INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:
Modeling Techniques and Analysis

Lecture - 39

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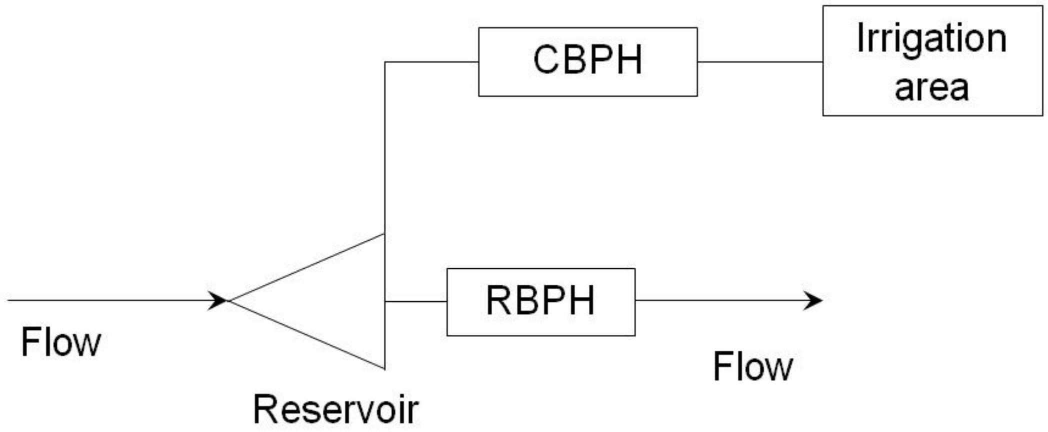
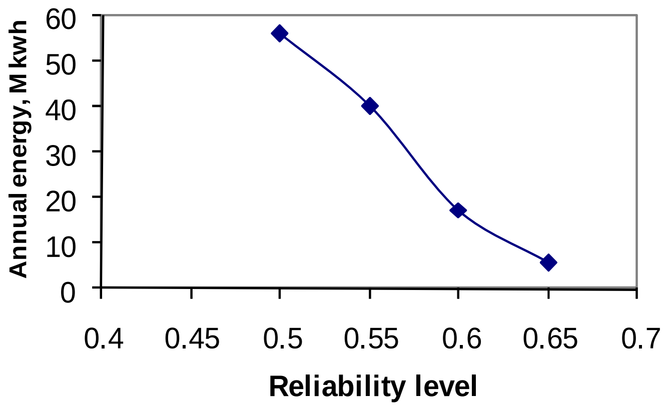
Summary of the previous lecture

Hydro power optimization:



CBPH: Canal Bed Powerhouse

RBPH: Riverbed Powerhouse



CROP YIELD OPTIMIZATION

Crop Yield Optimization

- The objective of an irrigation water management problem is to maximise the crop production, taking into account the response of the crop to the amount of irrigation applied.
- Typical crop production functions relate the yield ratio of a crop to a function of the evapotranspiration ratio over the growth stages.

Crop Yield Optimization

- Water allocation among crops

Influencing factors :Crop Areas, Rainfall, Soil Moisture, Irrigation Supply, Time of the year, PET, Crop sensitivity to deficit supply, Competition among crops for water.

Time →

Crop1	q_{11}				q_{51}
Crop2	q_{12}				q_{52}
Crop n	q_{1n}				q_{5n}
Supply:	Q1	Q2	Q3	Q4	Q5

Optimal allocation of a known quantity of water to maximize crop yield

Crop Yield Optimization

Crop production function

- –System performance measure
- –Response of the crop yield to water supply during intraseasonal periods.
- –A function of evapotranspiration deficits occurring during the growth stages of the crop.

Ky: Crop yield factor

$$\left(\frac{y}{y_m}\right)_c = \prod_{s=1}^{NS}\left[1 - ky_s^c\left(1 - \frac{ET_a^c}{ET_{max}^c}\right)_s\right]$$

….. Multiplicative production function

s=1	s=2	s=3	s=4	s=5

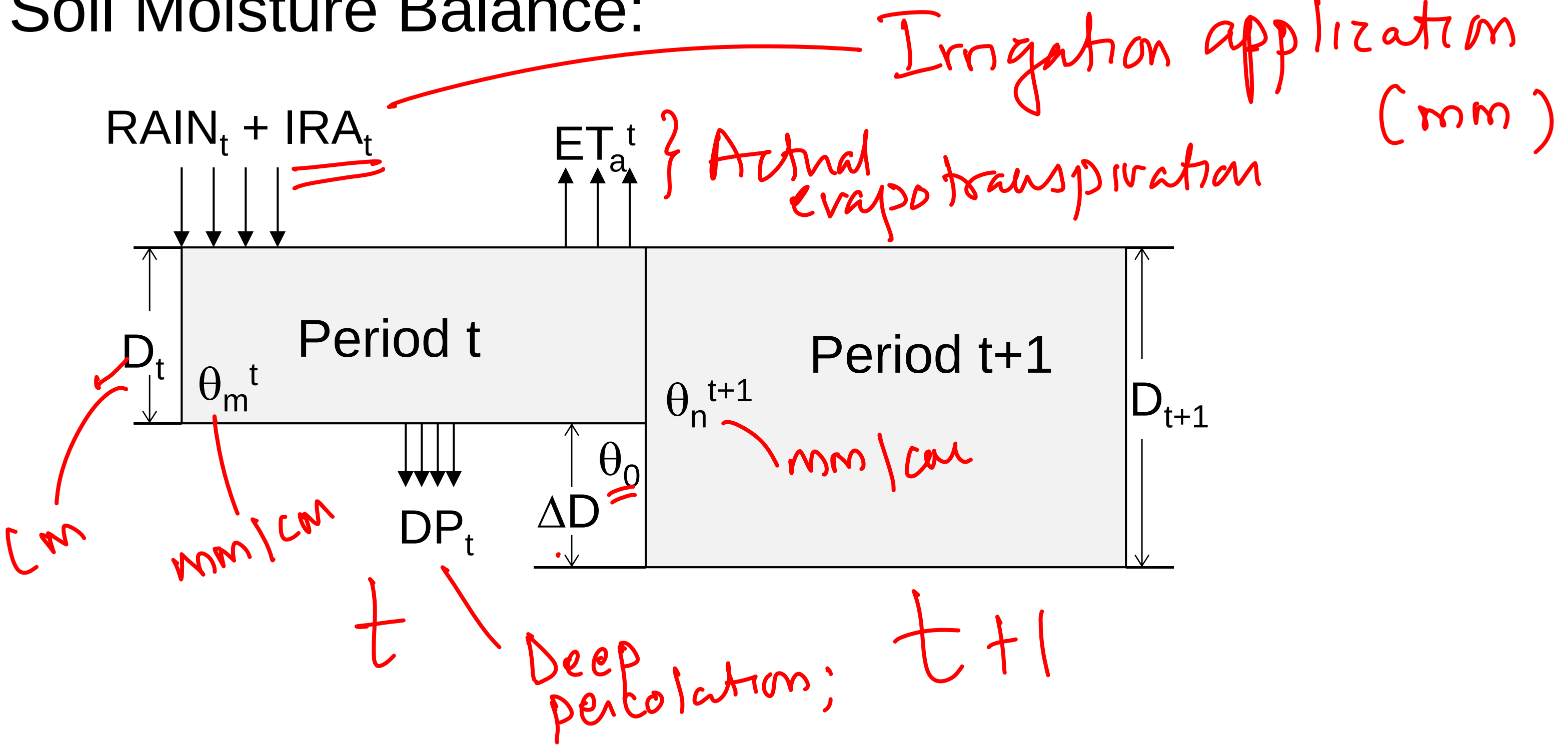
→ Crop Season

1. Establishment 2. Vegetative 3. Flowering

4. Yield Formation 5. Ripening

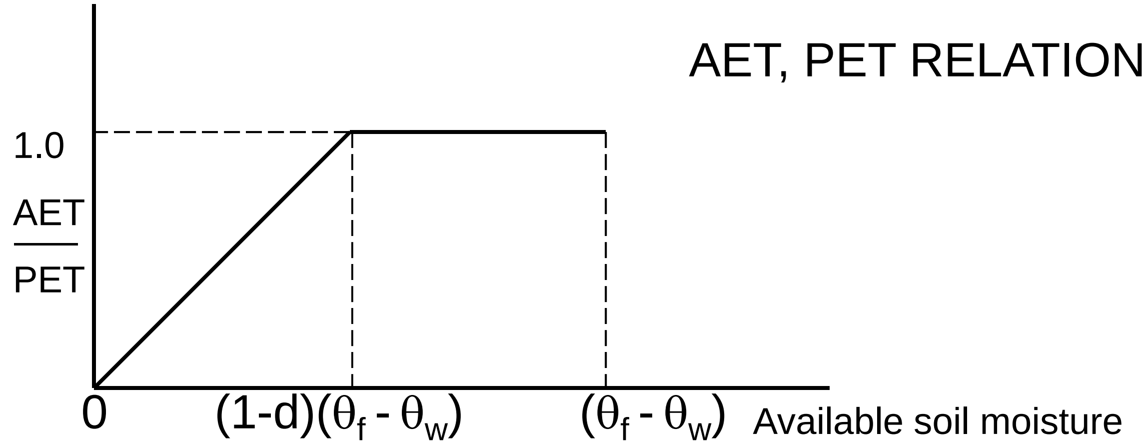
Crop Yield Optimization

Soil Moisture Balance:



$$\theta_n^{t+1} D_{t+1} = \theta_m^t \ D_t + RAIN_t + IRA_t \ - Et_a^t \ - DP_t + \theta_0 \Delta D$$

Crop Yield Optimization



$$AET = PET \quad \text{if } \theta_{avail} \geq (1-d)(\theta_f - \theta_w)$$

$$= \frac{\theta_{avail}\, PET}{(1-d)(\theta_f - \theta_w)} \quad \text{Otherwise}$$

$$\theta_{avail} = \theta_j + q_j + R_j - \theta_w$$

θ_{avail} = Available Soil Moisture

θ_f = Field Capacity

θ_w = Wilting Point

d = Depletion Factor

θ_j = Soil Moisture

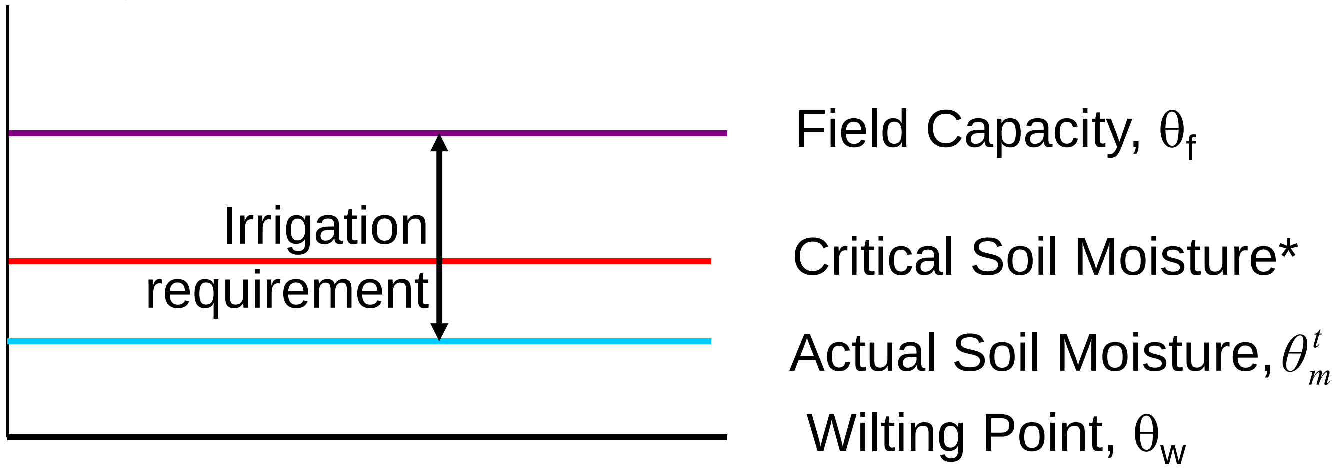
q_j = Irrigation Applied

R_j = Rainfall

Crop Yield Optimization

$$IRR_{c,m}^{t} = 0 \qquad if\ (\theta_m^t - \theta_w)\, D_c^t + RAIN_t \geq (1-d)(\theta_f - \theta_w)\, D_c^t$$

$$= [\,\theta_f D_c^t - (\theta_m^t\, D_c^t + RAIN_t\,)]A_c \qquad \text{otherwise}$$



Irrigation Requirement in period t : Amount of water required to raise the soil moisture to field capacity, when soil moisture is below a critical level.

* Critical Soil Moisture : $(1-d)(\theta_f - \theta_w)$

Crop Yield Optimization

$$\left[\frac{y}{y_m}\right] = 1 - \sum_{g=1}^{N_g}\left[Ky_g\left(1-\frac{AET}{PET}\right)_g\right]$$ ….. Additive production function

N_g : number of growth stages in the crop season.
g : growth stage index
Ky_g : yield factor for the growth stage g
y : actual yield of the crop
y_{max} : maximum yield of the crop
AET : actual evapotranspiration, and
PET : potential evapotranspiration

Ref. ET.

$PET_t = Kc_t \times ET_o$

$K_p \times E_{pan}$ — pan evap

The yield factor, Ky_g indicates the sensitivity of the crop yield to the evapotranspiration deficit in growth stage g

Crop Yield Optimization

- The soil moisture balance is written for a crop c as

$$\theta_c^{t+1} D_c^{t+1} = \theta_c^t D_c^t + RAIN^t + q_c^t - AET_c^t + \theta_0 (D_c^{t+1} - D_c^t) - DP_c^t$$

where θ^t_c is the soil moisture of crop c at the beginning of the period t,

D^t_c is the root depth of crop c during period t,

$RAIN^t$ is the effective rainfall (contribution of rainfall to soil moisture) in the command area in period t,

q^t_c is the irrigation application to crop c in period t,

AET^t_c is the actual evapotranspiration of crop c in period t,

θ_0 is the initial soil moisture in the soil zone into which the crop root extends at the beginning of period $t+1$, and

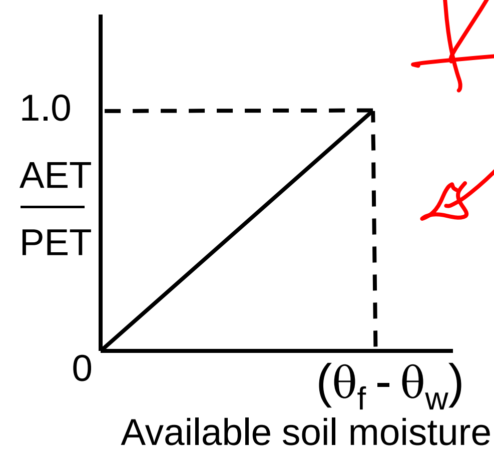
DP^t_c is the deep percolation

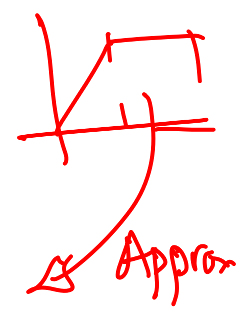
Crop Yield Optimization

- The relationship between AET/PET ratio and the available soil moisture is approximated by a linear relationship, with $AET = 0$, when the available soil moisture is zero and $AET = PET$ when the available soil moisture is equal to the maximum available soil moisture.
- θ_f and θ_w are soil moistures at field capacity and wilting point respectively.

$$AET_c^t \le \frac{(\theta_c^t D_c^t + RAIN^t + q_c^t) - \theta_w D_c^t}{(\theta_f - \theta_w) D_c^t} \times PET_c^t$$

$$AET_c^t \le PET_c^t$$





Crop Yield Optimization

$$Max \sum_{t=1}^{T} Ky^t \left(\frac{AET^t}{PET^t} \right)$$

s.t.

$$\theta^{t+1} = \theta^t + q^t + RAIN^t - AET^t - DP^t \quad \forall t$$

$$AET^t \le (\theta^t + q^t + RAIN^t - \theta_w) \times \frac{PET^t}{(\theta_f - \theta_w)} \quad \forall t$$

$$AET^t \le PET^t \quad \forall t$$

$$\theta^{t+1} \ge \theta_f \times \beta^t \quad \forall t$$

$$DP^t \le M \times \beta^t \quad \forall t$$

$$\theta_w \le \theta^{t+1} \le \theta_f \quad \forall t$$

$$\sum_{t=1}^{T} q^t \, A \le Q$$

Yield factor

Constant Root Depth

Integer (0,1)

Single Crop:

Known amount of water (Volume)

M: large no.

Area.

Crop Yield Optimization

Data:

- Crop type : two seasonal
- Crop period: 6 months
- Crop area: 3902.50 hectares
- Field capacity: 33.2% (3.32 mm/cm)
- Wilting point: 16.5% (1.65 mm/cm)
- Root Depth of the crop : 100 cm

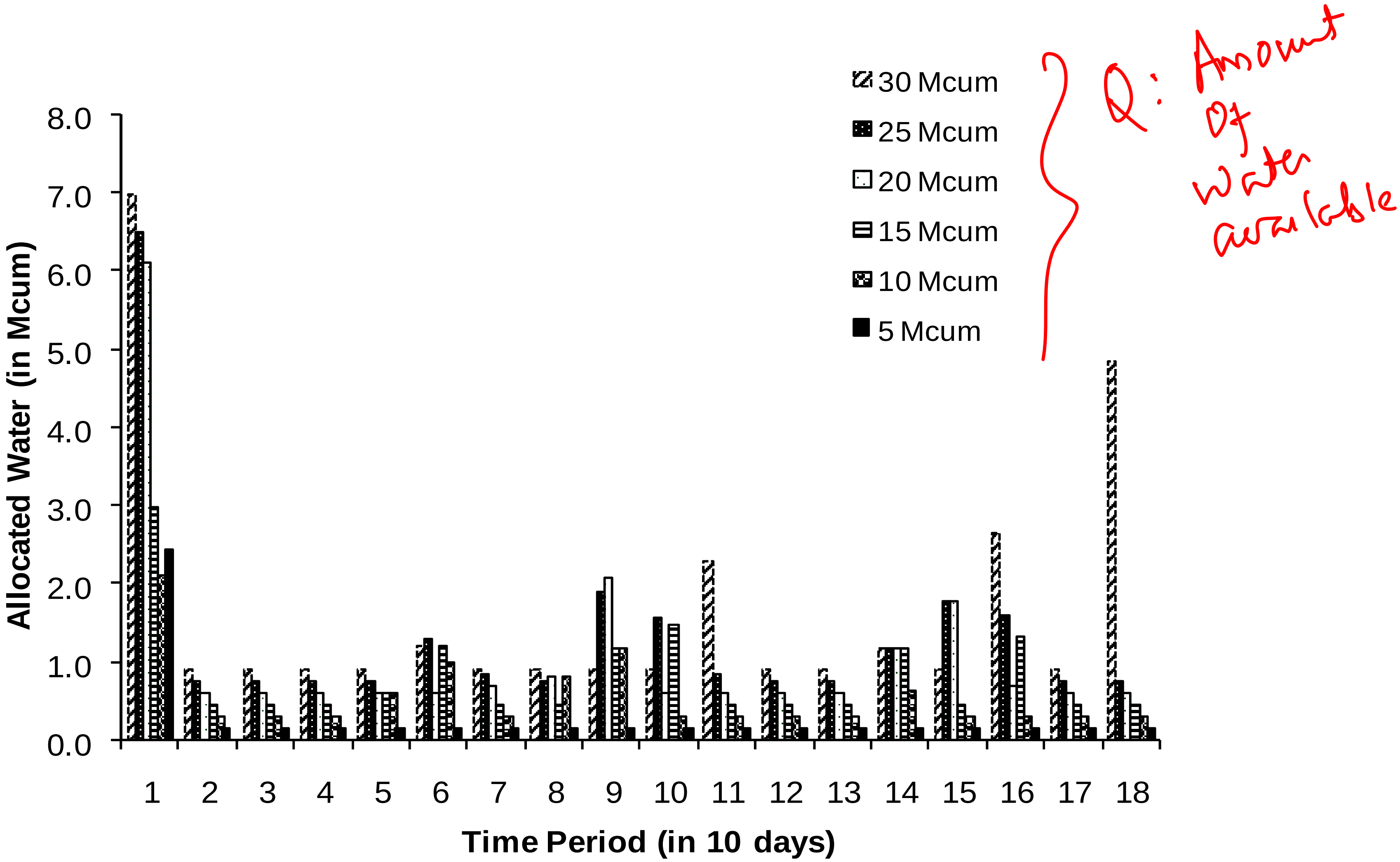
Time periods (10 days)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Rainfall ($Rain^t$) in mm	0.5	0.0	0.0	120.0	2.0	2.0	0.0	1.0	0.0	0.0	0.0	1.0	150.0	7.0	3.0	0.0	6.7	0.0

Crop Yield Optimization

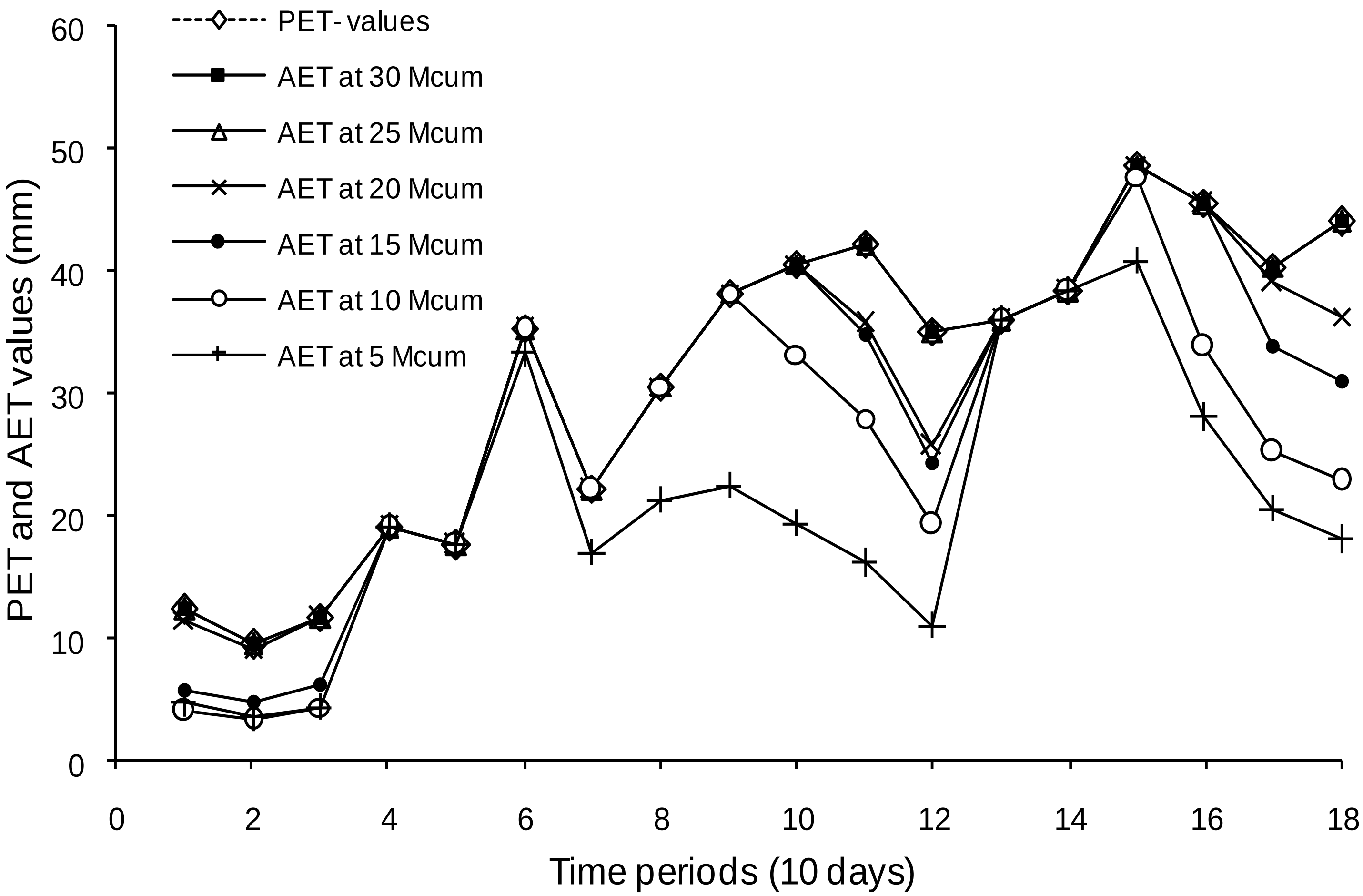
Time periods (10 days)	1	2	3	4	5	6	7	8	9	10
PET (mm)	12.32	9.49	11.68	19.06	17.59	35.35	22.20	30.65	37.22	40.65
Ky	0.20	0.20	0.20	0.25	0.25	0.25	0.25	0.25	0.25	0.25

Time periods (10 days)	11	12	13	14	15	16	17	18
PET (mm)	42.19	35.06	36.01	37.42	47.72	45.58	40.4	44.04
Ky	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

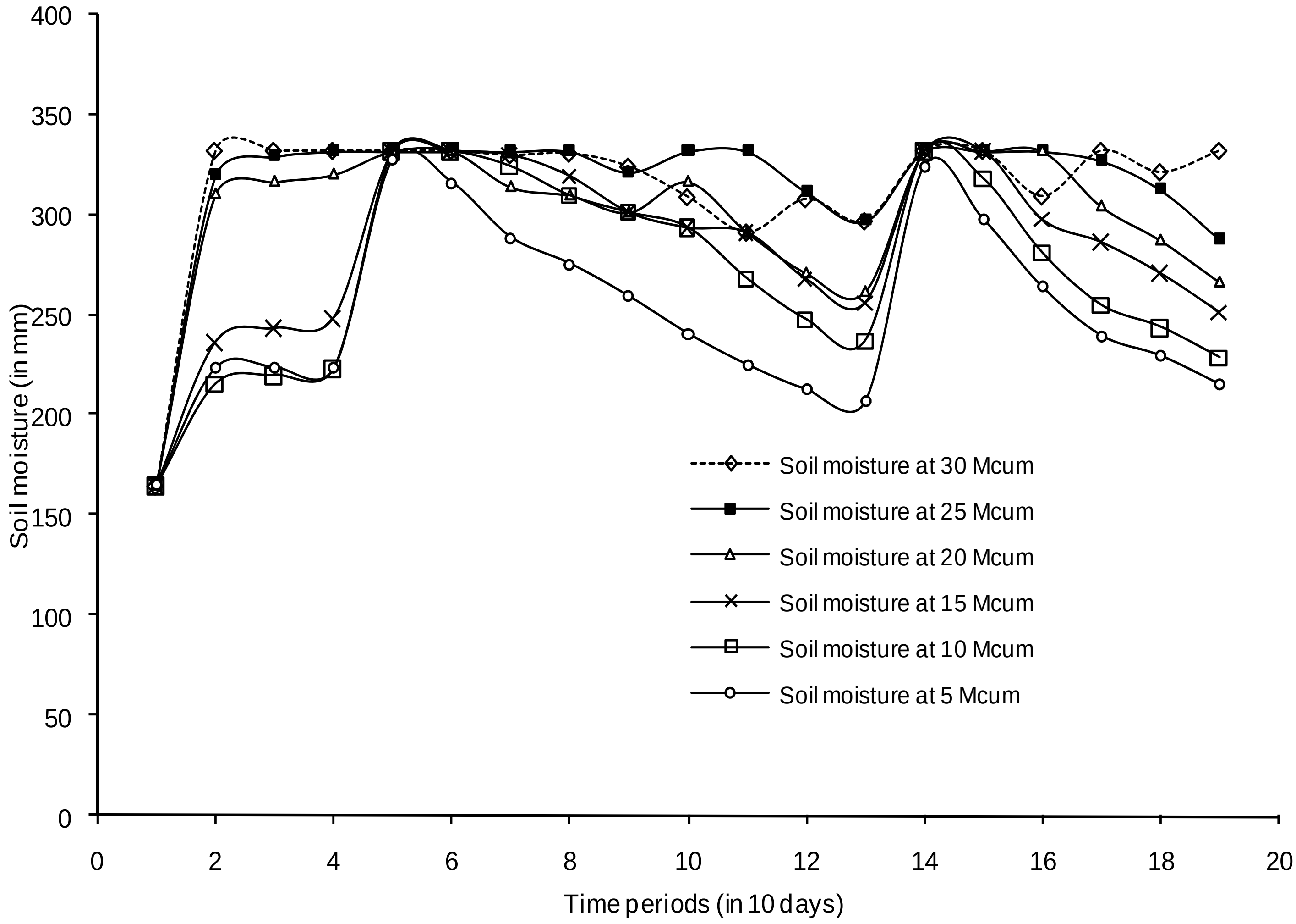
Crop Yield Optimization



Crop Yield Optimization



Crop Yield Optimization



Crop Yield Optimization

Variation of yield ratio with amount of available water (Q)

*****yield ratio = y/y_m

Sl No.	Q (Mm^3)	Yield Ratio*
1	30	1.000
2	25	1.000
3	20	0.823
4	15	0.465
5	10	0.085
6	5	0.0

Crop Yield Optimization

Irrigation water allocation to multiple crops

$$Max \sum_{c=1}^{N} \sum_{t=1}^{T} Ky_c^t \left(\frac{AET_c^t}{PET_c^t} \right)$$

s.t.

$$\theta_c^{t+1} = \theta_c^t + q_c^t + RAIN^t - AET_c^t - DP_c^t \quad \forall\ c,t$$

$$AET_c^t \le (\theta_c^t + q_c^t + RAIN_c^t - \theta_w) \times \frac{PET_c^t}{(\theta_f - \theta_w)} \quad \forall\ c,t$$

$$AET_c^t \le PET_c^t \quad \forall\ c,t$$

$$\theta_c^{t+1} \ge \theta_f \times \beta_c^t \quad \forall\ c,t$$

$$DP_c^t \le M \times \beta_c^t \quad \forall\ c,t$$

$$\theta_w \le \theta_c^{t+1} \le \theta_f \quad \forall\ c,t$$

$$\sum_{c=1}^{N} q_c^t \le Q^t \quad \forall\ t$$

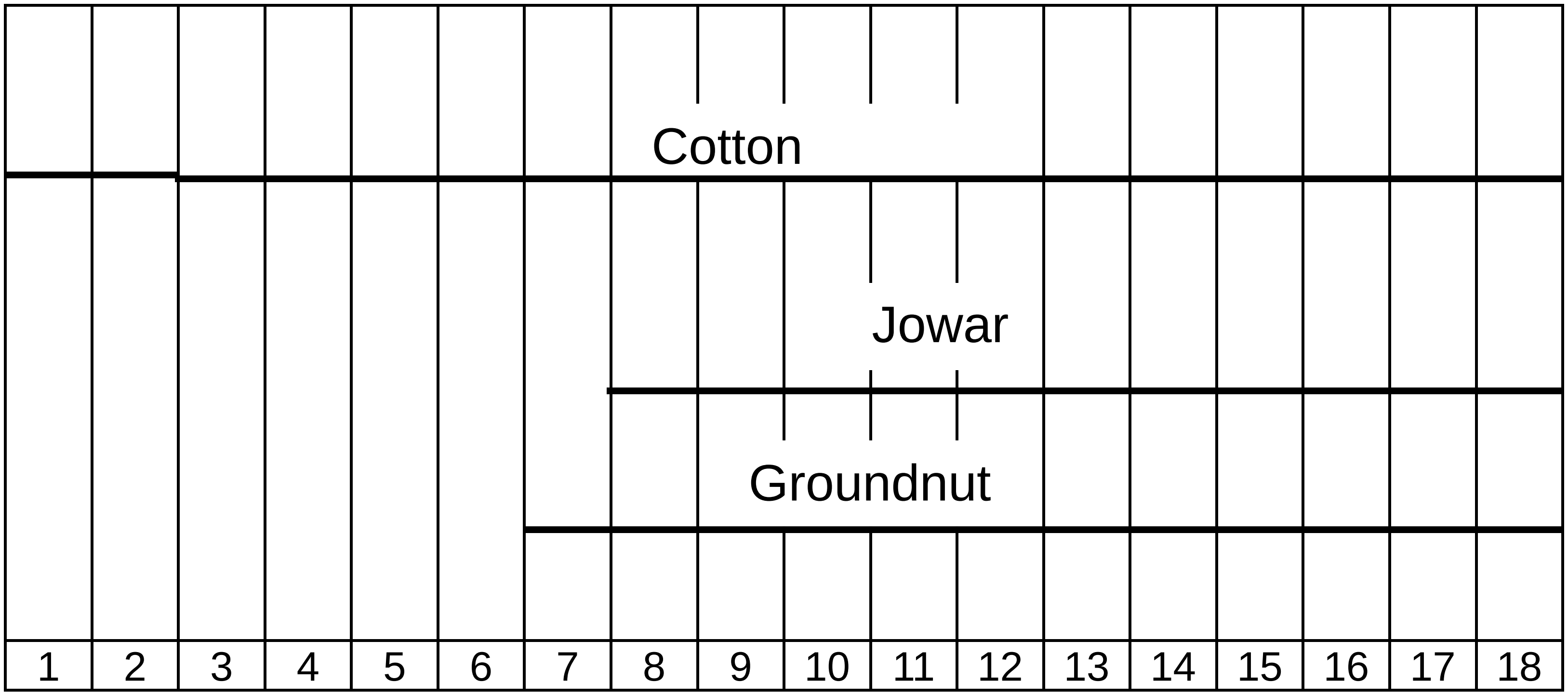
Water available in period t.

Crop Yield Optimization

- The crop areas are 3902.50 ha (Cotton), 1977.43 ha (Jowar) and 33.10 ha (Groundnut).
- The PET values for these crops are computed from pan evaporation data.
- The problem is to allocate the known amount of water Q^t, in period t, to the three crops, such that the total (relative) crop yield is maximized at the end of the season.
- The amount of water available in a period is a deterministic value, known in advance for all periods t.
- The field capacity is 33.20% and wilting point is 16.50%.
- The root depth is assumed to be 100 cm throughout the season for all crops

Crop Yield Optimization

Crop calendar



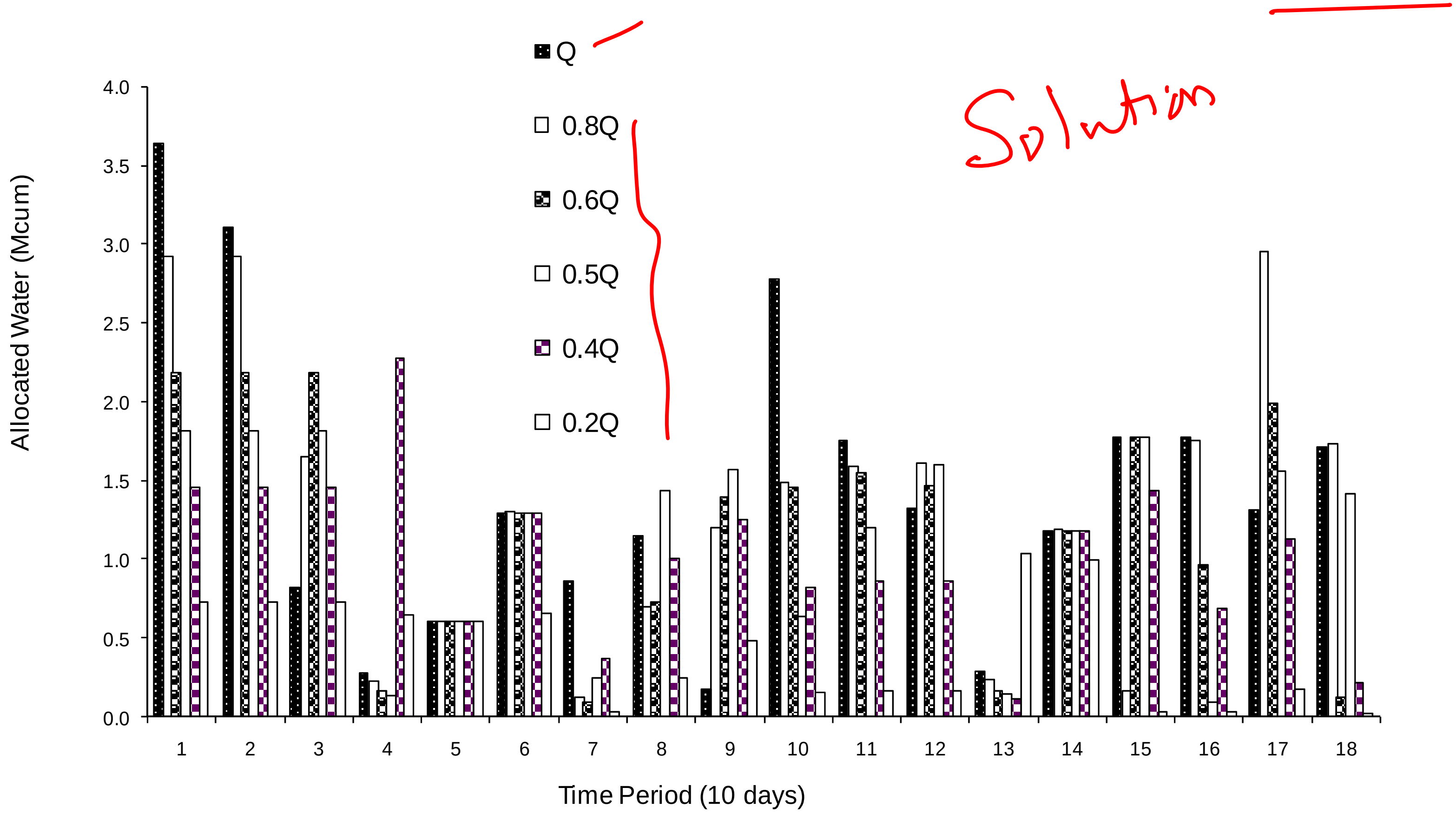
Crop Yield Optimization

Time periods (10 days)	1	2	3	4	5	6	7	8	9
Rainfall ($Rain^t$) (mm)	0.5	0.0	0.0	120.0	2.0	2.0	0.0	1.0	0.0
Available irrigation supply (Q^t) Mcum	3.65	3.65	3.65	5.70	3.70	3.50	3.20	3.20	3.50

Time periods (10 days)	10	11	12	13	14	15	16	17	18
Rainfall ($Rain^t$) (mm)	0.0	0.0	1.0	150.0	7.0	3.0	0.0	6.7	0.0
Available irrigation supply (Q^t) Mcum	3.50	3.70	3.70	5.80	5.60	4.00	4.00	5.00	3.00

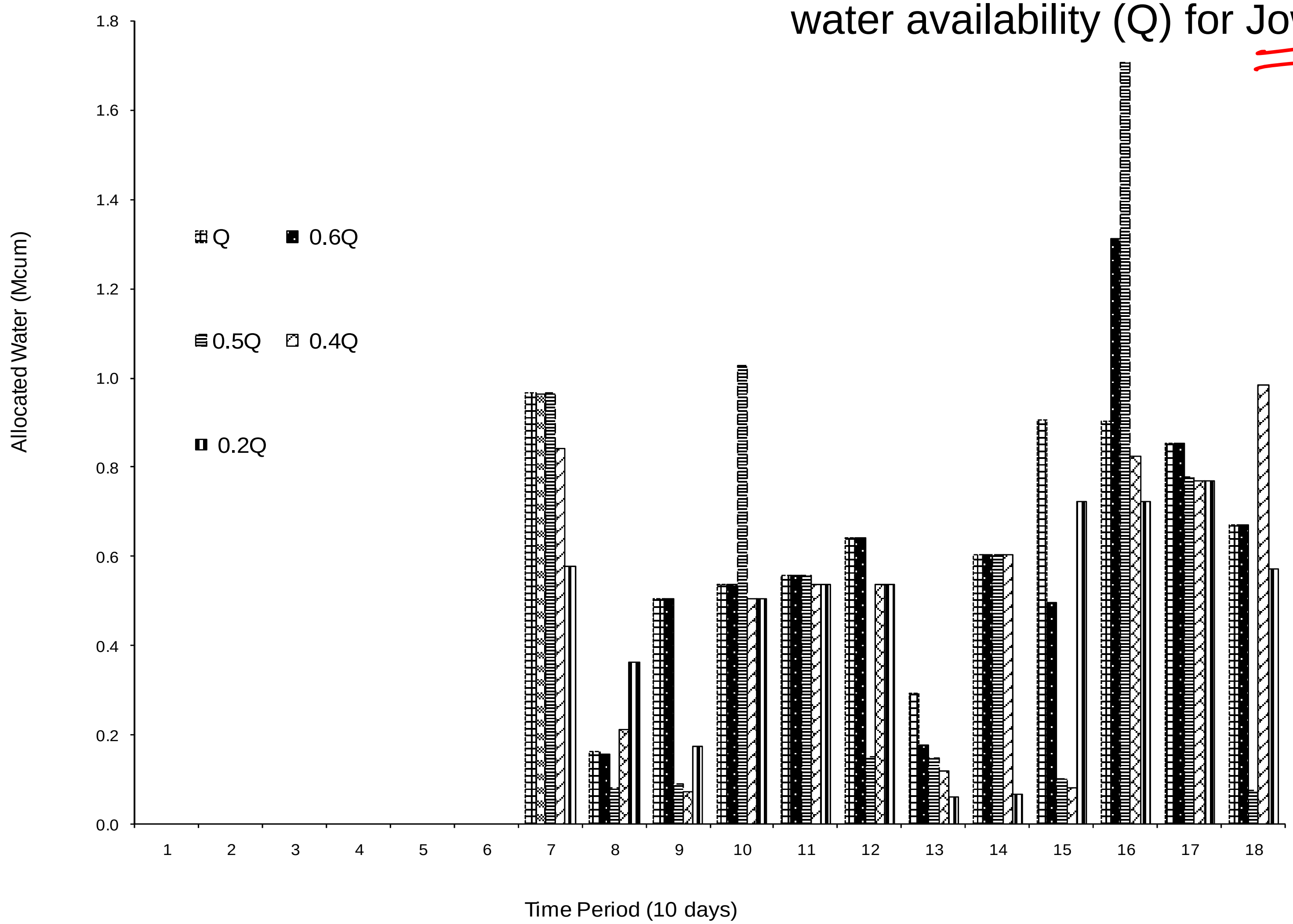
Crop Yield Optimization

Variation of water allocation with water availability (Q) for Cotton



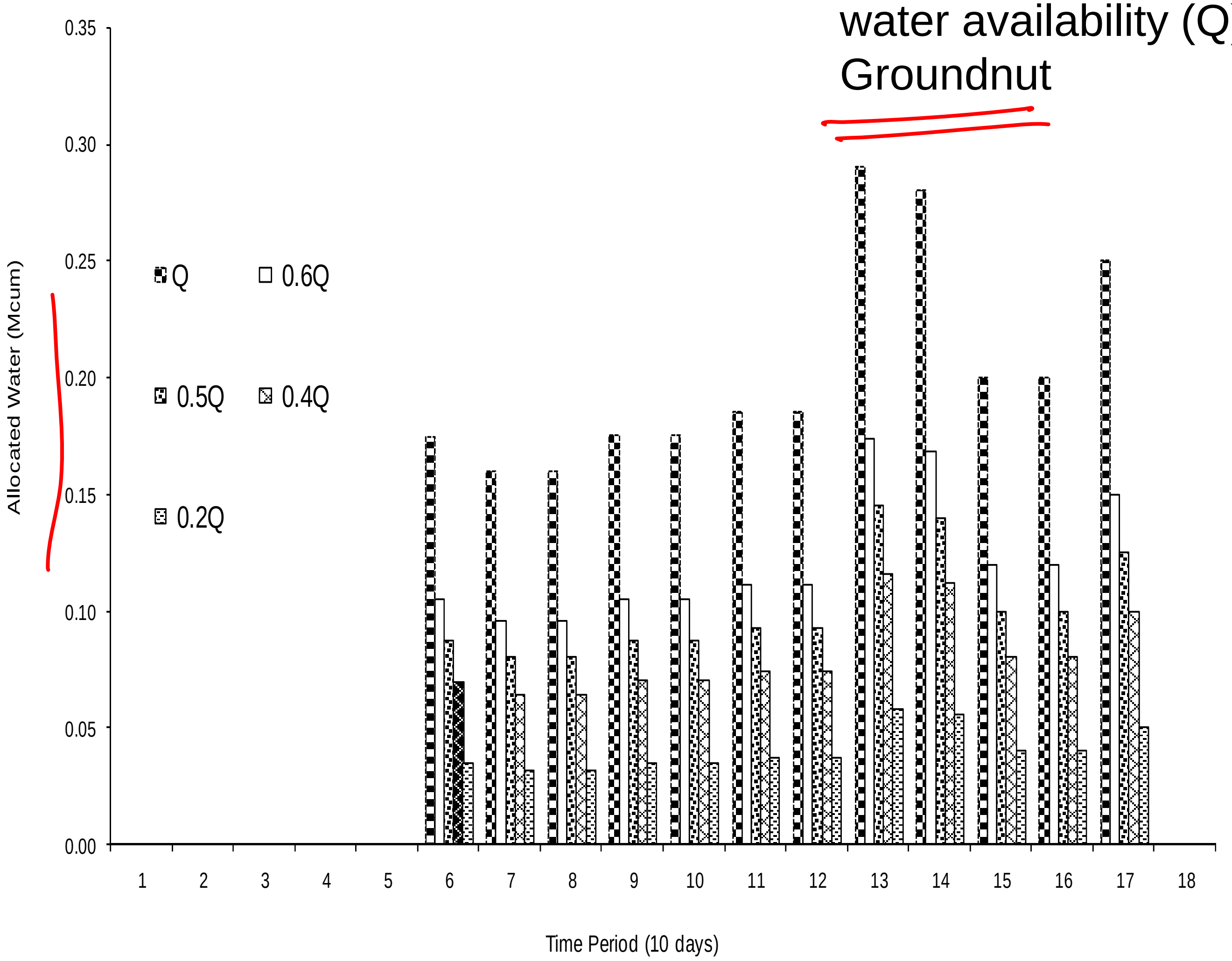
Crop Yield Optimization

Variation of water allocation with water availability (Q) for Jowar



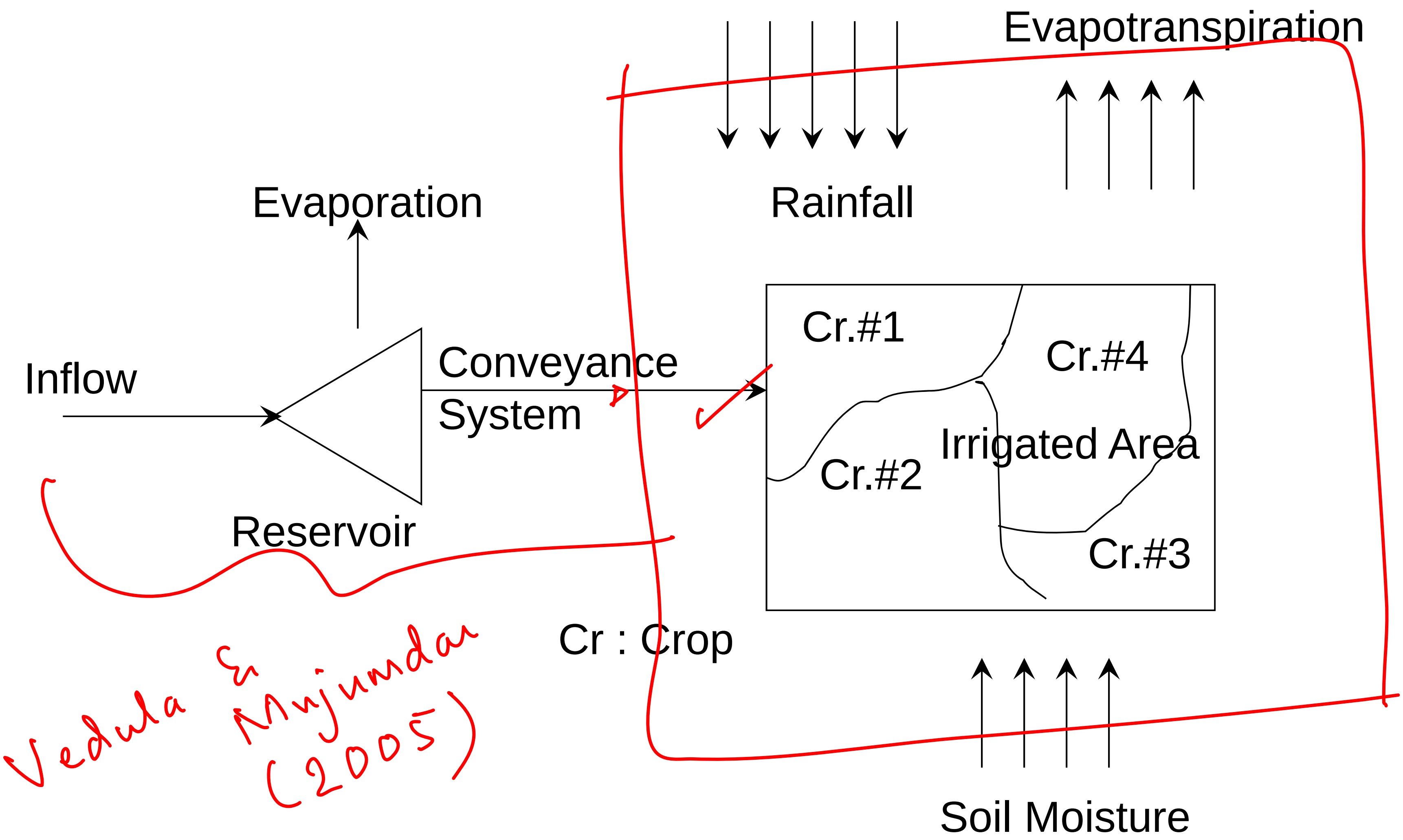
Crop Yield Optimization

Variation of water allocation with water availability (Q) for Groundnut



RESERVOIR OPERATION FOR IRRIGATION

Reservoir Operation for Irrigation



Vedula & Mujumdar (2005)