$$Ax = b$$

Does b satisfy $[C]$?





b does not satisfy (C)

No \downarrow sol for $Ax = b$

What \downarrow do we do?

No Solution $\Rightarrow Ax \neq b$ for any x

$\Rightarrow b - Ax \neq$ zero col. matrix
for any x

$b - Ax \neq \text{zero}$ set value
for any x

$$\text{Error} = b - Ax$$

Quantify Error

$$\text{Minimize } \sum_{i=1}^m (b_i - (Ax)_i)^2$$

Least SQUARE SOLUTION

b does not satisfy $[C]$

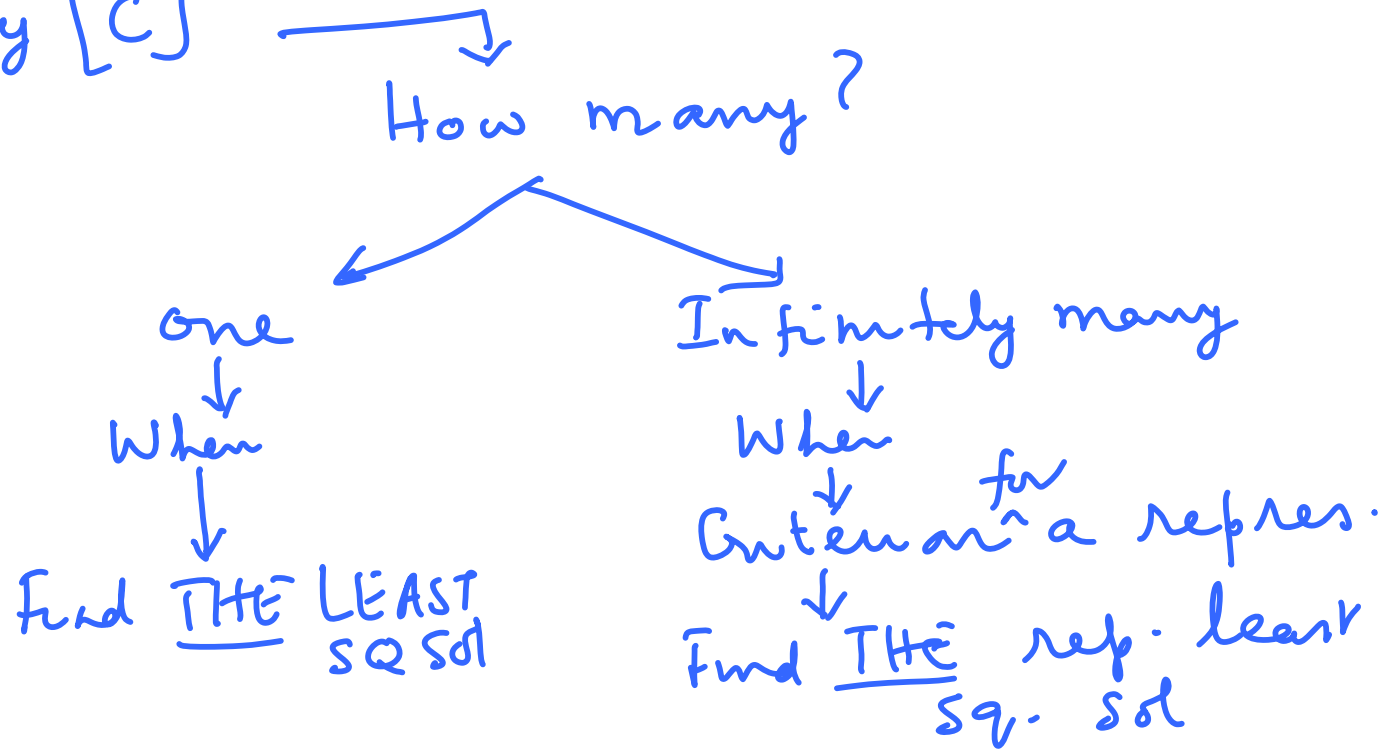
No solution exists for $Ax = b$

Then a column matrix x_l is called

a least square sol if

$$\sum_{i=1}^m (b_i - (Ax_l)_i)^2 \leq \sum_{i=1}^m (b_i - (Ax)_i)^2$$

We can show that we always can
get a least sq sol if b does not
satisfy $[C]$



What is the natural way to go about finding the answer?

What are "easy" systems?

Why are the gen systems difficult?

(Variables) — Unknown are all COUPLED
in a general system.

Uncouple them

A system where uncoupling is easy

— a system where there is no coupling

Each equation involves only one unknown

n equations

n unknowns

i^{th} eq. involves i^{th} unknown x_i