Linear Programming and its Extensions (NPTEL)
Assignment

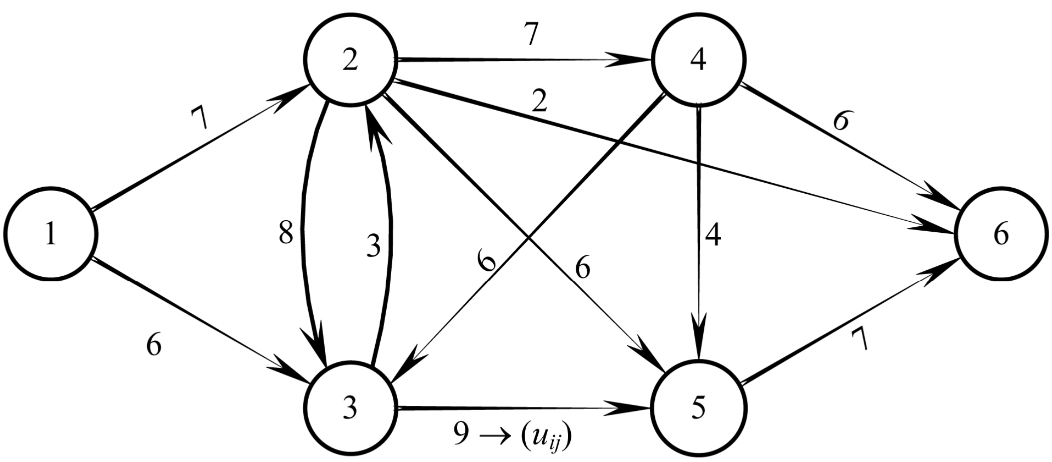
1. Consider a network $N=(V,A,u)$, where $U=\left[u_{ij}\right]$ is the $n\times n$ matrix of arc capacities, $\left|V\right|=n$ *and* $u_{ij}=\infty$ if arc$\left(i,j\right)\notin A$ and $u_{ii}=0\ \forall\ i=1,2,\ldots,n$. Define C_{ij}^{m} = capacity of maximum capacity path from node i to node j, when only nodes 1, 2,.., m-1 are used in the path.

   Note: Capacity of a path is the minimum of the capacities of its arcs.

   Begin with $C_{ij}=0\forall j$ and determine C_{ij}^{m} by the following recursive equation: $C_{ij}^{m+1} = \max(C_{ij}^{m}, \min(C_{im}^{m}, C_{mj}^{m}))$ …(1)

   Prove that equations (1) are valid for determining maximum capacity paths between all pairs of nodes (i,j).

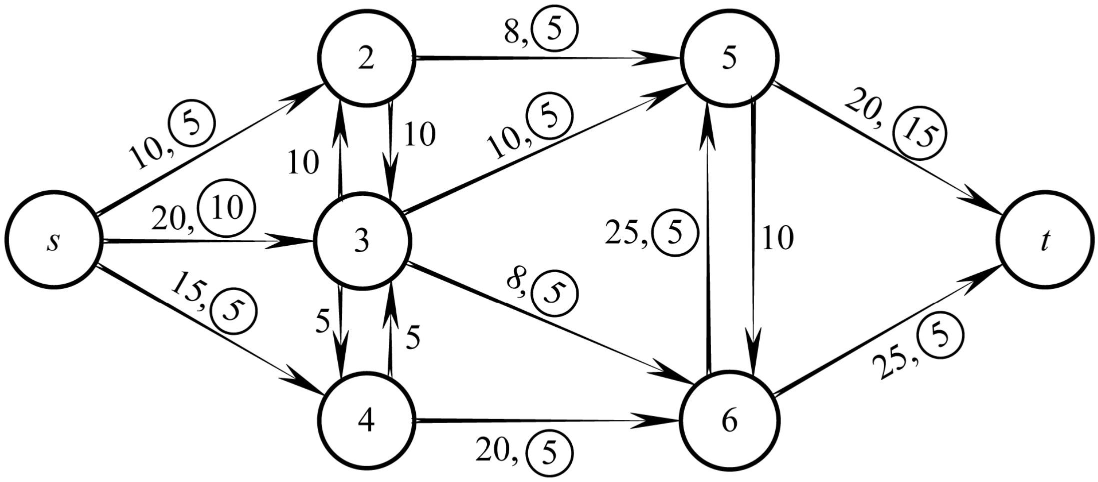
   For the network given below obtain C_{ij}^{7} for all $i,j\in V$ by the method outlined above:



2. Solve the longest path problem, i.e. find longest paths from node 1 to all the other nodes, for the network with are lengths given by the matrix

Nodes	1	2	3	4	5	6	7
1	0	2	3	4	∞	∞	∞
2	∞	0	∞	∞	−1	∞	∞
3	∞	−5	0	3	3	8	∞
4	∞	∞	∞	0	∞	∞	4
5	∞	∞	∞	∞	0	6	∞
6	∞	−6	∞	−7	∞	0	∞
7	∞	∞	∞	∞	∞	3	0

3. Consider the network shown below:

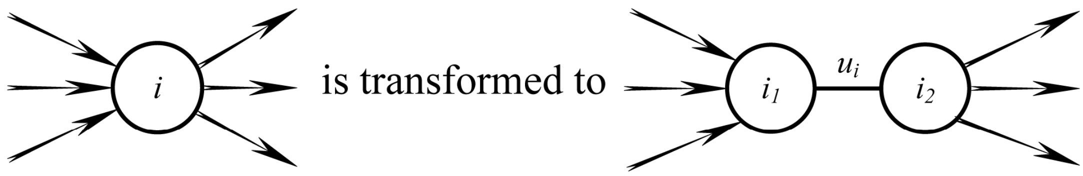


(i). Specify two s-t cuts , one cut with arc(6,5) as a forward arc.Compute net-flow across these cuts and verify that it is equal to the flow in the network.

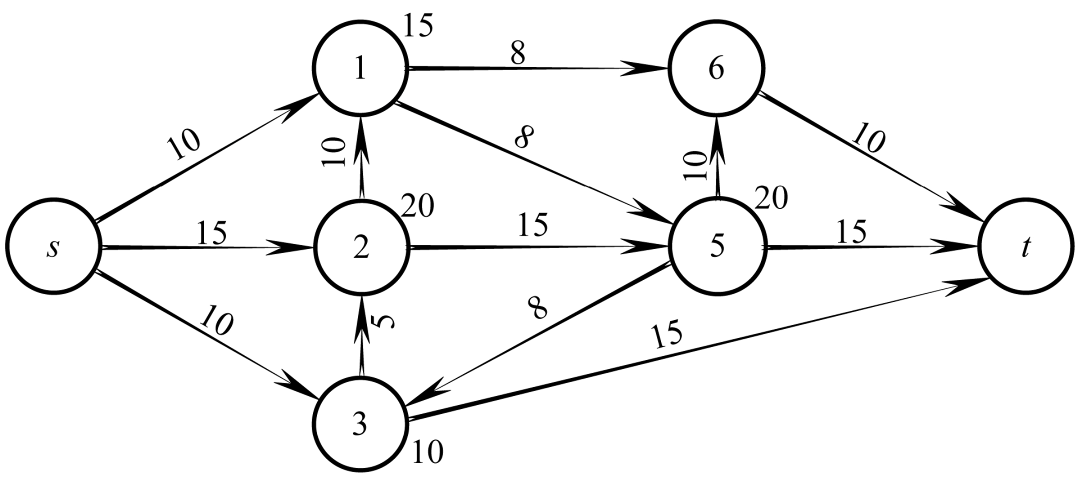
(ii). Find two augmenting paths and augment the flow along them.

4. <u>How to handle node-capacities</u>. Suppose in a max-flow algorithm we have the additional constraint that some nodes cannot handle more than a specified amount of flow, i.e. node i has capacity constraint of u_i units.

Split node i into two nodes i_1 and i_2. Incoming arcs to i are connected to node i_1. Outgoing arcs from i are now going out of node i_2. Capacity of arc(i_1, i_2) is u_i, i.e., the problem gets reduced to a max-flow problem with upper-bound constraints on arcs only.



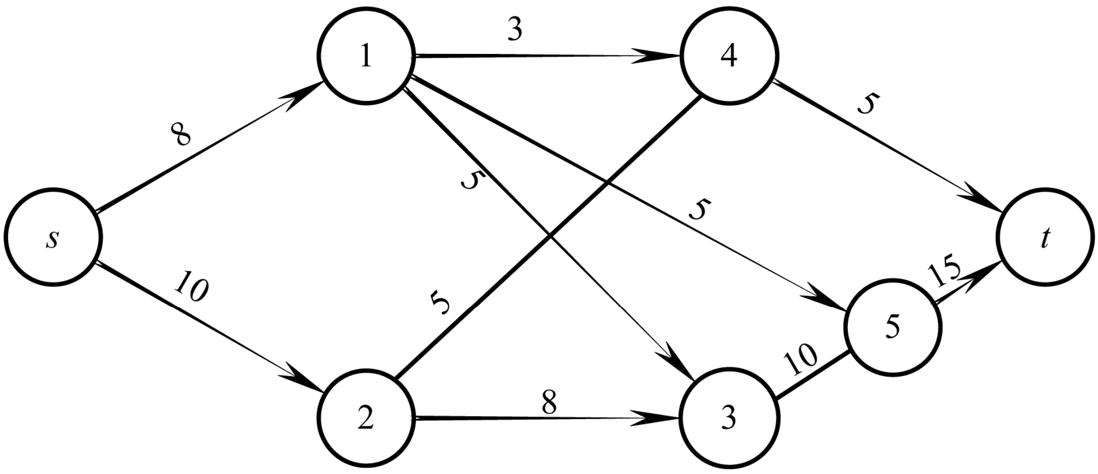
Transform the following max-flow problem with node capacities to a max-flow problem with no node capacities and obtain the max-flow .



Numbers next to the nodes indicate node capacities and nodes s, 6 and t have infinite capacities.

5. <u>Presence of undirected arcs</u>

Suppose edge (i, j) is undirected, with u_{ij} as its capacity. Replace edge (i, j) with two directed arcs (i, j) and (j,i) and $u_{ij} = u_{ji}$. Solve the max-flow problem. If $x_{ij} - x_{ji} > 0$ then replace the edge (i, j) with arc (i, j) and if $x_{ij} - x_{ji} < 0$ replace it by arc (j,i).
Solve the max-flow problem for the network shown below with edges (2,4) and (3,5) undirected.



Refer to the following books for more problems:

1. Linear & Combinatorial Programming by K.G. Murty.
2. Network Flows by R.K. Ahuja, T.L. Magnanti, J.B. Orlin.
3. Linear Programming and Network Flows by M.S. Bajaraa, J.J. Jarvis, H.D. Sherali.