INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 25

Course Instructor : Prof. P. P. MUJUMDAR

Department of Civil Engg., IISc.

Summary of the previous lecture

- Multi-reservoir operation

Max $$\sum_{i=1}^{3}\sum_{t=1}^{T}\left[B_t^1 R_t^i + B_t^2\left(K_i - S_t^i\right) + B_t^3 S_t^i\right]$$

s.t. $$S_{t+1}^i = S_t^i + Q_t^i - E_t^i - R_t^i - O_t^i \qquad \forall\ t,\ i=1,\ 2$$

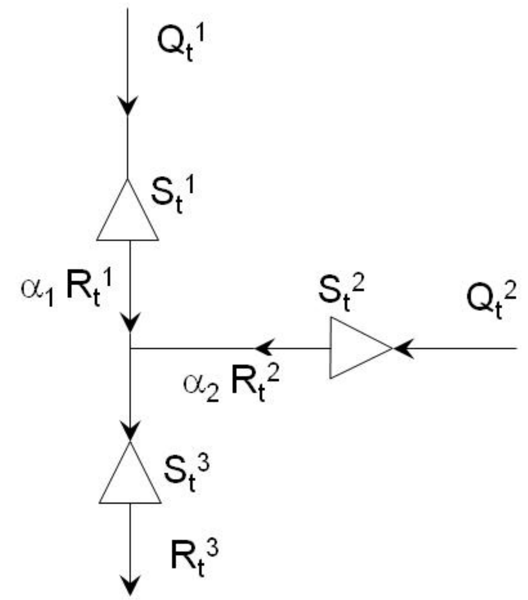
$$S_{t+1}^i = S_t^i + Q_t^i + \alpha_1 R_t^1 + \alpha_2 R_t^2 - E_t^i - R_t^i - O_t^i \qquad \forall\ t,\ i=3$$

$$S_t^i \le K_i \qquad i=1,\ 2,\ 3;\quad \forall\ t$$

$$K_i - S_t^i \ge F_{\min}^i \qquad i=1,\ 2,\ 3;\quad \forall\ t \in \text{Flood season}$$

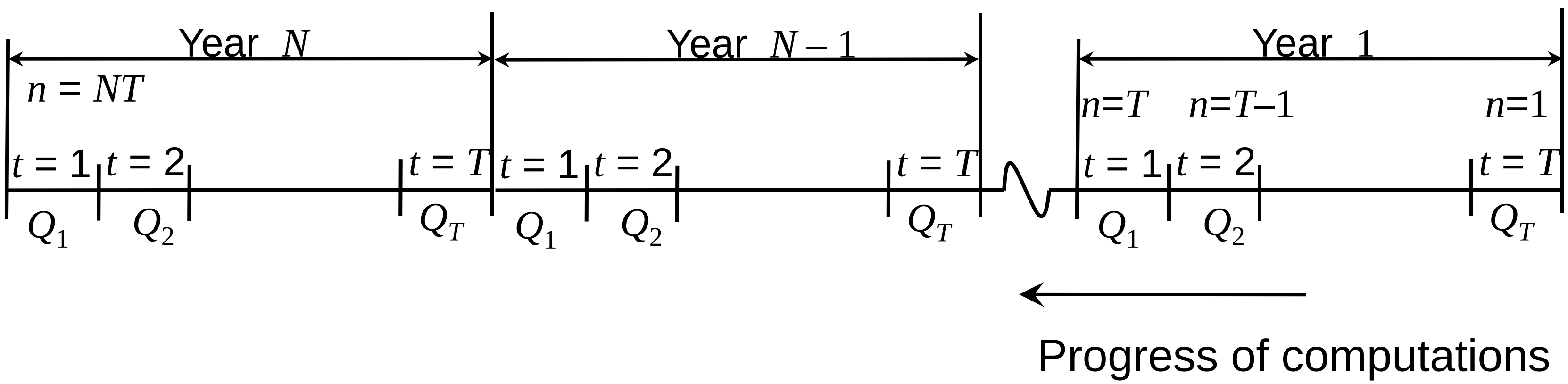
$$R_t^i \le R_{\max}^i\ ;\ R_t^i \ge 0\ ;\ S_t^i \ge 0 \qquad \forall\ t$$

$$S_{T+1}^i = S_1^i$$



- Stationary Policy Using DP

Stationary Policy Using DP



- State variable storage at the beginning of the time period t, S_t
- Computations start at some distant year in the future in the last time period.
- The computations are carried out until the solution reaches a steady state.

Stationary Policy Using DP

- The steady state is reached at stage n, when the annual net return given by

$$\left[f_t^{n+T}(S_t) - f_t^{n}(S_t) \right]$$

No. of time periods

State @ time t

converges to a constant value, for all S_t, where $f_t^n(S_t)$ is the accumulated net return up to stage n.

Stationary Policy Using DP

- The general recursive equation is

$$f_t^n(S_t) = \max\left[B_t(R_t, S_t) + f_{t+1}^{n-1}(S_t + Q_t - R_t) \right]$$

$$0 \le R_t \le S_t + Q_t$$

$$S_t + Q_t - R_t \le K$$

Accumulated System performance measure up to the previous stage

where $B_t(S_t, R_t)$ is the OF value in period t for specified storage, S_t, at the beginning of the period t, and R_t, the release during period t.

K is the known reservoir capacity and

Q_t is the inflow during period t.

Example – 1

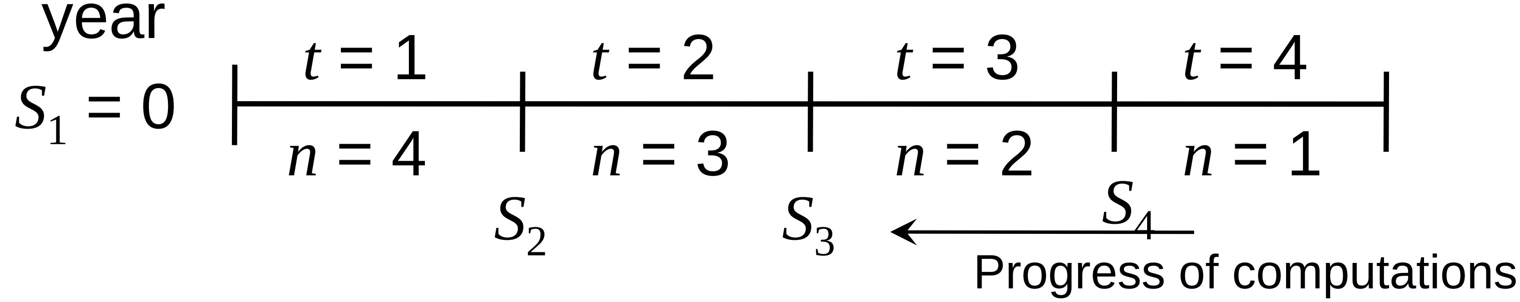
Inflows during four seasons to a reservoir with storage capacity of 4 units are 2, 1, 3 and 2 units resp.

Release from the reservoir during the season results in the following benefits which are same for all the four seasons.

Release	Benefits
0	-100
1	250
2	320
3	480
4	520
5	520
6	410
7	120

Example – 1 (Contd.)

- Refer to Lectures 16 and 17 of this course where the problem is solved over a time horizon of one year



Stage 1:

$Q_4 = 2 \quad t = 4 \quad \text{and} \quad n = 1$

$$f_4^1(S_4) = \text{Max}\left[B_4(R_4)\right]$$

$$0 \leq R_4 \leq (S_4 + Q_4)$$

$$S_4 + Q_4 - R_4 \leq 4$$

Stage 2:

$Q_3 = 3 \quad t = 3 \quad \text{and} \quad n = 2$

$$f_3^2(S_3) = \text{Max}\left[B_3(R_3) + f_4^1(S_3 + Q_3 - R_3)\right]$$

$$0 \leq R_3 \leq (S_3 + Q_3)$$

$$S_3 + Q_3 - R_3 \leq 4$$

For example, Stage-2 calculations $Q_3 = 3$ $t = 3$ and $n = 2$

S_3	R_3	$B_3(R_3)$	$S_3 + Q_3 - R_3$	$f_4^1(S_3+Q_3-R_3)$	$B_3(R_3) + f_4^1(S_3+Q_3-R_3)$	$f_3^2(S_3)$	R_3^*
2	1	250	4	520	770	1000	3, 4
	2	320	3	520	840		
	3	480	2	520	1000		
	4	520	1	480	1000		
	5	520	0	320	840		
3	2	320	4	520	840	1040	4
	3	480	3	520	1000		
	4	520	2	520	1040		
	5	520	1	480	1000		
	6	410	0	320	730		
4	3	480	4	520	1000	1040	4, 5
	4	520	3	520	1040		
	5	520	2	520	1040		
	6	410	1	480	890		
	7	120	0	320	440		

Refer to Lectures 16 and 17 of this course

Example – 1 (Contd.)

Year 1

n = 1	t = 4	$Q_t = 2$
S_4	$f_4^1(S_4)$	R_4^*
0	320	2
1	480	3
2	520	4
3	520	4, 5
4	520	4, 5

n = 2	t = 3	$Q_t = 3$
S_3	$f_3^2(S_3)$	R_3^*
0	800	2, 3
1	960	3
2	1000	3, 4
3	1040	4
4	1040	4, 5

Year 1

n = 3	t = 2	$Q_t = 1$
S_2	$f_2^3(S_2)$	R_2^*
0	1050	1
1	1210	1
2	1280	2, 3
3	1440	3
4	1480	3, 4

n = 4	t = 1	$Q_t = 2$
S_1	$f_1^4(S_1)$	R_1^*
0	1460	1
1	1530	1, 2, 3
2	1690	1, 2
3	1760	2, 3
4	1920	3

Example – 1 (Contd.)

Year 2

$n = 5$ $t = 4$ $Q_t = 2$

S_4	$f_4^5(S_4)$	R_4^*
0	1780	1, 2
1	1940	1, 3
2	2010	1, 2, 3
3	2170	1, 3
4	2240	2, 3

$n = 6$ $t = 3$ $Q_t = 3$

S_3	$f_3^6(S_3)$	R_3^*
0	2260	1, 2, 3
1	2420	1, 3
2	2490	1, 2, 3
3	2560	2, 3
4	2720	3

Year 2

$n = 7$ $t = 2$ $Q_t = 1$

S_2	$f_2^7(S_2)$	R_2^*
0	2510	1
1	2670	2
2	2740	1, 2, 3
3	2900	3
4	2970	1, 3

$n = 8$ $t = 1$ $Q_t = 2$

S_1	$f_1^8(S_1)$	R_1^*
0	2920	1
1	2990	1, 2, 3
2	3150	1, 3
3	3220	1, 2
4	3380	3

Example – 1 (Contd.)

Year 5

n = 17 t = 4 $Q_t = 2$

S_4	$f_4^{17}(S_4)$	R_4^*
0	6160	1, 2
1	6320	1, 3
2	6390	1, 2, 3
3	6550	1, 3
4	6620	2, 3

n = 19 t = 2 $Q_t = 1$

S_2	$f_2^{19}(S_2)$	R_2^*
0	6890	1
1	7050	1
2	7120	1, 2, 3
3	7280	1, 3
4	7350	1, 2, 3

n = 18 t = 3 $Q_t = 3$

S_3	$f_3^{18}(S_3)$	R_3^*
0	6640	1, 2, 3
1	6800	1, 3
2	6870	1, 2, 3
3	7030	3
4	7100	3

n = 20 t = 1 $Q_t = 2$

S_1	$f_1^{20}(S_1)$	R_1^*
0	7300	1
1	7370	1, 2, 3
2	7530	1, 3
3	7600	1, 2
4	7760	3

Example – 1 (Contd.)

Check for steady state:

n = 15   t = 2 Q_t = 1

S_2	$f_2^{15}(S_2)$	R_2^*
0	5430	1
1	5590	1
2	5660	1, 2, 3
3	5820	1, 3
4	5890	1, 2, 3

n = 19   t = 2 Q_t = 1

S_2	$f_2^{19}(S_2)$	R_2^*
0	6890	1
1	7050	1
2	7120	1, 2, 3
3	7280	1, 3
4	7350	1, 2, 3

n = 16   t = 1 Q_t = 2

S_1	$f_1^{16}(S_1)$	R_1^*
0	5840	1
1	5910	1, 2, 3
2	6070	1, 3
3	6140	1, 2
4	6300	3

n = 20   t = 1 Q_t = 2

S_1	$f_1^{20}(S_1)$	R_1^*
0	7300	1
1	7370	1, 2, 3
2	7530	1, 3
3	7600	1, 2
4	7760	3

Example – 1 (Contd.)

Check for steady state:

$$\left[f_t^{n+T}\left(S_t\right) - f_t^{n}\left(S_t\right) \right]$$

For example, n = 15, n+T = 15+4 = 19, t = 2

$$\left[f_2^{19}\left(S_t\right) - f_2^{15}\left(S_t\right) \right] = 7050 - 5590 = 1460$$

For S_t = 1

$$\left[f_2^{19}\left(S_t\right) - f_2^{15}\left(S_t\right) \right] = 7280 - 5820 = 1460$$

For S_t = 3

Verify that this condition for steady state is satisfied for all time periods, t, and for all states, S_t.

HYDROPOWER GENERATION

Hydropower Generation

- The kinetic energy produced by 1 m^3 of water falling through a distance of 1 m is

$$\rho g H = 1000 \times 9.81 \times 1$$

$$= 9810 \;\; N - m$$

ρ is density of water (1000 kg/ m^3)

g is acceleration due to gravity (9.81 m/s^2)

H is the height from which the water falls in m

Hydropower Generation

- The energy generated per second is called as power (9810 watts).
- A discharge of 1 m^3/sec produces a power of 9810 watts at a head of 1 m.
- An average flow of q_t m^3/sec, falling through a height of H_t m continuously in a period t will yield power of 9810 $q_t H_t$ watts (or 9.81 $q_t H_t$ kilowatts).
- General unit for power is kilowatt-hour

Hydropower Generation

$$kWH_t = \frac{9.81 \times 10^6 \times R_t \times H_t}{3600}$$

$$= 2725 R_t H_t$$

R_t is total flow in Mm^3 in period t and H_t is the head in m

- The equation assumes 100% conversion of energy.

$$kWH_t = 2725\ R_t\ H_t\ \eta$$

η is overall efficiency

Hydropower Generation

- Firm power: The amount of power that can be generated with certainty without interruption at site.
- The corresponding energy is Firm energy.
- Firm power can be produced with 100% reliability all the time.
- Secondary power: The power that can be generated more than 50% of time.
- Run-of-the-river power plants are those which produce power by using water directly without any requirements for water storage.

  For example, a natural drop in the channel.

Hydropower Generation

- The head available is nearly constant throughout the year and hence flow rate determines the generated power.
- Firm power corresponds to the minimum flow at that site.
- Major power plants accompany a storage reservoir.
- The head, flow and storage are all interdependent.
- For obtaining firm energy at a reservoir, the flows must be routed through the reservoir, simulating the energy production from period to period.

Hydropower Generation

- For example, a river with a minimum monthly flow of 20 Mm^3 has a drop of 50 m at a site along the river. Consider the efficiency as 75%

  Firm energy produced at that site is

$$kWH_t = 2725\ R_t\ H_t\ \eta$$

$$= 2725 \times 20 \times 50 \times 0.75$$

$$= 2043750\ \ kWH$$

$$= 2.044\ \ \ GWH$$ (giga watt hour)