$\chi(G) \geq r$, then G has

a K_r minor.

a Planar graph which needs

5 colors to color —

$\chi(G) \geq 5 \longrightarrow \boxed{K_5 \text{ minor}}$

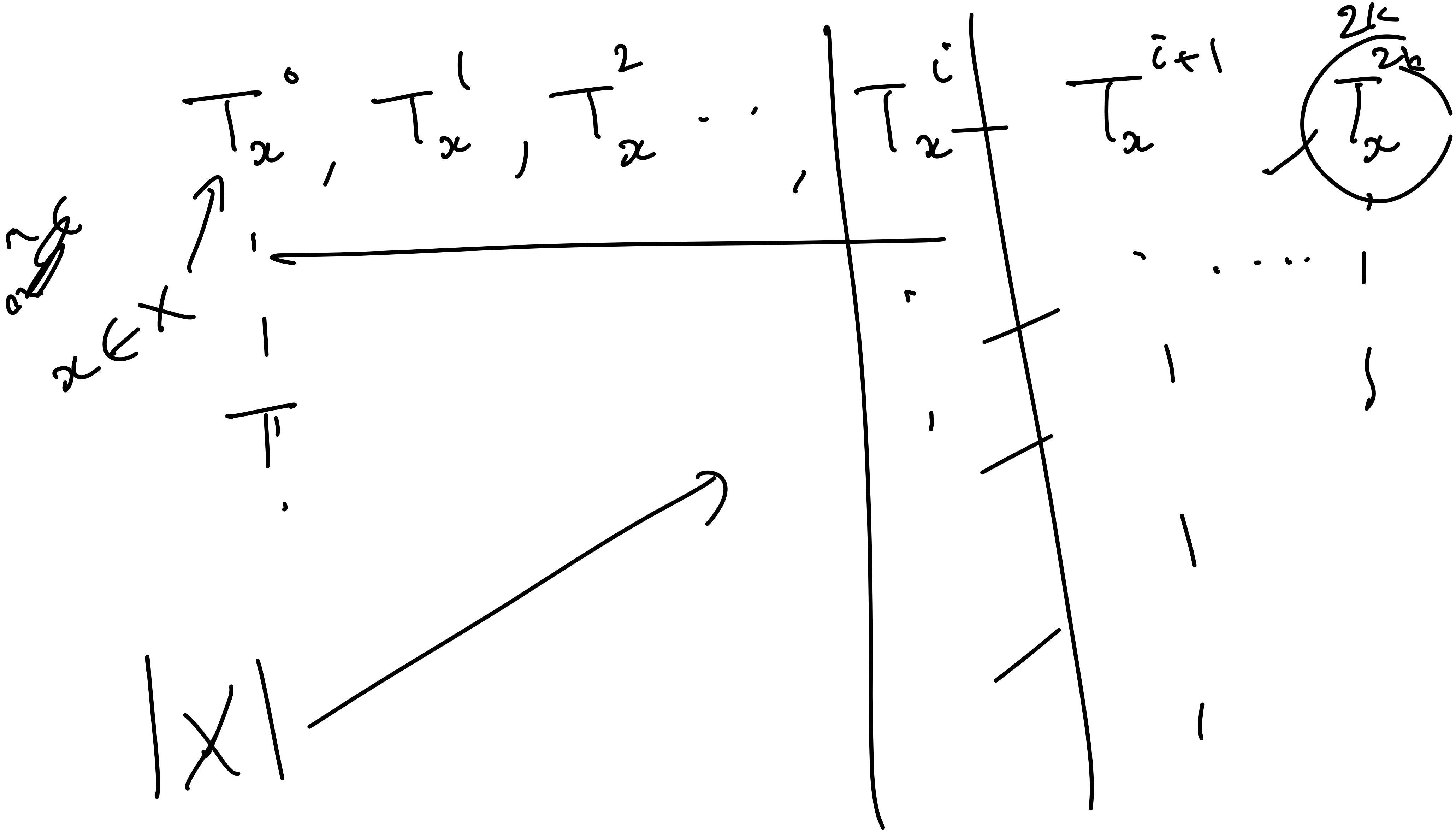
Hajós conjecture.

G, $\left.\begin{array}{l}\text{girth} \geq 8k+3 \\ \text{min. degree} \geq d\end{array}\right\}$

$X = \{ \quad \}$

$u, v \in X$, $\text{dist}(u,v) > 2k$

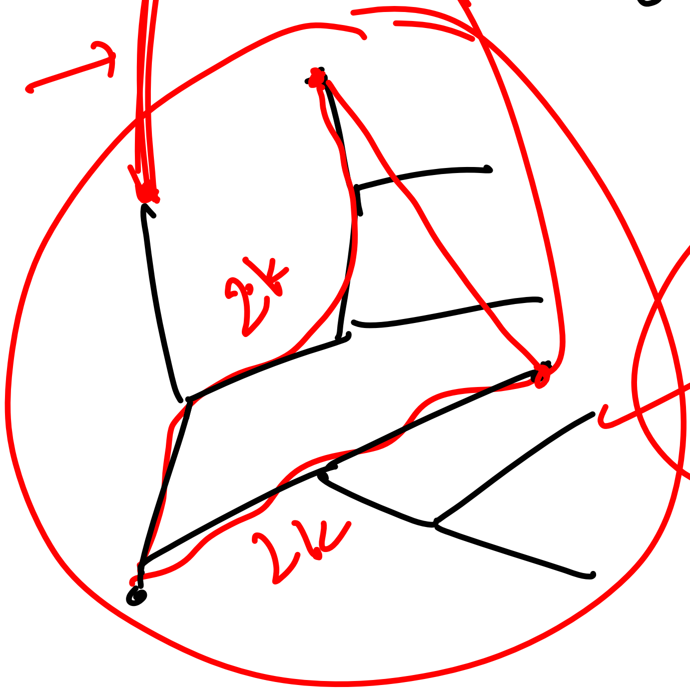

x
$2x$
T_x^0
T_x^1
y
T_y^0
T_y^1
T_z^0
T_z^1


$T_x^0, T_x^1, T_x^2, \ldots, T_x^i$
T_x^{i+1}
T_x^{2k}
$x \in X$
T'
$|X|$

$G \to \left\{ \cancel{T^{2k}_{x_1}},\ \cancel{T^{2k}_{x_2}},\ \dots,\ \cancel{T^{2k}_{x_t}} \right\} \to \cancel{H}$

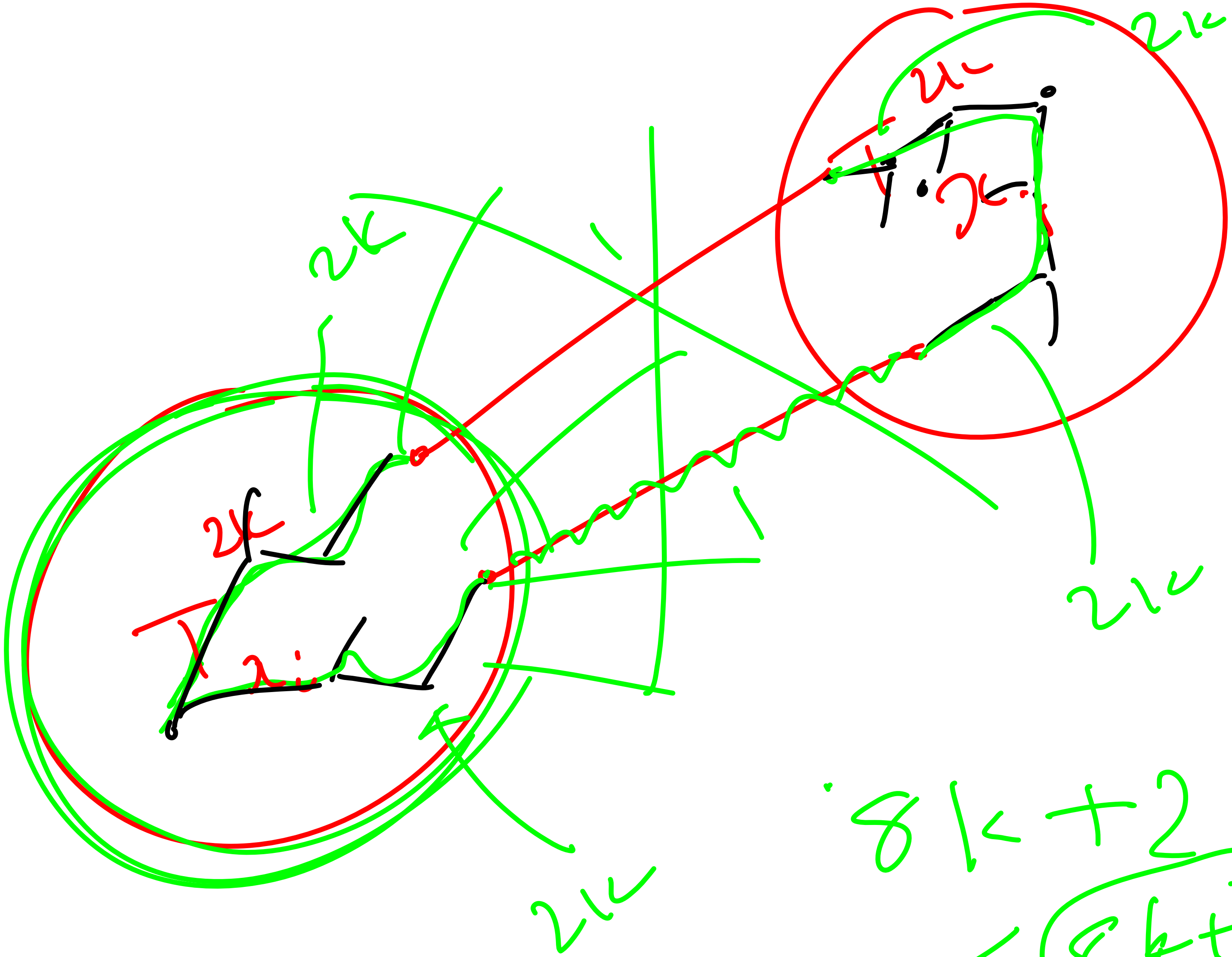
$$X = \overbrace{\{x_1, x_2, \dots, x_t\}}^{\geq d(d-1)^k}$$

$$t = |X|$$



$\cancel{4k+1}$

$8k+3$

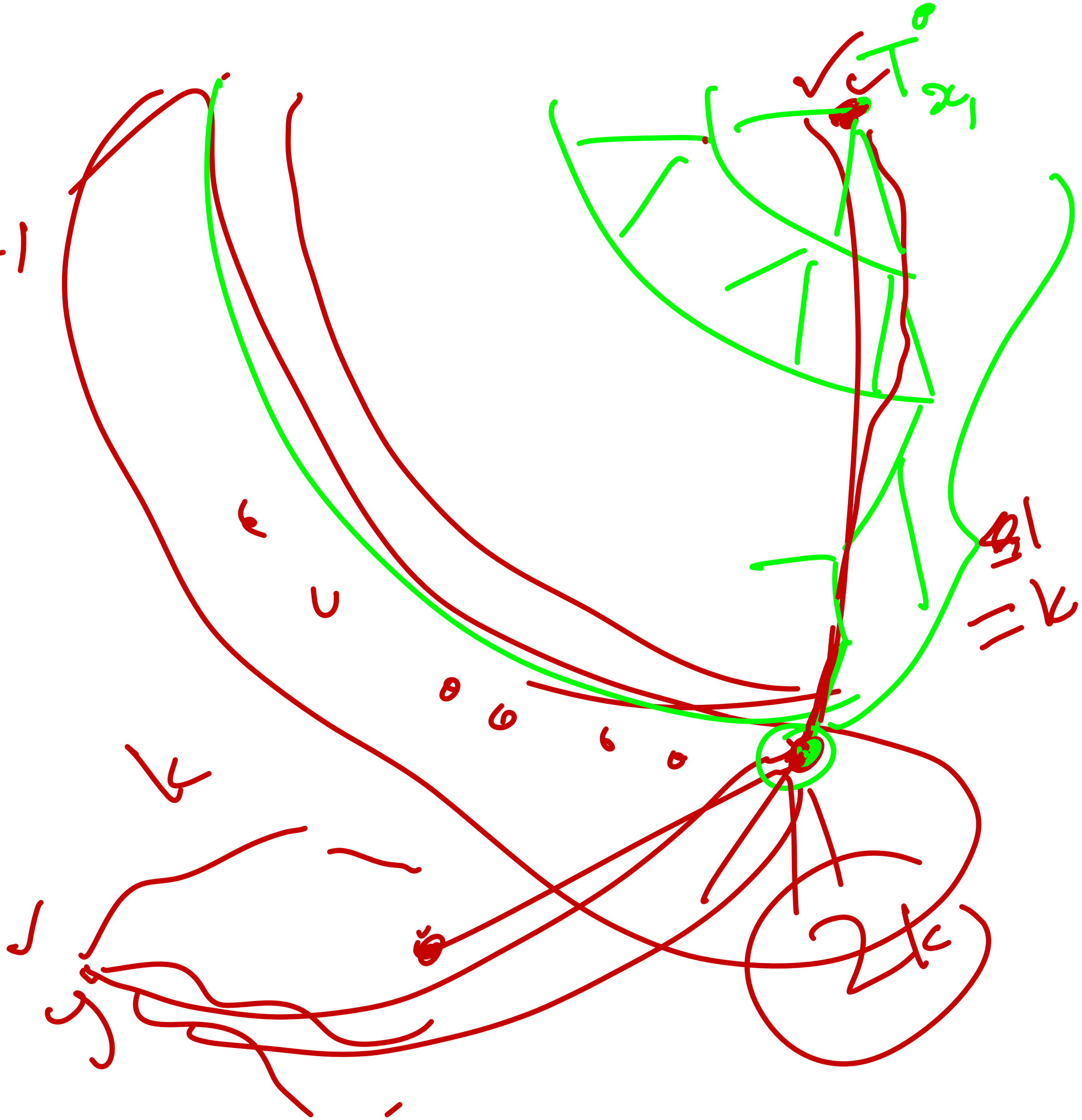


$8k+2 < 8k+3$

$$T_x^{2k}$$

$$d\,(d-1)^{k-1}$$

$$\underbrace{T_x^{k-1}}$$



$k = k(\gamma)$

$$3 \cdot 2^{\boxed{k}} \geq c\, \gamma \sqrt{\log \gamma}$$

$$\delta(H) \geq 3 \cdot 2^{k}$$

T_k ||

$d\,(d-1)$

K_γ

girth $\geq 8k+3$

k

$$\cancel{3}\; 2^{k} \geq \frac{c\, r \sqrt{\log r}}{3}$$

$$k \geq \log r + \frac{1}{2} \log\log r + \log\left(\frac{c}{3}\right)$$

$$8k + 3 \leq 8 \log r + 4 \log\log r + c' = f(r)$$

$\{K_5, K_{3,3}\}$

$K_5, K_{3,3}$

K_5 or $K_{3,3}$

$\{\ ,\ ,\ ,\ \}$

Tree decomposition

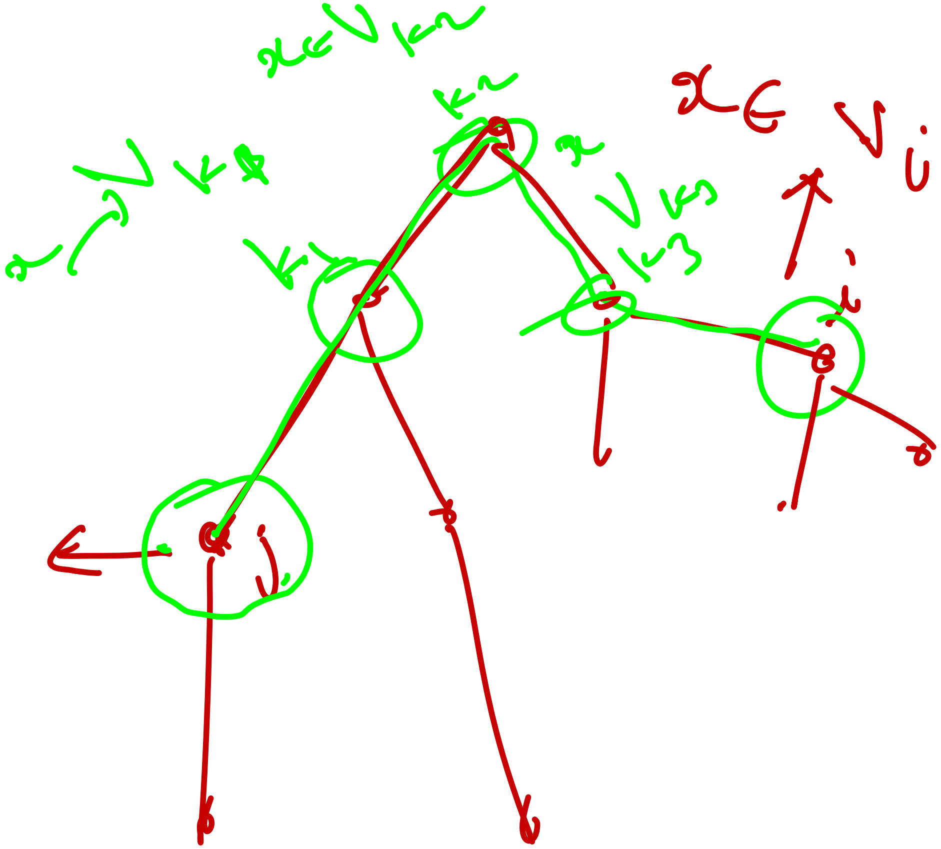
(1) $\bigcup V_i = \underline{V(G)}$

(2) $(u,v) \in E(G)$, there should be some i, $u, v \in V_i$ ✓

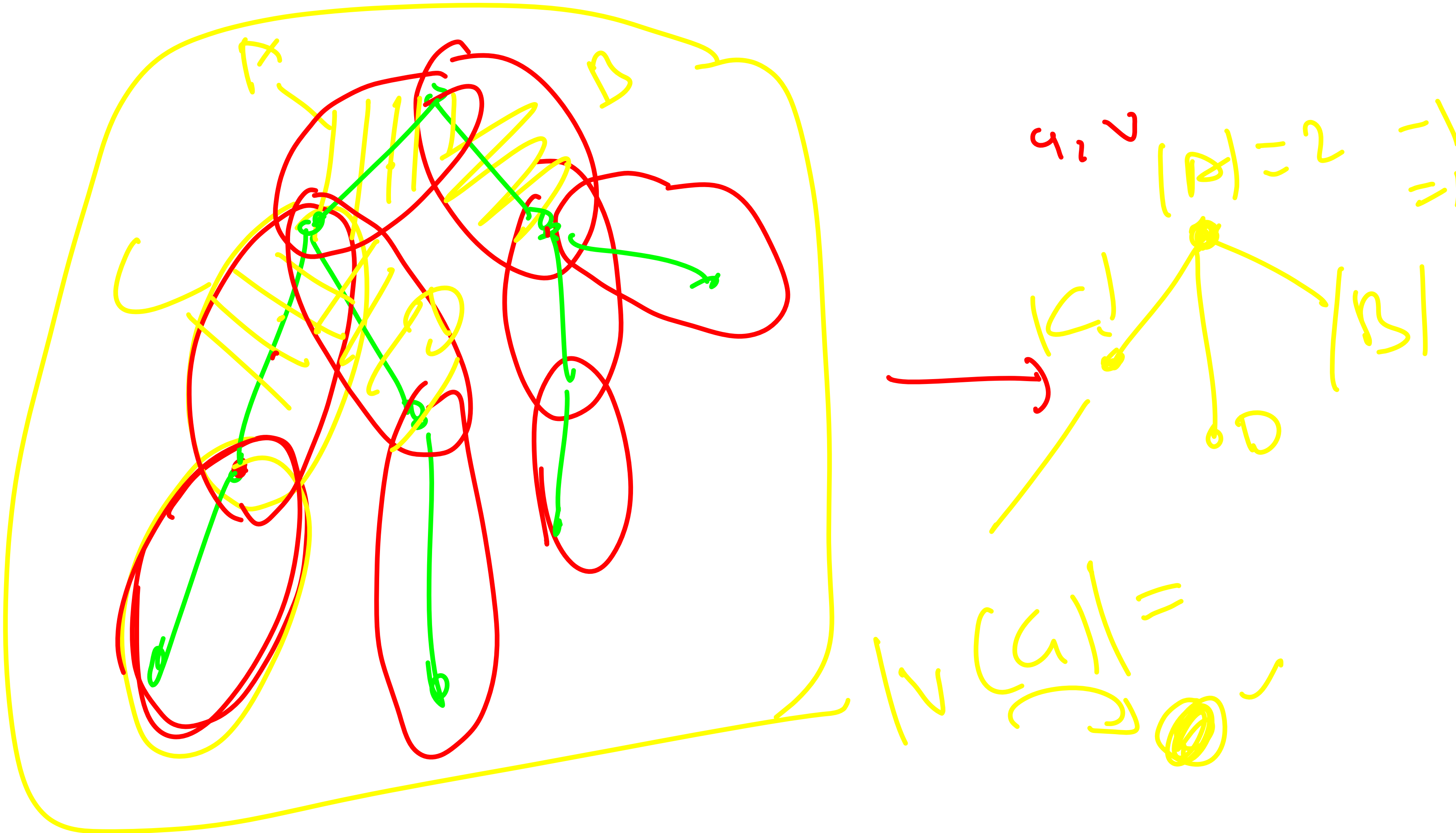
(3)

$x \in V_j$

$x \in V(G)$



$\boxed{V_j \cap V_i}$



A
D
u, v
|A| = 2
|C|
|B|
D
|V(G)| =