INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 24

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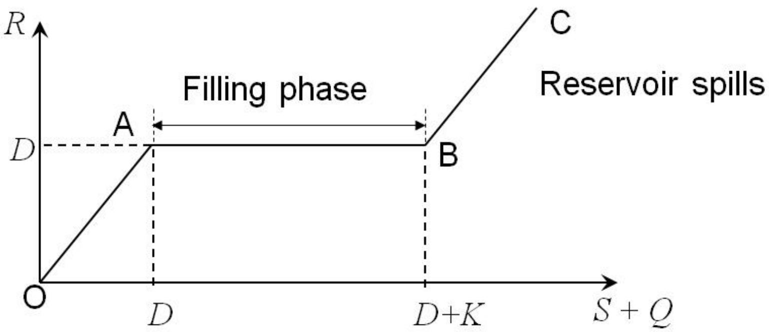
Summary of the previous lecture

- Reservoir operation
  - Standard operating policy



  - Optimal operating policy using LP

Max $\sum_t R_t$

s.t.

$$S_{t+1} = S_t + Q_t - E_t - R_t - O_t \qquad \forall\ t$$

$$R_t \leq D_t \qquad \forall\ t$$

$$S_t \leq K \qquad \forall\ t$$

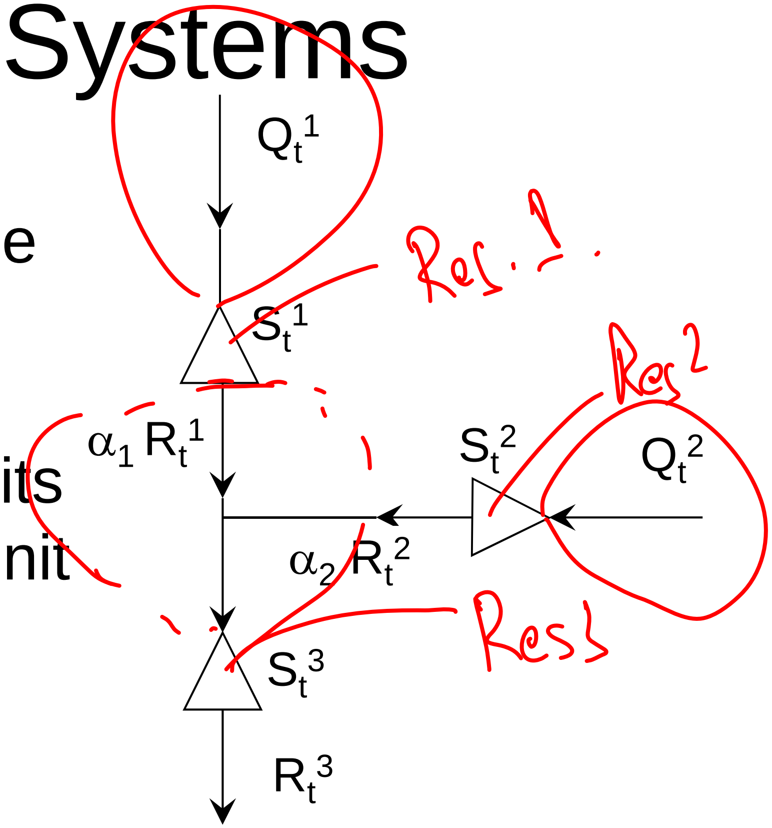
$$R_t \geq 0;\ S_t \geq 0 \qquad \forall\ t$$

$$S_{T+1} = S_1$$

  - Multi-reservoir operation

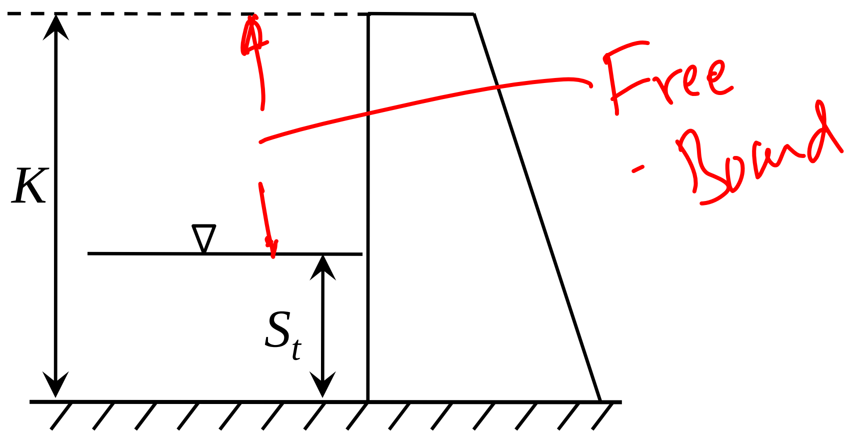
Multi-reservoir Systems

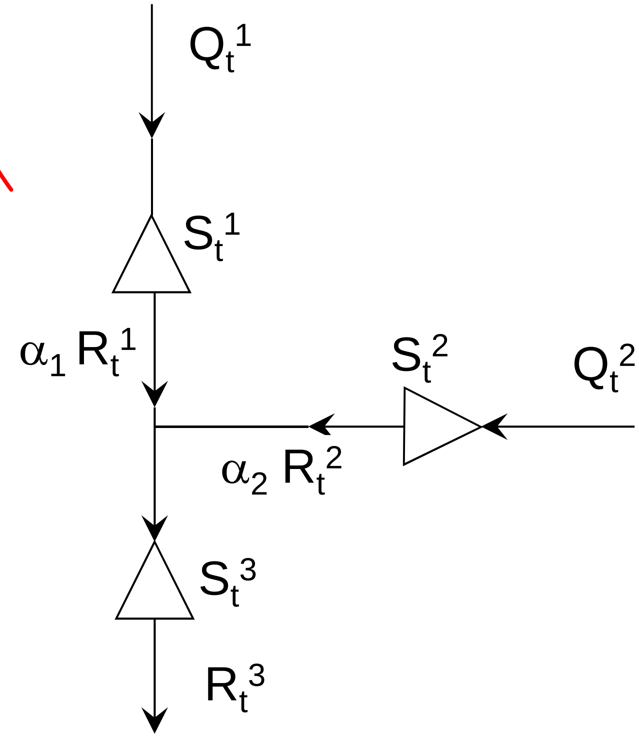
- The system serves the purpose of water supply, flood control and hydro power generation.
- B_{1t}^{i}, B_{2t}^{i} and B_{3t}^{i} are net benefits associated with unit release, unit available flood freeboard and unit storage for reservoir i in period t.



- A portion of release from reservoir 1 and 2 flows to reservoir 3.
- A minimum storage $F_{\min}^{i}$, needs to ensure flood control in flood season at the reservoir i.
- Maximum release at reservoir i is $R_{\max}^{i}$

Multi-reservoir Systems





- Assumption is

reservoir

$$B^i_{1t} = B^1_t \quad \forall\, t$$

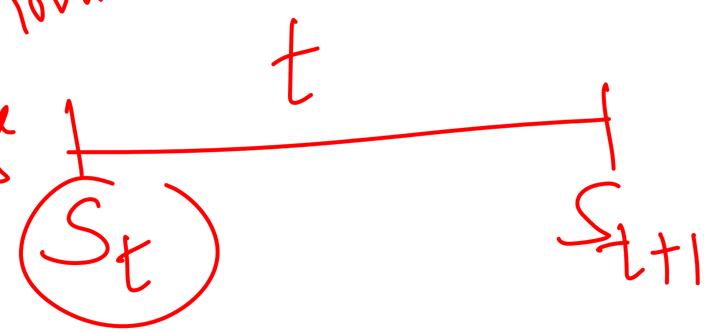
Release

Flood

$$B^i_{2t} = B^2_t \quad \forall\, t$$

Storage

$$B^i_{3t} = B^3_t \quad \forall\, t$$



Multi-reservoir Systems

No. of time periods

Capacity of reservoir i

Hydropower

Flood volume / storage

LP formulation:

Max $$\sum_{i=1}^{3}\sum_{t=1}^{T}\left[B_t^1 R_t^i + B_t^2 \left(K_i - S_t^i\right) + B_t^3 S_t^i \right]$$

s.t.

$$S_{t+1}^i = S_t^i + Q_t^i - E_t^i - R_t^i - O_t^i \qquad \forall\ t,\ i = 1,\ 2$$

Overflow / spill

$$S_{t+1}^i = S_t^i + Q_t^i + \alpha_1 R_t^1 + \alpha_2 R_t^2 - E_t^i - R_t^i - O_t^i \qquad \forall\ t,\ i = 3$$

u/s reservoirs

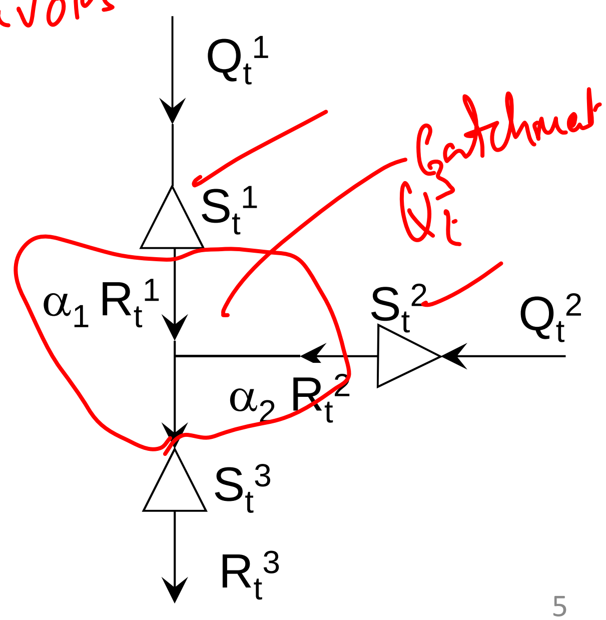
$$S_t^i \le K_i \qquad i = 1,\ 2,\ 3; \quad \forall\ t$$

$$K_i - S_t^i \ge F_{\min}^i \qquad i = 1,\ 2,\ 3; \quad \forall\ t \in \text{ Flood season}$$

$$R_t^i \le R_{\max}^i \qquad \forall\ t$$

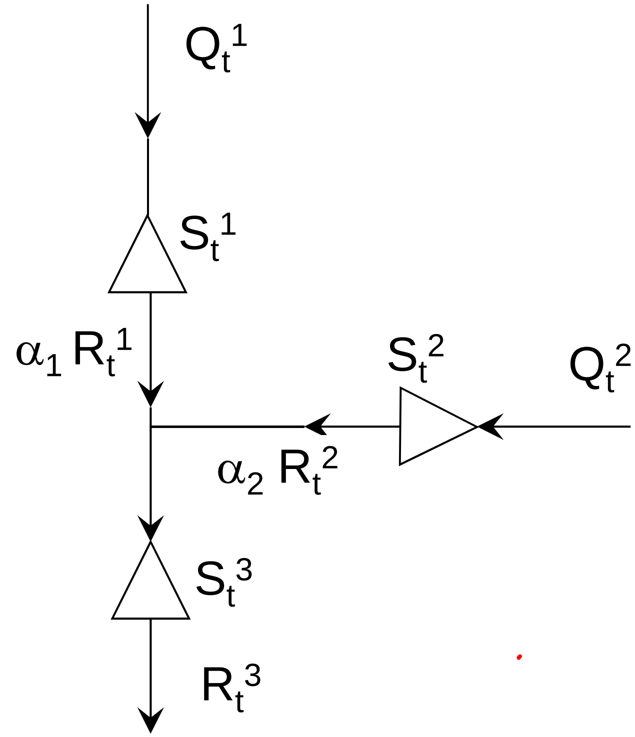
$$R_t^i \ge 0 \ ; \quad S_t^i \ge 0 \qquad \forall\ t$$

$$S_{T+1}^i = S_1^i$$



Multi-reservoir Systems

- i refers to reservoir (i = 1,2,3)
- t is time period (t = 1,2,…..,12)
- Q is inflow
- K is reservoir capacity
- S is storage
- R is release
- O is overflow or spill
- The storage S_t is the storage at the beginning of period t.



Example – 3

Consider the data given in table below for a three period, three reservoir system.

Reservoir	Inflow			K	F_{min}			B_1^*	B_2^*	B_3^*
	t = 1	t = 2	t = 3		t = 1	t = 2	t = 3			
1	25	10	15	10	3	2	7	50	10	25
2	10	30	15	15	2	3	4	60	10	30
3	20	12	15	20	2	3	5	70	10	35

$\alpha_1 = 0.2$ and $\alpha_2 = 0.3$

Example – 3 (Contd.)

LP formulation:

B_1^* B_2^* B_3^*

Max $$\sum_{i=1}^{3}\sum_{t=1}^{3}\left[B_t^1 R_t^i + B_t^2\left(K_i - S_t^i\right) + B_t^3 S_t^i\right]$$

s.t. $S_{t+1}^i = S_t^i + Q_t^i - E_t^i - R_t^i - O_t^i$ $i = 1,\ 2$; t = 1,2,3

$S_{t+1}^i = S_t^i + Q_t^i + \alpha_1 R_t^1 + \alpha_2 R_t^2 - E_t^i - R_t^i - O_t^i$ $i = 3$; t = 1,2,3

$S_t^i \le K_i$ $i = 1,\ 2,\ 3$; t = 1,2,3

$K_i - S_t^i \ge F_{\min}^i$ $i = 1,\ 2,\ 3$; t = 1,2,3

$F^i_{min,t}$

$t \in F.S$

$R_t^i \le R_{\max}^i$

$R_t^i \ge 0$; $S_t^i \ge 0$ } t = 1,2,3

$S_3^i = S_1^i$

Example – 3 (Contd.)

LINGO.

```
MODEL:
SETS: RES/1..3/: K;
      NSP/1..2/;
      NSP1/1..3/: B1, B2, B3;
      SP(RES,NSP1) : R, E, L, BETA, S, Q, FMIN;
ENDSETS

MAX = @SUM(RES(I): @SUM(NSP1(T): B1(T)*R(I,T) + B2(T)*(K(I) - S(I,T)) +
B3(T)*S(I,T);));
@FOR(NSP(T):
S(1,T+1) = S(1,T) + Q(1,T) - R(1,T) - E(1,T) - L(1,T);
);

S(1,1) = S(1,3) + Q(1,3) - R(1,3) - E(1,3) - L(1,3);

@FOR(NSP(T):
S(2,T+1) = S(2,T) + Q(2,T) - R(2,T) - E(2,T) - L(2,T);
);

S(2,1) = S(2,3) + Q(2,3) - R(2,3) - E(2,3) - L(2,3);
```

Example – 3 (Contd.)

```
@FOR(NSP(T):
S(3,T+1) = S(3,T) + Q(3,T) + ALFA1*R(1,T) + ALFA2*R(2,T) - R(3,T) - E(3,T) -
L(3,T);
);

S(3,1) = S(3,3) + Q(3,3) + ALFA1*R(1,3) + ALFA2*R(2,3) - R(3,3) - E(3,3) - L(3,3);

@FOR(RES(I):
        @FOR(NSP1(T):
        S(I,T) < K(I);
        K(I) - S(I,T) > FMIN(I,T);
        ));

DATA:
K = 10, 15, 20;
FMIN = 3 2 7     2 3 4     2 3 5;
Q = 25 10 15     10 30 15     20 12 15;
E = 0 0 0     0 0 0     0 0 0;
ALFA1 = 0.2;  ALFA2 = 0.3;  B1 = 50, 60, 70;  B2 = 10, 10, 10; B3 = 25, 30, 35;
ENDDATA
END
```

Example – 3 (Contd.)

Solution:

	Reservoir 1			Reservoir 2			Reservoir 3		
	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$	$t = 1$	$t = 2$	$t = 3$
S_t	0	8	3	2	12	11	0	17	15
R_t	17	15	18	0	31	24	6.4	26.3	40.8
$(K–S_t)$	10	2	7	13	3	4	20	3	5

Flood freeboard.

S + Q

8 + 10 − 15

= 3

Stationary Policy Using DP

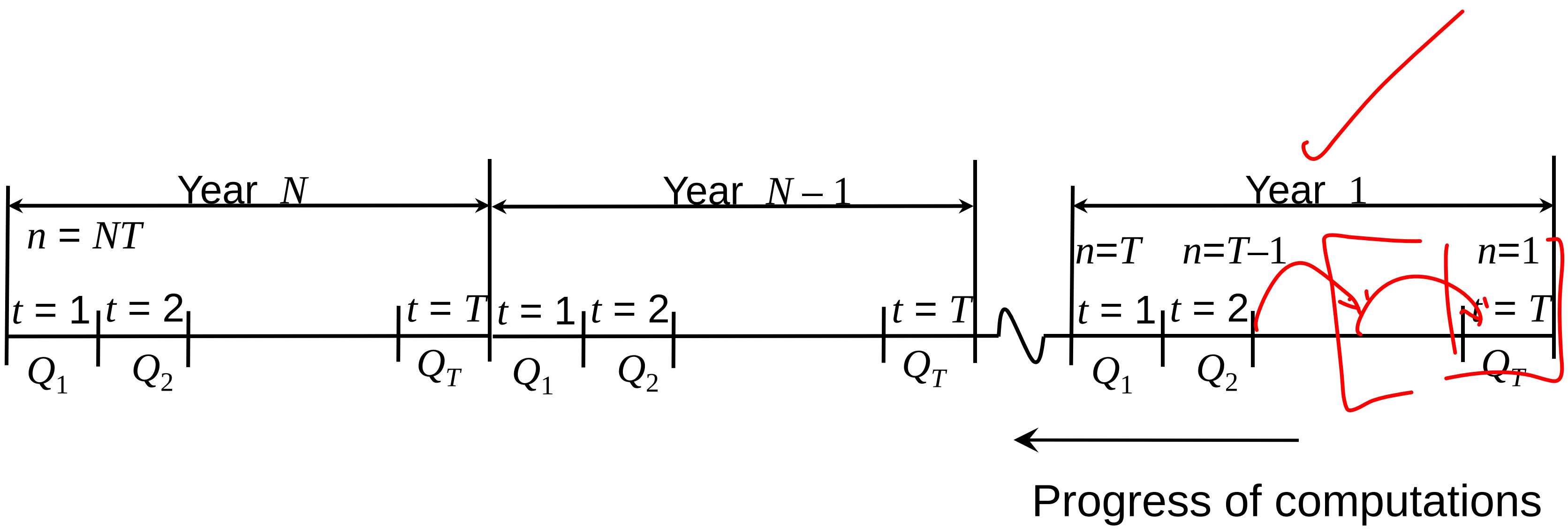
- The stationary policy derived using DP specifies the release as a function of storage in a period.
- Objective is to derive an operating policy which results in the maximized annual net benefit in the long run.

return

State

Stationary Policy Using DP

- Computations start at some distant year in the future in the last time period.
- The choice of this year is such that the computations yield a steady state solution.



Stationary Policy Using DP

1. There are no boundary conditions; the initial or the final storage values are not specified and the policy for all possible storage states are sought.
2. The computations extend beyond the one year horizon, with the stage index n continuously increasing from n=1,2,…,T,T+1,T+2,…., and period index, t, keeps track of the position of the particular stage within the year t=T,T-1,…,1, T,T-1,…,1….
3. The computations are carried out until the solution reaches a steady state.