Advanced Engineering Mathematics
Exercises on Module 4: Probability and Statistics

1. A survey of people in given region showed that 25% drank regularly. The probability of death due to liver disease, given that a person drank regularly, was 6 times the probability of death due to liver disease, given that a person did not drink regularly. The probability of death due to liver disease in the region is 0.005. If a person dies due to liver disease what is the probability that he/she drank regularly?

2. In a production line ICs are packed in vials of 5 and sent for inspection. The probabilities that the number of defectives in a vial is $0,1,2,3$ are $1/3$, $1/4$, $1/4$, $1/6$ respectively. Two ICs are drawn at random from a vial and found to be good. What is the probability that all ICs in this vial are good?

3. An insurance company floats an insurance policy for an eventuality taking place with probability 0.05 over the period of policy. If the sum insured is Rs. 100000 then what should be the premium so that the expected earning of the insurance company is Rs. 1000 per policy sold?

4. Suppose X is a discrete random variable with pmf given by $P(X=-1)=\dfrac{1-\alpha}{3}$, $P(X=0)=\dfrac{1}{3}$, $P(X=1)=\dfrac{1+\alpha}{3}$, where α is a real number. Find the range of α for which this is a valid pmf. Also determine the maximum and minimum values of *Var (X)*. Write the cdf for $\alpha=\dfrac{1}{2}$. Hence show that the median is not unique.

5. Let X be a continuous random variable with the pdf

$$f(x)=\begin{cases} \dfrac{b\,x}{(1+x^2)}, & \sqrt{e-1}<x<\sqrt{e^2-1}, \\ 0, & \text{otherwise.} \end{cases}$$

   Find $b, P(2<X^2<2.5)$ and $V(X)$. Also find the cdf and quantiles.

6. Let X be a random variable with moment generating function

$$M_X(t)=\left(\frac{2e^t}{3-e^t}\right)^4.$$

   Find $P(5\le X\le 7)$ and $E(X)$.

7. The average number of accidents during 9:00 a.m. to 9:00 p.m. is thrice the average number during 9:00 p.m. to 9:00 a.m. at a traffic junction. Given no accidents are recorded during a day (24 hour period), the conditional probability that the recording time was between 9:00 p.m. to 9:00 a.m. is thrice the

conditional probability that the recording time was between 9:00 a.m. to 9:00 p.m. What is average number of accidents during the day (24 hour period)?

8. Questions are asked to Girish in a quiz competition one by one until he fails to answer correctly. The probability of his answering correctly a question is p. The probability that he will quit after answering an odd number of questions is 0.9. Find the value of p.

9. A missile can successfully hit a target with probability 0.75. If three successful hits can destroy the target completely, how many missiles must be fired so that the probability of the completely destroying the target is not less than 0.95?

10. A parallel system has 3 independent components. The lifetime X_i (in hours) of the ith component is exponentially distributed with mean 3^i, for $i = 1, 2, 3$. What is the probability that the system is working even after 27 hours? If the system is working after 27 hours, what is the probability that only third component is working?

11. The time to failure (in years), X, of the electronic tubes produced at two manufacturing plants I and II follows a gamma distribution. For the tubes produced at plant I, the mean is 2 and the variance 4, whereas for the tubes produced at plant II, the mean is 4 and variance 8. Plant II produces four times as many tubes as plant I. The tubes are intermingled and supplied. What is the probability that a tube selected at random will work for at least 6 years?

12. The length of a door handle (in cm), manufactured by a factory, is normally distributed with $\mu = 6.0$ and $\sigma = 0.2$. The place to fix on the door can allow error in length up to 0.3 cm. What percentage of handles manufactured in the factory will be defective? How much should be the value of σ, so that the number of defectives in reduced to only 5%?

13. The marks (out of 100) of students in a class are normally distributed. Grade 'A' is awarded for more than 80 marks, grade 'B' between 60-80, grade 'C' between 40-60 and 'F' below 40 marks. If 10% students get 'A' and 15% get 'F', find the percentage of students getting grade 'B'.

14. Five percent of items produced by a company are defective. Items are sold in boxes of 100 and a guarantee is given that a box contains not more than 2 defectives. Using Poisson approximation to binomial distribution, determine the probability that a box will fail to meet the guarantee?

15. The probability that a student joining a Ph.D. program in IIT Kharagpur will complete within 4 years is 0.25. Find the approximate probability that out of next 1000 students more than 260 complete within 4 years?

16. Fatal accidents are observed to occur at a stretch of highway according to a Poisson process at a rate of 15 per month. Find the approximate probability that in a year there are less than 150 fatal accidents at that stretch.

17. Let (X, Y) have the joint density function given by

$$f(x,y) = \left(\frac{ax+by}{y^3}\right), \quad 0 \le x \le 1, \ y \ge 1,$$
$$= 0, \quad \text{elsewhere.}$$

If $E(Y)$ exists, then find a and b. For these values of a and b find the marginal and conditional distributions of X and Y. Are X and Y independent? Also find $E(X)$, $E(Y)$, $Var(X)$, $Var(Y)$ and $Cov(X,Y)$. Also find $P(X \le 0.25)$, $P(Y \le 3)$, $P(X \le 0.5 | Y = 2)$, $P(Y > 2 | X = 0.25)$ $P(X \le Y/2)$, $P(X+Y < 2)$ and $P(Y - X > 1)$.

18. From a store containing 2 defective, 3 partially defective and 3 good computers, a random sample of 4 computers is selected. Find the expected number of defective and partially defective computers in the sample. What will be the covariance between the number of defective and number of partially defective computers?

19. Let (X, Y) have bivariate normal distribution with density function

$$f(x,y) = \frac{3}{4\pi\sqrt{2}} e^{-\frac{9}{16}\left[(x-1)^2 - \frac{2}{3}(x-1)(y-1) + (y-1)^2\right]}, \quad -\infty < x, y < \infty.$$

Find the correlation coefficient between X and Y, $P(1 < X < 2 \mid Y = 2)$, and $P(4 < 2X + 3Y < 6)$.

20. Let X_1, X_2, X_3 be independent exponential random variables with the probability density $f(x) = \lambda e^{-\lambda x}, \quad x > 0$. Define random variables Y_1, Y_2 and Y_3 as

$$Y_1 = X_1 + X_2 + X_3, \ Y_2 = \frac{X_1 + X_2}{X_1 + X_2 + X_3}, \ Y_3 = \frac{X_1}{X_1 + X_2}.$$

Find the joint and marginal densities of Y_1, Y_2 and Y_3. Are they independent?

21. Suppose that for a certain individual, calorie intake at breakfast is random variable with mean 500 and standard deviation (s.d.) 50, calorie intake at lunch is a random variable with mean 900 and s.d. 100, and calorie intake at dinner is a random variable with mean 2000 and s.d. 180. Assuming that intakes at different meals are independent of one another, what is the approximate probability that the average calorie intake per day over the next year (365 days) is at most 3410?

22. Let $X_1, X_2, ..., X_n, X_{n+1}$ be independent $N(0,1)$ random variables and $Y = \frac{1}{n}\sum_{i=1}^{n} X_i^2$.

Find the distribution of $X_{n+1}/\sqrt{Y}$.

23. Let $X_1, X_2, ..., X_{2n}$ be independent $N(0,1)$ random variables and $W = \frac{1}{2}\sum_{i=1}^{n}\left(X_{2i-1} - X_{2i}\right)^2$. What is the distribution of W?

24. Lengths of pins (in mm) produced by a machine follow a $N(\mu,\sigma^2)$ distribution. Find the maximum likelihood estimators of μ and σ^2 based on a random sample of size 10 with observations: 7.12, 7.13, 7.01, 6.95, 6.89, 6.97, 6.99, 6.93, 7.05, 7.02.

25. Let $X_1, X_2, ..., X_n$ be independent $N(\mu,\sigma^2)$ random variables. Find unbiased estimator of μ^2. Is it consistent?

26. Let $X_1, X_2, ..., X_n$ be independent $Exp(\mu,\sigma)$ random variables. Find the method of moments and maximum likelihood estimators of μ and σ^2. Compare them by evaluating their mean squared errors.

27. Breaking strength (in kg) of the front part of a new vehicle is normally distributed. In 10 trials the breaking strengths were found to be 578, 572, 570, 568, 572, 570, 570, 572, 596, 584. Find a 95% confidence interval for the mean breaking strength. Can we say that the mean breaking strength is significantly less than 570 at 1% level of significance? Further test the hypothesis that the variance is less than 8 sq kg. At 5% level of significance.

28. Carbon emissions on 8 randomly selected vehicles of brand A were recorded as 150, 250, 240, 280, 290, 210, 220, 180, whereas those of 10 randomly selected of brand B were recorded as 140, 230, 270, 190, 270, 200, 150, 200, 190, 170. Set up 90% confidence intervals for the difference in the means and ratio of variances of the two populations. Also test the hypothesis that the variances of the two populations are equal (at 10% level of significance). Based on the result of this test, conduct an appropriate test for the hypothesis that the average emission from vehicles of brand B is less than the average emission from vehicles of brand A (at 5% level of significance).

29. An experiment was conducted to compare the recovery time (in days) of patients from a serious disease using two different medications. The first medicine was given to a random sample of 15 patients and the sample mean and sample variance of the recovery time were observed to be 16 and 1.4 respectively. The second medicine was administered to a random sample of 19 patients and sample mean and sample variance of the recovery time were observed to be 20 and 2.0 respectively. Test the hypothesis that the average recovery time using the first medicine is significantly less than the one by using the second (at 5% level). Assume the two populations to be normal with equal variances.

30. In a random sample of 200 families watching television in Bombay at any given time, it was found that 45 were watching Network A. At the same time, in a random sample of 110 families watching television in New Delhi, it was found that 32 were watching Network A. Test the hypothesis that Network A is equally popular in both states (at this time) at 1% level of significance.

Advanced Engineering Mathematics
Hints to Exercises on Module 4: Probability and Statistics

1. Let A denote the event that the person drinks regularly, and let B denote the event that the death is due to liver disease. Given
$P(A) = 0.25,\ P(B) = 0.005,\ P(B \mid A) = 6P(B \mid A^C) = \alpha$, say.
By theorem of total probability
$P(B) = P(B \mid A)P(A) + P(B \mid A^C)P(A^C)$, OR
$$0.005 = \frac{\alpha}{4} + \frac{3}{4}.\frac{\alpha}{6} \Rightarrow \alpha = \frac{8}{3} \times 0.005$$
So $P(A \mid B) = \dfrac{P(B \mid A)P(A)}{P(B)} = \dfrac{2}{3}$

2. Let B_j denote the event that a vial has j defective ICs, for $j = 0, 1, 2, 3$. Let A denote the event that two ICs drawn at random from a vial are good. It is given that $P(B_0) = \frac{1}{3},\ P(B_1) = \frac{1}{4}, P(B_2) = \frac{1}{4}, P(B_3) = \frac{1}{6}.$ Also we find that
$$P(A \mid B_0) = 1,\ P(A \mid B_1) = \binom{4}{2}\bigg/\binom{5}{2} = \frac{3}{5}, P(A \mid B_2) = \binom{3}{2}\bigg/\binom{5}{2} = \frac{3}{10},$$
$$P(A \mid B_3) = \binom{2}{2}\bigg/\binom{5}{2} = \frac{1}{10}.$$
Using Bayes Theorem,
$$P(B_0 \mid A) = \frac{P(A \mid B_0)P(B_0)}{\sum_{j=0}^{3} P(A \mid B_j)P(B_j)} = \frac{40}{69}.$$

3. Let P be the premium and let X denote the earning of the insurance company per policy. Then
$$X = \begin{cases} P, & \text{if there is no eventuality during period of the policy} \\ P\text{-}100000, & \text{if there is eventuality during period of the policy} \end{cases}$$
Now $E(X) = .95\,P + .05\,(P - 100000) = 1000$ yields $P = 6000.$ So Rs. 6000 should be the premium.

4. We must have $0 \le 1 - \alpha \le 3$ and $0 \le 1 + \alpha \le 3$. This yields $-1 \le \alpha \le 1.$ Also $V(X) = \frac{2}{3} - \frac{4\alpha^2}{9}$. This has minimum value $\frac{2}{9}$ and maximum value $\frac{2}{3}$.
For $\alpha = \frac{1}{2}$, the cdf is described by
$$F_X(x) = \begin{cases} 0, & x < -1, \\ 1/6, & -1 \le x < 0, \\ 1/2, & 0 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

Let M be the median, then $0 \le M \le 1$.

5. We should have $\int_{-\infty}^{\infty} f(x)\,dx = 1.$ It gives b = 2.

Now $P(2 < X^2 < 2.5) = \int_{-\infty}^{\infty} f(x)\,dx = \log_e(7/6).$

$$E(X) = \int_{-\infty}^{\infty} x\ f(x)\,dx = 2\int_{\sqrt{e-1}}^{\sqrt{e^2-1}} \frac{x^2}{1+x^2}\,dx$$
$$= 2\left[\left(\sqrt{e^2-1} - \sqrt{e-1}\right) + \left(\tan^{-1}\sqrt{e-1} - \tan^{-1}\sqrt{e^2-1}\right)\right].$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2\ f(x)\,dx = 2\int_{\sqrt{e-1}}^{\sqrt{e^2-1}} \frac{x^3}{1+x^2}\,dx$$
$$= 2\int_{\sqrt{e-1}}^{\sqrt{e^2-1}} \left(x - \frac{x}{1+x^2}\right)dx = (e^2 - e - 1).$$

$$V(X) = (e^2 - e - 1) - 4\left[\left(\sqrt{e^2-1} - \sqrt{e-1}\right) - \left(\tan^{-1}\sqrt{e^2-1} - \tan^{-1}\sqrt{e-1}\right)\right].$$

The cdf is given by

$$F_X(x) = \begin{cases} 0, & x < \sqrt{e-1}, \\ \log_e(1+x^2) - 1, & \sqrt{e-1} \le x < \sqrt{e^2-1}, \\ 1, & x \ge \sqrt{e^2-1}. \end{cases}$$

$$Q_1 = \sqrt{e^{5/4}-1},\ \ M = \sqrt{e^{3/2}-1},\ \ Q_3 = \sqrt{e^{7/4}-1}.$$

6. This is mgf of NB(4, 2/3) distribution. So the pmf of X is given by

$$P(X = k) = \binom{k-1}{3}\left(\frac{1}{3}\right)^{k-4}\left(\frac{2}{3}\right)^4,\quad k = 4, 5, \ldots\ldots$$

$$P(5 \le X \le 7) = \frac{1376}{2187} \cong 0.629. \quad E(X) = \frac{4}{2/3} = 6.$$

7. Let X be the number of accidents recorded during a 12 hour period. Let D denote the period between 9:00 a.m. to 9:00 p.m. and N denote the period between 9:00 p.m. to 9: a.m. Then it is given that X/D ~$\mathscr{P}(\lambda)$, X/N ~$\mathscr{P}(3\lambda)$, for some $\lambda > 0$.

$$P(N \mid X = 0) = \frac{P(X = 0 \mid N)P(N)}{P(X = 0 \mid N)P(N) + P(X = 0 \mid D)P(D)} = \frac{e^{-\lambda}/2}{e^{-\lambda}/2 + e^{-3\lambda}/2}$$
$$= (1 + e^{-2\lambda})^{-1} = \alpha,\ say$$

$$P(D \mid X = 0) = \frac{P(X = 0 \mid N)P(D)}{P(X = 0 \mid N)P(N) + P(X = 0 \mid D)P(D)} = \frac{e^{-3\lambda}/2}{e^{-\lambda}/2 + e^{-3\lambda}/2}$$
$$= (1 + e^{2\lambda})^{-1} = \beta,\ say$$

Given $\alpha = 3\beta \Rightarrow \lambda = \frac{1}{2}\log_e 3 \approx 0.5493.$ So the average number of accidents in a 24 hour period is $4\lambda = 2\log_e 3 \approx 2.1972.$

8. Let X denote the number of questions asked to Girish (the last one he answers wrongly or fails to answer).

$$P(X = k) = p^{k-1}q, \quad k = 1, 2, ...$$

Given $\sum_{k=0}^{\infty} P(X = 2k+1) = 0.9 \quad \Rightarrow \frac{q}{1-p^2} = 0.9 \;\Rightarrow p = \frac{1}{9}.$

9. Let X be the number of successful hits. Suppose n missiles are fired. Then $X \sim Bin(n, 0.75).$ We want n so that $P(X \geq 3) \geq 0.95.$ This is equivalent to

$\sum_{i=0}^{2} P(X = i) \leq 0.05$, or, $\left(\frac{1}{4}\right)^n + n.\frac{3}{4}.\left(\frac{1}{4}\right)^{n-1} + \binom{n}{2}\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^{n-2} \leq 0.05,$ or,

$10\,(9n^2 - 3n + 2) \leq 4^n.$ The minimum value of n for which this is satisfied is $n = 6.$

10. Let X denote the system life. Then

$$P(X > 27) = 1 - P(X \leq 27) = 1 - \prod_{i=1}^{3} P(X_i \leq 27) = 1 - (1-e^{-9})(1-e^{-3})(1-e^{-1}).$$

$$P(X_1 < 27, X_2 < 27, X_3 \geq 27 \mid X \geq 27)$$
$$= \frac{P(X_1 < 27, X_2 < 27, X_3 \geq 27)}{P(X \geq 27)}$$
$$= \frac{(1-e^{-9})(1-e^{-3})\, e^{-1}}{1-(1-e^{-9})(1-e^{-3})\,(1-e^{-1})}.$$

11. Given $E(X \mid I) = 2 = \frac{r}{\alpha}, Var(X \mid I) = 4 = \frac{r}{\alpha^2}$. So $r = 1, \alpha = \frac{1}{2}.$

Similarly $E(X \mid II) = 4 = \frac{r}{\alpha}, Var(X \mid I) = 8 = \frac{r}{\alpha^2}$. So $r = 2, \alpha = \frac{1}{2}.$

Therefore $P(X > 6 \mid I) = \int_6^{\infty} \frac{1}{2} e^{-x/2} dx = e^{-3}$ and

By theorem of total probability

$$P(X > 6) = P(X > 6 \mid I)P(I) + P(X > 6 \mid II)P(II) = \frac{17}{3}e^{-3} \cong 0.1693.$$

12. $P\left(\text{handle is defective}\right) = 1 - P(6 - 0.3 < X < 6 + 0.3) = 1 - P(-1.5 < \frac{X-6}{0.2} < 1.5)$

$= 2\Phi(-1.5) = 2 \times 0.0667 = 0.1334$

So the percentage of defective handles is 13.34%.

If the percentage of defectives is to be only 5%, then we must have

$1 - P(6 - 0.3 < X < 6 + 0.3) = 0.05$, or, $2\Phi\left(-\frac{0.3}{\sigma}\right) = 0.05$,

or, $\Phi\left(-\frac{0.3}{\sigma}\right) = 0.025 \Rightarrow -\frac{0.3}{\sigma} = -1.96 \Rightarrow \sigma = 0.153$

13. Let X be the marks out of 100. Then $X \sim N(\mu, \sigma^2)$.

$$P(X > 80) = 0.1 \Rightarrow P\left(Z > \frac{80-\mu}{\sigma}\right) = 0.1 \Rightarrow \frac{80-\mu}{\sigma} = 1.28, \text{ or,}$$

$$\mu + 1.28\,\sigma = 80 \qquad \cdots(1)$$

$$P(X < 40) = 0.15 \Rightarrow \Phi\left(\frac{40-\mu}{\sigma}\right) = 0.15 \Rightarrow \frac{40-\mu}{\sigma} = -1.037, \text{ or,}$$

$$\mu - 1.037\,\sigma = 40 \qquad \cdots(2)$$

Solving (1) and (2), we get $\mu = 57.9025, \sigma = 17.2637.$ Now

$$P(\text{grade } B) = P(60 \le X < 80) = P(X < 80) - P(X < 60)$$

$$= 0.9 - \Phi\left(\frac{60-\mu}{\sigma}\right) = 0.9 - \Phi(0.12) = 0.9 - 0.5478 = 0.3522.$$

So percentage of students getting grade ‘B’ is 35.22%.

14. Let X denote the number of defectives in a box. Then $X \sim Bin\,(100, 0.05)$.

Since $n = 100,\ p = 0.05 \Rightarrow \lambda = np = 5.$ Therefore,

$$P(\text{box will fail to meet the guarantee}) = P(X > 3) = 1 - P(X \le 2)$$

$$= 1 - (e^{-5} + 5e^{-5} + \frac{25}{2}e^{-5}) = 1 - \frac{37}{2}e^{-5} = 0.8753.$$

15. Let X denote the number of students joining the Ph.D. program in IIT Kharagpur.

Then $X \sim Bin\,(1000, 0.25)$.

So $P(X > 260) \simeq P\left(\frac{X-250}{\sqrt{187.5}} > \frac{259.5-250}{\sqrt{187.5}}\right) \simeq P(Z > 0.69)$

$= \Phi(-0.69) = 0.2451.$

16. Let X denote the number of fatal accidents over the year.

Then $X \sim \mathrm{P}(180)$

Hence

$$P(X < 100) \simeq P\left(\frac{X-180}{\sqrt{180}} \le \frac{159.5-180}{\sqrt{180}}\right) \simeq P(Z \le -1.53) = 0.063$$

17. $f_Y(y) = \int_0^1 f(x,y)\,dx = \frac{a}{2y^3} + \frac{b}{y^2}, \quad y \ge 1.$ So

$E(Y) = \frac{a}{2}\int_1^\infty \frac{1}{y^2}\,dy + b\int_1^\infty \frac{1}{y}\,dy < \infty \Rightarrow b = 0.$ Therefore, $f_Y(y) = \frac{a}{2y^3}, \quad y \ge 1.$

Now $\int_1^\infty f_Y(y)\,dy = 1 \Rightarrow a = 4.$ Hence, $f_Y(y) = \frac{2}{y^3}, \quad y \ge 1.$ Also

$f_{X,Y}(x,y) = \frac{4x}{y^3},\ 0 \le x \le 1,\ y \ge 1.$ The marginal pdf of X is

$f_X(x) = 2x,\ 0 \le x \le 1.$ Clearly, X and Y are independent. Other parts can now be answered easily.

18. Let X be the number of defective computers in the sample and let Y be the number of partially defective computers in the sample. The joint pmf of (X,Y) is given by

	Y					
		0	1	2	3	$p_X(x)$
X	0	0	3/70	9/70	3/70	15/70
	1	2/70	18/70	18/70	2/70	40/70
	2	3/70	9/70	3/70	0	15/70
	$p_Y(y)$	5/70	30/70	30/70	5/70	

Other parts can now be answered easily.

19. $\mu_1 = 1, \mu_2 = 1, \sigma_1 = 1, \sigma_2 = 1, \rho = 1/3$
The conditional distribution of $X \mid Y = 2$ is $N(4/3, 8/9)$. So
$P(1 < X < 2 \mid Y = 2) = 0.3979.$
Also $2X + 3Y \sim N(5,17)$. So $P\left(4 < 2X + 3Y < 6\right) = 0.1896.$

20. Apply the Jacobian approach to get the required distributions. They are independent. The joint pdf of $\underline{Y} = (Y_1, Y_2, Y_3)$ is

$$f_{\underline{Y}}(\underline{y}) = \frac{1}{2}\lambda^3 y_1^2\, e^{-\lambda y_1},\ y_1 > 0,\ 0 < y_2 < 1,\ 0 < y_3 < 1.$$

The marginals can be found easily now.

21. Let X_1 be the calorie intake at breakfast, X_2 be the calorie intake at lunch and X_3 be the calorie intake at dinner. Let $Y = X_1 + X_2 + X_3$. Then $\mu = E(Y) = 3400, \sigma^2 = Var(Y) = 44900.$ Let Y_i denote the intake on i-th day. Here $n = 365.$ Applying Central Limit Theorem

$$\sqrt{\frac{365}{44900}}\left(\overline{Y} - 3400\right) \xrightarrow{L} Z \sim N(0,1)$$

So $P(\overline{Y} \le 3410) \cong P(Z \le 0.90) = 0.8159.$

22. The distribution of $X_{n+1}/\sqrt{Y}$ is t_n.

23. The distribution of W is χ_n^2.

24. $\hat{\mu}_{ML} = \overline{x} = 7.006,\ \hat{\sigma}^2_{ML} = 0.0054$

25. An unbiased estimator for μ^2 is $\overline{X}^2 - \dfrac{S^2}{n}$. It is consistent.

26. $\hat{\mu}_{MM} = \overline{x} - \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{x}^2},\ \hat{\sigma}_{MM} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \overline{x}^2}$

$\hat{\mu}_{ML} = x_{(1)},\ \hat{\sigma}_{ML} = \overline{x} - x_{(1)}$

The MSEs can be calculated using the form of the distribution.

Problems 27-30: Solutions based on the formulae and methods given in the lectures.