Linear Programming and its Extensions (NPTEL)
Assignment

1. Use of complimentary slackness conditions for the transportation problem.
   Consider the following balanced transportation problem.

Markets

		1	2	3	4	5	S_i
Sources	1	11	10	3	2	-7	17
	2	6	9	10	11	12	7
	3	3	0	6	7	7	9
	4	-4	-1	2	2	4	7
	D_j	10	12	7	3	8	40

$\bar{u}=(-7,8,3,0)$ and $\bar{v}=(-4,-3,2,1,4)$ is a dual feasible solution.
Use complementary slackness conditions to obtain an optimum basic feasible solution for the above problem and its dual. If there are alternate optimum solutions, find them all.

2. In the transportation problem tableau given below a feasible solution is indicated.

   (i) Obtain a basic feasible solution from it.

	Markets						
	1	2	3	4	5	6	S_i
Sources	2	3 (7)	6	7 (3)	11	6 (3)	13
	0 (4)	8	5 (5)	12	1 (6)	10	15
	5	8	3 (2)	4 (5)	15	4 (3)	10
	0	1 (2)	3 (3)	10	4 (4)	11	9
D_j	4	9	10	8	10	6	

(ii) Check if the basic feasible solution obtained is optimal.

3. Formulate the following unbalanced transportation problem as balanced transportation problems

(i)

		Markets					
		1	2	3	4	5	S_i (Maximum Amount available)
Sources	1	6	-4	9	3	11	5
	2	1	8	3	1	-6	7
	3	-3	-10	7	5	7	9
	4	8	1	13	15	4	18
	D_j	1	13	3	2	18	

(ii) Markets

		1	2	3	4	5	S_i (Maximum Amount available)
Sources	1	6	-4	9	3	11	5
	2	1	8	3	1	-6	7
	3	-3	-10	7	5	7	9
	4	8	1	13	15	4	18
D_j (Minimum requirement)		1	13	3	2	18	

For (i) verify that the basic set

$\{(1,2),(1,4),(2,3),(2,5),(3,2),(4,2),(4,5)\}$ constitutes an optimum solution when the supply at source 4, $S_4 = 6$.

Obtain an optimum solution for the given problem.

For (ii) Consider the basic set:
$\{(1,1),(1,3),(2,3),(2,4),(3,1),(3,5),(4,2),(4,3),(4,6)\}$

Using the basic set above obtain a basic feasible solution $\{\bar{x}_{ij}\}$ for the balanced transportation problem, obtained by adding 6th market with costs as $C_{i,\,n+1}$, $i=1,\cdots m$ and $D_6 = 2$.
For each $i=1$ to m, let $C_{i,\,n+1} = \min\{0, C_{i1}, C_{i2}, \cdots C_{in}\}$.
For each i such that $C_{i,\,n+1} < 0$, let $j = r_i$ satisfy

$$C_{i,\,n+1} = \min\{C_{i1}, C_{i2}, \cdots C_{in}\} = C_{i r_i}.$$

Now construct the solution $\left\{\hat{x}_{ij}\right\}$ as follows:

$\hat{x}_{ij} = \bar{x}_{ij}$ for all i such that $C_{i,\, n+1} \geq 0$.

$= \bar{x}_{ij}$ for all i such that $C_{i,\, n+1} < 0, \quad j \neq r_i$

$= \bar{x}_{ij} + \bar{x}_{i,\, n+1}$ for all i such that $C_{i,\, n+1} < 0, \quad j = r_i$

Show that $\left\{\hat{x}_{ij}\right\}$ is optimal for the original unbalanced problem.

4. <u>Transportation Paradox</u>. Modified Version of problem 11-36 (from K G Murty)

Let $\{S\}, \{D\}$ and $\{\hat{S}\}, \{\hat{D}\}$ denote two sets of supply and Demand vectors for the balanced transportation problem with m supply points and n demand points,

$$\text{i.e. } S = \begin{pmatrix} S_1 \\ \vdots \\ S_m \end{pmatrix}, \quad \hat{S} = \begin{pmatrix} \hat{S}_1 \\ \vdots \\ \hat{S}_m \end{pmatrix}, \quad D = \begin{pmatrix} D_1 \\ \vdots \\ D_n \end{pmatrix} \ \textit{and}\ \hat{D} = \begin{pmatrix} \hat{D}_1 \\ \vdots \\ \hat{D}_n \end{pmatrix}$$

Let $S \geq \hat{S}$ and $D \geq \hat{D}$.

Let $Z(S, D)$ denote the objective function value for the transportation problem with S as the supply vector and D as the demand vector. If $C_{ij} \geq 0 \ \forall (I_{ij})$, one would expect that $Z(\hat{S}, \hat{D}) \leq Z(S, D)$ i.e. with all the costs non-negative one would expect that higher the volume of goods transported, higher is the cost. But this need not be true as demonstrated by the following example.

Unit Transportation costs. (Rs/Ton)

Sources	Markets 1	2	3	4	5	Supplies (Tons)
1	30	11	5	35	8	30
2	2	5	2	5	1	$10+\delta$
3	35	20	6	40	8	35
4	19	2	4	30	10	30
Demands (Tons)	25	$25+\delta$	16	12	27	

Table given below is optimal for $0 \leq \delta \leq 2$

	$20+\delta$	8	$2-\delta$	
			$10-\delta$	
		8		27
25	5			

Optimal Cost = $1129-19\delta$.
This cost decreases as δ increases from 0 to 2.

Obtain an optimal solution when $\delta > 2$. Show that for $2 \leq \delta \leq 27$, the cost continues to decrease as δ increases. This is known as the Transportation Paradox.

5. Apply Bland's anti-cycling algorithm to the transportation problem from Thapa & Dantzig discussed in the lecture, that showed cycling phenomenon. Compare the number of iterations with that of the ε - perturbation method discussed in the lecture.