CS 6210: Performance Evaluation of Computer Systems; Aug. 2011, Prof. Krishna Sivalingam
Sample Exercises: Set 3
Please try to solve these by yourself or in groups. You are always welcome to contact the instructors or TAs for clarifications on whether your approach/solution is correct. You will learn better by attempting to solve these, rather than simply going through the solutions.

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1. Consider a tandem system of three M/M/1 queues with arrival rate of λ and respective service rates of μ_1 and μ_2. From the basic product-form joint probability distribution property, i.e.

   $P(n_1, n_2) = (1 - \rho_1)\rho_1^{n_1}(1 - \rho_2)\rho_2^{n_2}$

   determine the expression for $E[n]$, i.e. the total number of customers in the system.

2. Consider the M/G/1 system with the dierence that before the first customer in each busy period starts service, it takes some time for restart the server from idle state. Let this setup time be represented by the random variable Δ, which has a given general distribution and is independent of all other random variables in the model. Let $\rho = \lambda\overline{X}$ be the utilization factor. Further, assume that while setup is in progress the customer waits in queue. Show using Residual Lifetime analysis that:

   - P(the server is idle) is
     $$\frac{(1-\rho)}{(1+\lambda\overline{\Delta})}$$
     (Note that setting up service (routes) still implies server is busy, though it is not serving customers yet).
   - P(the server is in setup phase) is
     $$\frac{(1-\rho)\lambda\overline{\Delta}}{(1+\lambda\overline{\Delta})}$$
   - Average length of the busy period is
     $$\frac{(\overline{X}+\overline{\Delta})}{(1-\rho)}$$
   - Determine the average waiting time in queue, $E[w]$.

3. Raj Jain's book, Questions 25.2, 33.2, 33.3, 33.5, 33.7

4. Consider a simulation run that is being terminated using the "Batch Means" method. The ratio of the 95% confidence interval width to the mean value of the computed metric should be less than 0.05. Given that ten batches have been computed and the respective7 means are: 8, 9, 4, 7, 6, 7, 9, 8, 8, 10, should the simulation stop or continue further?

   Note: for $n < 30$ samples, the Student's t-distribution table value of $t_{1-\alpha/2;n-1}$ should be used, where the second term in the subscript denotes the degrees of freedom. Please consult Table A.4 in Jain's book. Microsoft Excel provides the CONFIDENCE and CONFIDENCE.T functions, fyi.