NPTEL Course on Numerical Optimization
Module 1 : Introduction
Practice Problems

1. Sketch the contours of the functions,

    (a) $f(\mathbf{x}) = x_1^2 + 4x_2^2 - 4x_1 - 8x_2$

    (b) $f(\mathbf{x}) = 8x_1 + 12x_2 + x_1^2 - 2x_2^2$

    Deduce the value $\mathbf{x}^*$ which minimizes/maximizes f.

2. Find the point (x, y) on the graph of $y = \sqrt{x}$ nearest to the point $(4, 0)$. Formulate this as a constrained optimization problem.

3. Let $S = \{\mathbf{x} : x_1^2 + x_2^2 \leq 1, x_1 - x_2^2 \geq 0\}$ and $\mathbf{y} = (\frac{1}{2}, 5)^T$. We want to find a point $\mathbf{x}^*$ in the set S which is "closest" to $\mathbf{y}$. Formulate this as a constrained minimization problem. Also, solve this problem graphically.

4. An isosceles triangle with largest area is to be inscribed in a circle of radius 10 cm. Formulate this as a constrained maximization problem.

5. Of all the triangles with a given perimeter, we want to find the one with maximum area. Use arithmetic mean-geometric mean inequality to solve this problem. Also, formulate this as a constrained maximization problem.

6. A publisher has orders for 400 and 200 copies of a certain book from the places P_1 and P_2 respectively. The company has 500 and 300 copies of this book available at warehouses W_1 and W_2 respectively. It costs $Rs.30$ to ship a book from W_1 to P_1, but it costs $Rs.50$ more to to ship it to P_2. It costs $Rs.50$ to ship a book from W_2 to P_1, but it costs $Rs.20$ to ship it to P_2. How many copies of the book should the company ship from each warehouse to the places P_1 and P_2 to fill the order at the least cost? Formulate this as an optimization problem.