

**CS 6210: Perf. Evaluation of Comp. Systems**

**Final Exam: Nov. 18, 2011, 2:00 PM - 5:00 PM, Prof. Krishna Sivalingam**

*Closed Book and Closed Notes; No Cell Phone; ONLY Scientific Calculator; No Sharing of Calculator; No discussions with students enrolled in class.*

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Answer the following **EIGHT** questions. Include all calculations and mathematical expression derivations in your answer sheets, for complete credit. Please write legibly. **If you use additional sheets, please write your name and roll number on the additional sheets AND Staple the booklets.**

If you do not present your IIT Madras ID during the Quiz, you have to present it to me within ONE hour of the Quiz; else, the Quiz will not be considered for grading.

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1. (15) Random Variables:

- (a) (10) Let  $X_1, X_2, \dots, X_n$  be mutually independent exponentially distributed random variables with parameters  $\mu_1, \mu_2, \dots, \mu_n$  ( $\mu_i \neq \mu_j, i \neq j$ ) respectively. If

$$X = \sum_{i=1}^n X_i$$

Derive the Laplace transform of  $X$  and using the Laplace transform, derive its mean and variance.

- (b) (5) Jobs arriving to a compute server have been found to require CPU time that can be modeled by an exponential distribution with parameter  $\frac{1}{140} \text{ ms}^{-1}$ . A round-robin scheduler with time quantum of 100 ms is used. A job that does not finish within a given time quantum will be routed back to the tail of the waiting jobs queue (i.e. Ready Queue). Find the probability that an arriving job is forced to wait for the second quantum. Out of 800 jobs arriving in a day, how many are expected to finish within the first time quantum?

2. (15) Markov Chains: Consider the slotted Aloha MAC protocol used for a channel shared by  $N$  nodes. Let  $\lambda$  denote the Poisson packet arrival rate per slot, to a node's transmitter queue. Let  $P_b$  denote the retransmission probability of a backlogged node in a given slot. Assume that each node's transmit buffer can hold at most **THREE** packets, including the one for which transmission is being attempted. Use a Markov model to represent the node's behavior. Using this model, explain how we can determine the system throughput and average packet delay.

You can state any other simplifying assumptions as needed (e.g. the probability of more than one packet arriving in a slot is negligibly small.)

3. (15) Queuing Theory:

- (a) (8) Consider an  $M/M/\infty$  queuing system with an infinite number of servers, with parameters  $\lambda$  and  $\mu$ . In such a system, all arriving jobs being receiving service immediately. The service rate is  $n\mu$  when there are  $n$  jobs in the system.

Draw the birth-death process model for this system. Derive the expressions for  $p_n$ ,  $E[n]$ ,  $Var[n]$  and  $E[r]$ . Is the system always stable?

- (b) (7) Consider an IP router's output queue to which packets are arriving at the Poisson rate of 12,500 packets/second; Packet lengths are exponentially distributed with average length of 1,000 bytes. The packets are served by a set of three parallel transmission links sharing a common queue, with each link operating at 100 Mbps. Assume that the queue capacity is infinite. Determine the total mean packet delay.

4. (15) Convolution: Consider a closed time-sharing system with a CPU and 3 storage disks (A, B and G) and a set of  $N = 4$  jobs circulating in the system. The disk service times per request were 25, 16 and 30 milliseconds respectively for A, B and G; the average service time per CPU visit was 12 milliseconds. Each job requested disk A **four** times, disk B **five** times and disk G **ten** times. The think time  $Z = 1$  second. Using the convolution algorithm, determine the system throughput ( $X(N)$ ) and system response time ( $R(N)$ ), and device utilizations.

For what value of  $N = N^*$ , do you expect queuing to appear in the system.

5. (15) MVA: Consider a closed central sub-system with a CPU and 3 storage disks (A, B and G) and a set of  $N = 3$  jobs circulating in the system. The disk service times per request were 25, 15 and 30 milliseconds respectively for A, B and G; the average service time per CPU visit was 12 milliseconds. Each job requested disk A **four** times, disk B **five** times and disk G **ten** times. Assume that disk B is the designated network. For the "shorted" network, determine the system throughput ( $X(N)$ ), system response time ( $R(N)$ ), and the per-device throughputs using exact MVA algorithm.

6. (15) Comparing Systems:

- (a) (8) Consider two systems, A and B, for which the mean time to failures is measured over several failures. For System A, 972 failures were recorded with mean time between failures being 124.10 and the standard deviation being 198.20; For System B, 153 failures were recorded with mean time between failures being 141.47 and the standard deviation being 226.11. Which system is better (with 95% confidence) using the  $t$ -test? Present all details that support your answer. Ask the TA for the desired  $t$  value when you are ready.

- (b) (3) Sachin has got out for a "duck" in 14 out of the 301 Test match innings that he has played. If we would like to compute his "duck" scoring proportion with a half-width accuracy of 10% at the 90% confidence level, how many more innings should be play? Do you think it is really feasible?

- (c) (4) Two algorithms are run on the same set of 10 datafiles and the respective mean runtimes are 10.34 and 23.90 milliseconds (with corresponding standard deviation of 5.6 and 2.8 milliseconds). If we would like to state that the first algorithm is better than the second with 95% confidence, how many datafiles should be compared?

7. (15)  $2^k$  Factorial Designs

- (a) (5) Provide that for a  $2^2$  factorial design:  $SST = SSA + SSB + SSAB$ .
- (b) (10) Consider the results of a performance study where network throughput (MB) is measured varying Buffer Size (KB), Packet Size (Bytes) and Scheduling Algorithm (FCFS, WFQ).
- 4KB Buffer, Packet size 1024, FCFS: 11
  - 4KB Buffer, Packet size 4096, FCFS: 18
  - 8KB Buffer, Packet size 1024, FCFS: 23
  - 8KB Buffer, Packet size 4096, FCFS: 38
  - 4KB Buffer, Packet size 1024, WFQ: 17
  - 4KB Buffer, Packet size 4096, WFQ: 22
  - 8KB Buffer, Packet size 1024, WFQ: 29
  - 8KB Buffer, Packet size 4096, WFQ: 45

Determine the proportion of variation that can be explained by the different effects.

8. (15) Miscellaneous:

- (a) (8) Derive the “Utilization Law” and the “Forced Flow Law” from the basic definitions.
- (b) (7) Consider an experiment with the first set of measurements as follows: 6, 9, 8, 7, 8, 7, 9, 8, 8, 9. Using the initial data deletion method, determine the length of the transient interval  $l$ .

Random hint: If you are trying to determine  $Q$ ,  $P(n_X = j)$ , etc., are you sure these are needed?

Random Hint: The convolution theorem: the MGF of a sum of independent variables is the product of the individual MGFs.

Random values:

**Unit Normal Variate Quantiles**

p	z
0.995	2.576
0.990	2.326
0.975	1.960
0.95	1.645