# NPTEL Course on Numerical Optimization

## Module 8 : Duality

### Practice Problems

1. Find the Lagrangian saddle points of the problem:

$$\begin{array}{ll} \min & x^2 \\ \text{s.t.} & 0 \leq x \leq 1 \end{array}$$

2. Write the dual of the problem:

$$\begin{array}{ll} \max & f(\boldsymbol{x}) \\ \text{s.t.} & h_j(\boldsymbol{x}) \leq 0 \\ & \mathbf{A}\boldsymbol{x} = \mathbf{b} \\ & \boldsymbol{x} \geq \mathbf{0} \end{array}$$

where $f$ and $-h_j$ are concave functions and $\mathbf{A} \in \mathbb{R}^{m \times n}$.

3. Find the global maximum of the dual function of the following problem:

$$\begin{array}{ll} \min & -x_1 x_2 \\ \text{s.t.} & (x_1 - 3)^2 + x_2^2 = 5 \end{array}$$

4. Give an example of a convex programming problem where Slater's condition does not hold and there is no duality gap at optimality.

5. Solve the following problem by writing its dual and solving it.

$$\begin{array}{ll} \min & x_1^2 + x_2^2 \\ \text{s.t.} & x_1 + x_2 \geq 1 \\ & (x_1, x_2)^T \in \mathbb{R}^2 \end{array}$$

6. Write the dual of the following quadratic programming problem:

$$\begin{array}{ll} \min & \boldsymbol{x}^T \boldsymbol{H} \boldsymbol{x} + \boldsymbol{c}^T \boldsymbol{x} \\ \text{s.t.} & \boldsymbol{H}\boldsymbol{x} + \boldsymbol{c} \geq \mathbf{0} \\ & \boldsymbol{x} \geq \mathbf{0} \end{array}$$

where $\boldsymbol{H}$ is a symmetric and positive semi-definite matrix.