Linear Algebra

$$x + 3y = 6$$
$$x - y = 2$$

$x = 3 \quad y = 1$ sol

THE ONLY SOL.

$$a_1 x + a_2 y = b_1'$$

$$b_1 x + b_2 y = b_2'$$

1) Lines intersect at one pt Unique

2) Lines coincide infinite sol

3) Lines are parallel No Sol

m equations n unknown

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$- \quad - \quad - \quad - \quad -$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Matrix Notation

$$A = (a_{ij})_{m \times n}$$

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$$

$$Ax = b$$

$$Ax = b$$

Problem Given $m \times n$ m —

For different b find sol.

1st Question : What is the condition that b should satisfy for a sol to exist.

Suppose we have this condition [C]

$$Ax = b$$

Does b satisfy [C] ?

YES

NO

Solution exists

How many ?

ONE

Infinitely many

When ?

one

↓

When?

↓

Find THE sol

inf. many

↓

When?

↓

Structure of all the sol?

↓

Criterion to choose a unique repr.

↓

FIND THE unique rep

b does not satisfy (C)

No sol for $Ax = b$

What do we do?

No solution $\Longrightarrow$ $Ax \neq b$ for any x

$\Longrightarrow$ $b - Ax \neq$ zero col. matrix for any x

$b - Ax \neq$ zero col matrix
for any x

Error $= b - Ax$

Quantify Error

Minimize $\sum_{i=1}^{m} \left(b_i - (Ax)_i \right)^2$

Least SQUARE SOLUTION

b does not satisfy [C]

No solution exists for $Ax = b$

Then a column matrix x_l is called

a least square sol if

$$\sum_{i=1}^{m} \left(b_i - (Ax_l)_i\right)^2 \leq \sum_{i=1}^{m} \left(b_i - (Ax)_i\right)^2$$

We can show that we always can get a least sq sol if b does not satisfy [C] → How many?

- One → When → Find <u>THE</u> LEAST SQ SOL
- Infinitely many → When → Criterion for a repres. → Find <u>THE</u> rep. least sq. sol

What is the natural way to go about finding the Answer?

What are "easy" systems?

Why are the gen systems difficult?

(Variables) — Unknowns are all COUPLED in a general system.

Uncouple them

A system where uncoupling is easy

— a system where there is no coupling

Each equation involves only one unknown

n equations

n unknowns

i^{th} eq. involves i^{th} unknown x_i