Graph Theory: Lecture No. 40

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Kühn and Osthus: For every integer s, there is an integer r_s such that Hadwiger Conjecture holds for all graphs $G \not\supseteq K_{s,s}$ and $r \geq r_s$.

There is a constant g such that all graphs G of girth at least g satisfy the implication $\chi(G) \geq r \rightarrow G \supseteq TK_r$ for all r.

There is a constant $c \in R$ such that for $r \in N$, every graph G of average degree $d(G) \geq cr^2$ contains K_r as a topological minor.

Kostochka, 1982: There exists a constant $c \in R$ such that for every $r \in N$, every graph G of average degree $d(G) \geq cr\sqrt{\log r}$ contains K_r as a minor. Up to the value of c, this bound is best possible as a function of r.

Let $d, k \in N$ **with** $d \geq 3$ **and let** G **be a graph of minimum degree** $\delta(G) \geq d$ **and girth** $g(G) \geq 8k + 3$**. Then** G **has a minor** H **of minimum degree** $\delta(H) \geq d(d-1)^k$**.**

Thomassen, 1983: There exists a function $f : N \to N$, such that every graph of minimum degree at least 3 and girth at least $f(r)$ has a K_r minor, for all $r \in N$.

Take $f(r) = 8 \log r + 4 \log \log r + c$ **for some constant** $c \in R$**. Take** $k = k(r)$ **minimal with** $3.2^k \geq c' r \sqrt{\log r}$**, where** c' **is the constant from Kostochka's Lemma.**

There exists a constant g such that $G \supseteq TK_r$ for every graph G satisfying $\delta(G) \geq r - 1$ and girth $\geq g$.

Let G be a graph, T a tree, and let $\nu = (V_t)_{t \in T}$ be a family of vertex sets $V_t \subseteq V(G)$ indexed by the vertices t of T. The pair (T, ν) is called a tree decomposition of G if it satisfies the following 3 conditions.

1. $V(G) = \cup_{t \in T} V_t$
2. **for every edge $e \in G$, there exists a $t \in T$, such that both ends of e lie in V_t.**
3. $V_{t_1} \cap V_{t_2} \subseteq V_{t_3}$ **whenever t_3 belongs to the unique path between t_1 and t_2 in T.**

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