INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 22

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Summary of the previous lecture

- Reservoir capacity using LP
  - With storage-dependent evaporation losses

Minimize K

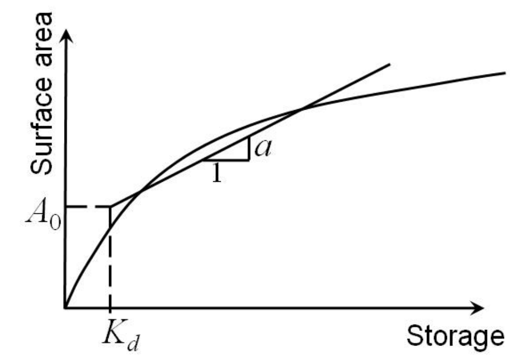
s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - R_t = (1 + a_t)\, S_{t+1} \qquad \forall t$$

$$S_t \le K \qquad \forall t$$

$$R_t \ge D_t \qquad \forall t$$

$S_{T+1} = S_1$, where T is the last period



- Storage yield function

Maximize R

s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - R \ge (1 + a_t)\, S_{t+1} \qquad \forall t$$

and

$$S_t \le K \qquad \forall t$$

with $S_{T+1} = S_1$, where T is the last period

Known

$R_1 = R_2 = \cdots = R$

Example – 1

The monthly inflows (Q_t) in Mm^3 and evaporation rate (e_t) in mm for a reservoir are given below

	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Q_t	70.61	412.75	348.40	142.29	103.78	45.00	19.06
e_t	231.81	147.57	147.57	152.14	122.96	121.76	99.89

	Jan.	Feb.	Mar.	Apr.	May
Q_t	14.27	10.77	8.69	9.48	18.19
e_t	97.44	106.14	146.29	220.97	246.75

Area corresponding to dead storage level, A_0 = 37.01 Mm^2
Slope of the area-capacity curve beyond dead storage,
a = 0.117115 m^2/m^3

Obtain the storage yield function.

Example – 1 (Contd.)

LP Formulation

Maximize R

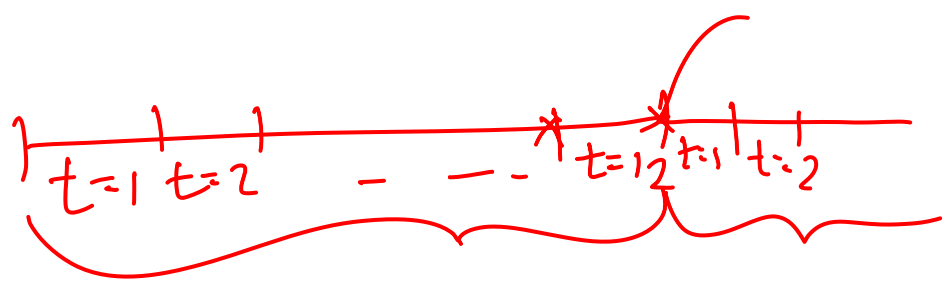
s.t.

$$(1 - a_t)\, S_t + Q_t - L_t - R \geq (1 + a_t)\, S_{t+1} \qquad t = 1, 2, \ldots .12$$

$$S_t \leq K \qquad t = 1, 2, \ldots .12$$

Given.

$$S_{13} = S_1$$



Example – 1 (Contd.)

e_t converted to 'm'

Month	Q_t (Mm^3)	e_t mm	$a_t = a * e_t / 2$	$L_t = A_o * e_t$ (Mm^3)	$(1 - a_t)$	$(1 + a_t)$
Jun	70.61	231.81	0.01357	8.58	0.9864	1.0136
Jul	412.75	147.57	0.00864	5.46	0.9914	1.0086
Aug	348.4	147.57	0.00864	5.46	0.9914	1.0086
Sep	142.29	152.14	0.00891	5.63	0.9911	1.0089
Oct	103.78	122.96	0.00720	4.55	0.9928	1.0072
Nov	45	121.76	0.00713	4.51	0.9929	1.0071
Dec	19.06	99.89	0.00585	3.70	0.9942	1.0058
Jan	14.27	97.44	0.00571	3.61	0.9943	1.0057
Feb	10.77	106.14	0.00622	3.93	0.9938	1.0062
Mar	8.69	146.29	0.00857	5.41	0.9914	1.0086
Apr	9.48	220.97	0.01294	8.18	0.9871	1.0129
May	18.19	246.75	0.01445	9.13	0.9856	1.0144

Example – 1 (Contd.)

$K = 600$ Mm^3

Maximize R

s.t. $(1 - a_t)\, S_t + Q_t - L_t - R = (1 + a_t)\, S_{t+1}$ $\qquad S_t \leq K$

$0.9864 * S_1 + 70.61 - 8.58 - R = 1.0136 * S_2$	$S_1 \leq 600$
$0.9914 * S_2 + 412.75 - 5.46 - R = 1.0086 * S_3$	$S_2 \leq 600$
$0.9914 * S_3 + 348.4 - 5.46 - R = 1.0086 * S_4$	$S_3 \leq 600$
$0.9911 * S_4 + 142.29 - 5.63 - R = 1.0089 * S_5$	$S_4 \leq 600$
$0.9928 * S_5 + 103.78 - 4.55 - R = 1.0072 * S_6$	$S_5 \leq 600$
$0.9929 * S_6 + 45 - 4.51 - R = 1.0071 * S_7$	$S_6 \leq 600$
$0.9942 * S_7 + 19.06 - 3.7 - R = 1.0058 * S_8$	$S_7 \leq 600$
$0.9943 * S_8 + 14.27 - 3.61 - R = 1.0057 * S_9$	$S_8 \leq 600$
$0.9938 * S_9 + 10.77 - 3.93 - R = 1.0062 * S_{10}$	$S_9 \leq 600$
$0.9914 * S_{10} + 8.69 - 5.41 - R = 1.0086 * S_{11}$	$S_{10} \leq 600$
$0.9871 * S_{11} + 9.48 - 8.18 - R = 1.0129 * S_{12}$	$S_{11} \leq 600$
$0.9856 * S_{12} + 18.19 - 9.13 - R = 1.0144 * S_1$	$S_{12} \leq 600$

Solution:

$R = 89.35$ Mm^3

$\sum_t Q_t \geq \sum_t (D_t + E_t)$

Reservoir Capacity Using LP

Storage yield function:

- A plot of K for different values of R is storage yield function.

- This is a increasing function of R, up to some maximum feasible value of R.

Example – 1 (Contd.)

```
MODEL:
SETS:
periods/1..12/: Q, L, e, a;
   nsp1/1..13/:S;
ENDSETS

  Max = R;

@FOR(periods(t):
L(t)=e(t)*A0/1000;
a(t)=slope*e(t)/2000;
(1+a(t))*S(t+1) < (1-a(t))*S(t) + Q(t) - R - L(t);
S(t) < K;
S(13)=S(1);
);
```

1) LINGO
Software
Linear and General
Optimization
— LINDO Systems
2) MATLAB

Example – 1 (Contd.)

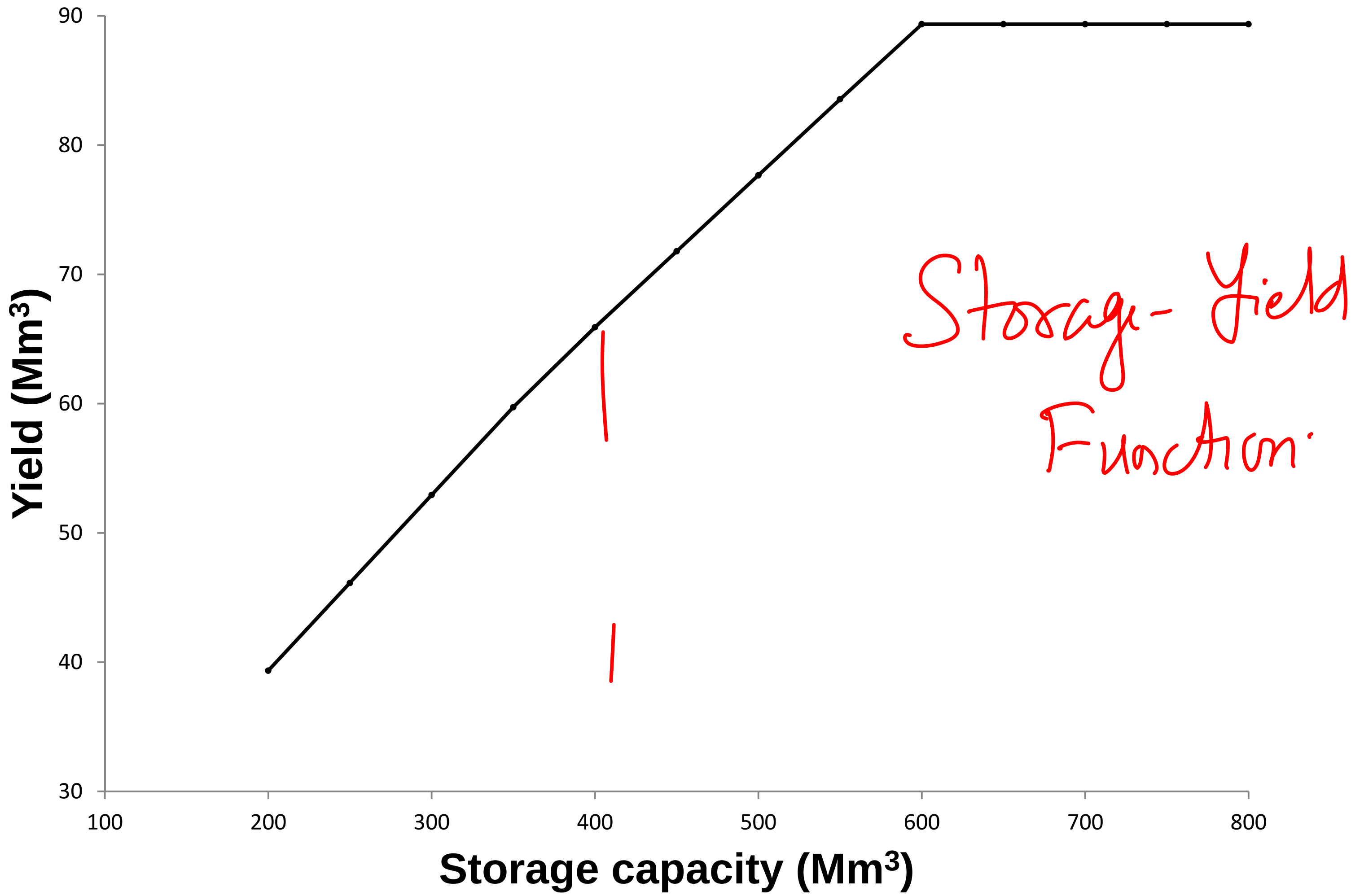
```
DATA:

Q =
70.61,412.75,348.40,142.29,103.78,45.0,19.06,14.27,10.77,8.69,9.48,1
8.19;

e = 231.81, 147.57, 147.57, 152.14, 122.96, 121.76, 99.89, 97.44,
106.14, 146.29, 220.97, 246.75;

A0 = 37.01;
slope = 0.117115;
K=600;

ENDDATA
END
```

Example – 1 (Contd.)

Storage capacity, K (Mm^3)	Yield, R (Mm^3)
200	39.34
250	46.13
300	52.93
350	59.72
400	65.91
450	71.78
500	77.66
550	83.54
600	89.35
650	89.35
700	89.35
750	89.35

Constant

Example – 1 (Contd.)



Reservoir Capacity Using LP

Mixed integer LP formulation for maximizing yield:

- An elegant way to solve a problem with equality constraints accounting for spills is by the use of integer variables in LP formulation.
- Mixed integer formulation ensures that spill will not occur unless reservoir is full.
- Introduce additional constraints using integer variables.

Reservoir Capacity Using LP

- Constraints may be specified as

Large no.

Binary integer Variable, β_t (0, 1)

$$Spill_t \leq \beta_t M$$

Capacity

$$\beta_t \leq \frac{S_{t+1}}{K}$$

β_t is integer ≤ 1

when S_{t+1} is less than K, then $\beta_t = 0$ and $Spill_t = 0$

when S_{t+1} is higher than K, then $\beta_t > 1$

β_t is forced to be equal to 1 ($S_{t+1} = K$) in order to make spill +ve

Reservoir Capacity Using LP

Mixed integer LP formulation for maximizing yield is

Maximize R

s.t.

$$\left.\begin{aligned}
&(1 - a_t)\, S_t + Q_t - L_t - R - Spill_t = (1 + a_t)\, S_{t+1} \\
&Spill_t \leq \beta_t\, M \\
&\beta_t \leq \frac{S_{t+1}}{K} \\
&\beta_t \text{ is integer} \leq 1 \\
&S_t \leq K
\end{aligned}\right\}\ \forall\, t$$

and $S_{T+1} = S_1$

Reservoir Capacity Using LP

Points to be noted:

- Total annual inflow in all periods is greater than the sum of demands and evaporation losses; otherwise the problem will be infeasible.
- The problem of determining K for a given R cannot be solved using mixed integer formulation, the constraint

$$\beta_t \le \frac{S_{t+1}}{K} \quad \text{or} \quad \beta_t K \le S_t$$

Decision Variable

becomes nonlinear when both are variables.

Example – 2

With the same data as in Example-1

```
MODEL:
SETS:
periods/1..12/: Q, L, e, a, B, spill;
   nsp1/1..13/:S;
ENDSETS

Max = R;

@FOR(periods(t):
@GIN(B(t));
spill(t) < B(t)*M;
B(t) < S(t+1)/K;
B(t) < 1;
L(t)=e(t)*A0/1000;
a(t)=slope*e(t)/2000;
(1+a(t))*S(t+1) = (1-a(t))*S(t) + Q(t) - R - L(t) - spill(t);
S(t) < K;
S(13)=S(1);
);
```

LINGO Software

Large no.

Mixed Integer problem.

Example – 2 (Contd.)

DATA:

Q =
70.61,412.75,348.40,142.29,103.78,45.0,19.06,14.27,10.77,8.69,9.48,1
8.19
;
e = 231.81, 147.57, 147.57, 152.14, 122.96, 121.76, 99.89, 97.44,
106.14,
146.29, 220.97, 246.75;
A0 = 37.01;
slope = 0.117115;
K=600;
M=990000;
ENDDATA
END

K = 400
K = 300
K = 200

Non-zero spills.

Solution:

R = 89.35 Mm^3

RESERVOIR OPERATION

Reservoir Operation

- A reservoir operating policy is a sequence of release decisions in operational periods, specified as a function of the state of the system.

- State of the system: storage at beginning of a period; inflow during the period etc.

- Most common policy implemented in practice – Standard Operating Policy (SOP)