

# NPTEL Course on Numerical Optimization

## Module 2 : Mathematical Background

### Practice Problems

1. Are the three vectors  $(1, 2)^T$ ,  $(3, 4)^T$  and  $(5, 6)^T$  linearly independent? Justify your answer.
2. Find the values of  $x$  for which the following matrix is not positive definite.

$$\begin{pmatrix} e^x & e^{-x} \\ e^{-x} & e^{-2x} \end{pmatrix}$$

3. Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by  $f(x_1, x_2) = x_1^2 + x_1x_2$ , where  $x_1 = \sin t_1 + t_2^2$  and  $x_2 = (t_1 + t_2)^2$ . Define  $h(t) = f(x(t))$ . Find  $\nabla h(t)$  and  $\nabla^2 h(t)$ .
4. Solve the following linear system using LU factorization:

$$\begin{pmatrix} 1 & 2 & -1 \\ 3 & 8 & -4 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 9 \\ -1 \end{pmatrix}$$

5. Write the second order truncated Taylor series for the function  $\cos(\frac{1}{x})$  around a nonzero point  $x$  and evaluate it for the case when  $x = 1$ .