INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 20

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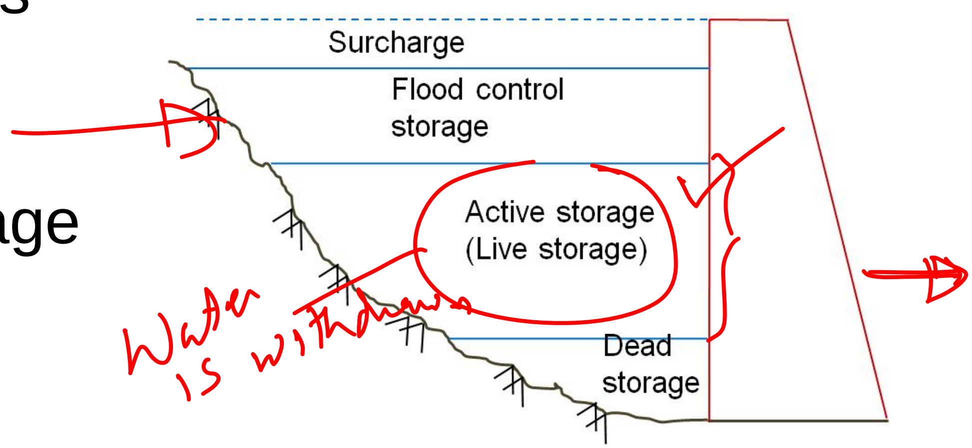
Summary of the previous lecture

- Multi-objective optimization
  - Weighting method
    - Attach weights to each objective

$$Z = w_1 Z_1 + w_2 Z_2 + \ldots\ldots\ldots + w_p Z_p$$

  - Constraint method
    - One objective is maximized with lower bounds on all the others

- Reservoir System
  - Flood control storage
  - Active storage
  - Dead storage



Reservoir Systems – Deterministic Inflows

Reservoir systems

- Reservoir modeling with deterministic inputs.
- Model formulations for two important aspects:
  - Reservoir sizing
  - Reservoir operation

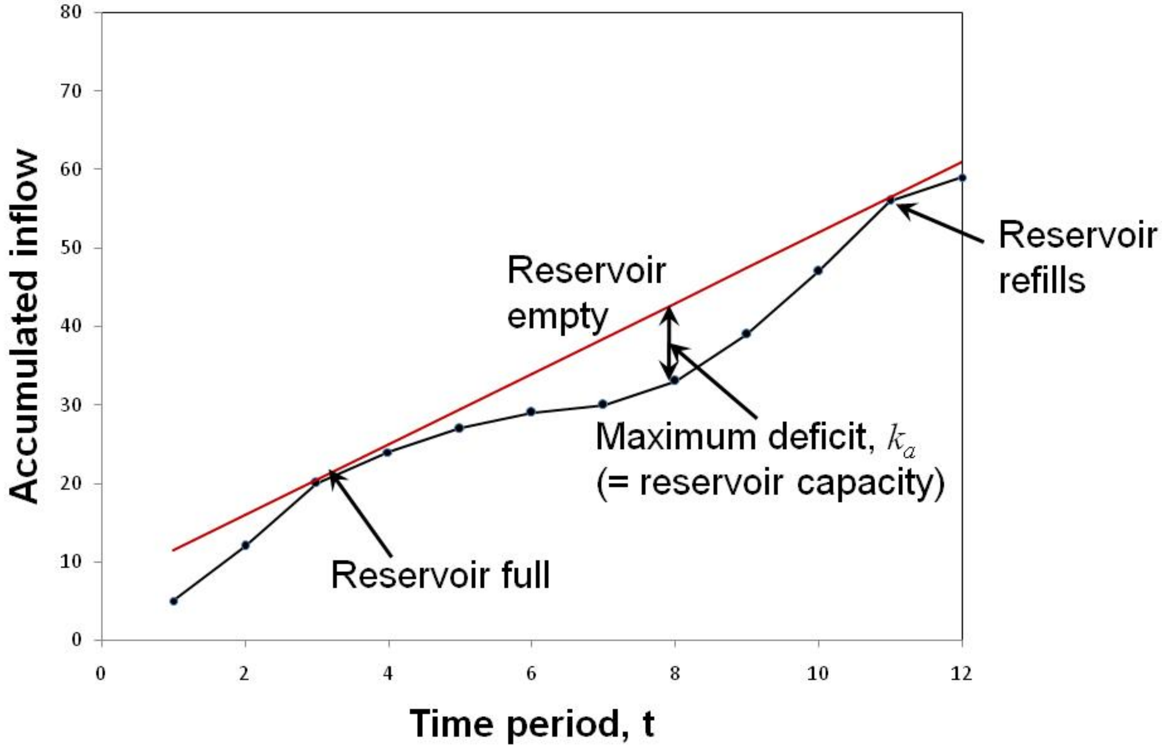
Reservoir Sizing

- The problem of reservoir sizing involves determination of the required storage capacity of the reservoir for given inflows and demands in a sequence of periods.

- The inflow sequence is assumed to repeat, i.e., if the inflow sequence is a year, the inflow in a given period (within the year) is same in all years.
    - Deterministic inflows

Reservoir Sizing

- Determine active storage capacity using Ripple diagram or the mass diagram.
- Plot with time as abscissa and cumulative inflow as ordinate.
- For constant release R_t, line with slope R_t placed tangential.
- Maximum distance between the two gives active storage.



Reservoir Sizing
(Sequent Peak Analysis)

Sequent peak analysis :

- Used for constant or varying demands.
- Find the maximum cumulative deficit over adjacent sequences of deficit periods and determine the maximum of these cumulative deficits.
- The inflow sequence is assumed to repeat and the analysis is carried out over two cycles (when necessary).
- Two cycles are required in case the critical period lies towards the end of an inflow sequence.

Reservoir Sizing

Sequent peak algorithm:

Let t denote the time period and K_t be defined as follows

$$K_t = K_{t-1} + R_t - Q_t \qquad \text{......... if positive}$$
$$= 0 \qquad \text{......... otherwise}$$

where Q_t is the inflow,

R_t the required release or demand in period t.

K_0 set equal to zero ($K_0 = 0$).

Reservoir Sizing

K_t may be expressed conveniently as

$$K_t = \text{Max}\ [\ 0,\ K_{t-1} + R_t - Q_t]$$

K_t values are computed for each time period for two successive cycles of inflow sequence.

Let K^* = Max $\{K_t\}$ over all t, then K^* is the required active storage capacity of the reservoir

Reservoir Sizing

- If the value of K_t is zero at the end of the last period of first cycle, then computations over the second cycle are not necessary.
- This happens when the critical period is entirely contained in the first cycle.
- Beyond a time period t for which the value of K_t is exactly same as that in the first cycle for the corresponding period, then also the computations can be ceased.

Example – 1

Determine the required capacity of a reservoir whose inflows and demands over a 6-period sequence are as given below

Period, t	1	2	3	4	5	6
Inflow, Q_t	4	8	7	3	2	0
Demand, R_t	5	0	5	6	2	6

Total inflows = total demand = 24 units

Example – 1 (Contd.)

K_t = Max [0, $K_{t-1} + R_t - Q_t$]

t	R_t	Q_t	K_{t-1}	$K_t = K_{t-1} + R_t - Q_t$
1	5	4	0	1
2	0	8	1	0
3	5	7	0	0
4	6	3	0	3
5	2	2	3	3
6	6	0	3	9
1	5	4	9	10
2	0	8	10	2
3	5	7	2	0
4	6	3	0	3
5	2	2	3	3

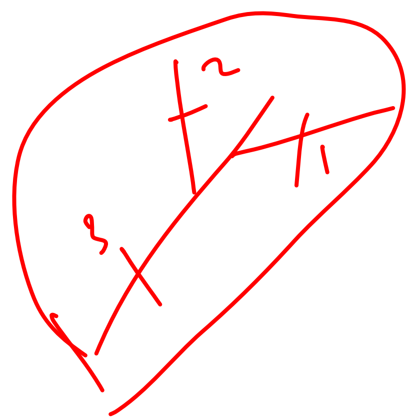
Reservoir capacity = Max {K_t} = 10

Lele (1987) → Modified Sequent Peak algorithm

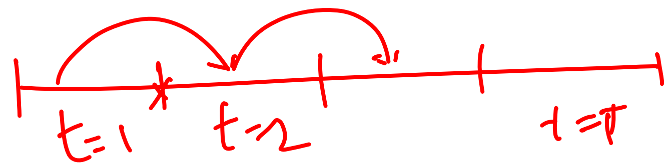
repeats

Reservoir Sizing

- Sequent peak method does not require any optimization software.
- Inclusion of evaporation losses is not easy.
- The algorithm is not readily adaptable to a system with more than one reservoir.
- Mathematical programming tools provide such a capability.



Reservoir Capacity Using LP



Introduction:

- An alternative and more elegant method to sequent peak method.
- Assumption: Inflows are deterministic.
- In LP, the linearity assumption simplifies incorporating the evaporation loss function easily into storage continuity relationships.
- Two sets of constraints to be satisfied: one relates to storage continuity and the other to capacity.
- Let R_t be the release and D_t the specific demand.

Reservoir Capacity Using LP

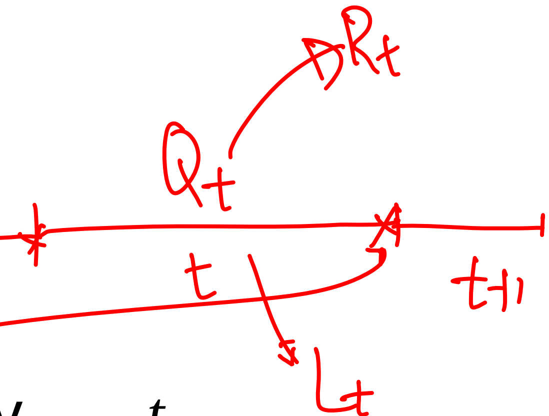
Optimization model for active storage:

Min K_a

s.t.

a. Mass balance

$$S_t + Q_t - R_t - L_t = S_{t+1} \qquad \forall \quad t$$

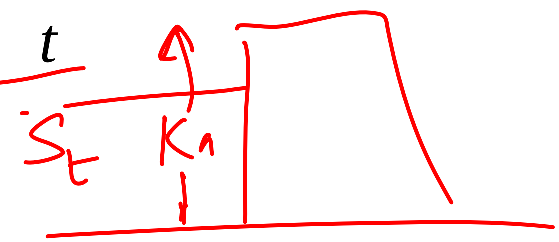


b. Maximum active storage

$$S_t \le K_a \qquad \forall \quad t$$

c. Non-negativity

$$S_t \ge 0; \;\; K_a \ge 0$$



Reservoir Capacity Using LP

In this,

R_t is pre-specified (known) release

Q_t is known inflow

L_t is estimated storage loss

S_t : storage at beginning of period t

K_a : active storage capacity

} Decision variables

Example – 2

Using LP, determine the required capacity of a reservoir whose inflows and demands over a 6-period sequence are as given below

Period, t	1	2	3	4	5	6
Inflow, Q_t	4	8	7	3	2	0
Demand, R_t	5	0	5	6	2	6

Total inflows = total demand = 24 units

Example – 2 (Contd.)

Minimize K

s.t.

$$S_1 + 4 - 5 = S_2$$
$$S_2 + 8 - 0 = S_3$$
$$S_3 + 7 - 5 = S_4$$
$$S_4 + 3 - 6 = S_5$$
$$S_5 + 2 - 2 = S_6$$
$$S_6 + 0 - 6 = S_1$$
$$S_1 \leq K \,;\; S_2 \leq K \,;\; S_3 \leq K \,;\; S_4 \leq K \,;\; S_5 \leq K \,;\; S_6 \leq K$$

Solution:

$K = 10$

$S_1 = 1$

$S_2 = 0$

$S_3 = 8$

$S_4 = 10$

$S_5 = 7$

$S_6 = 7$

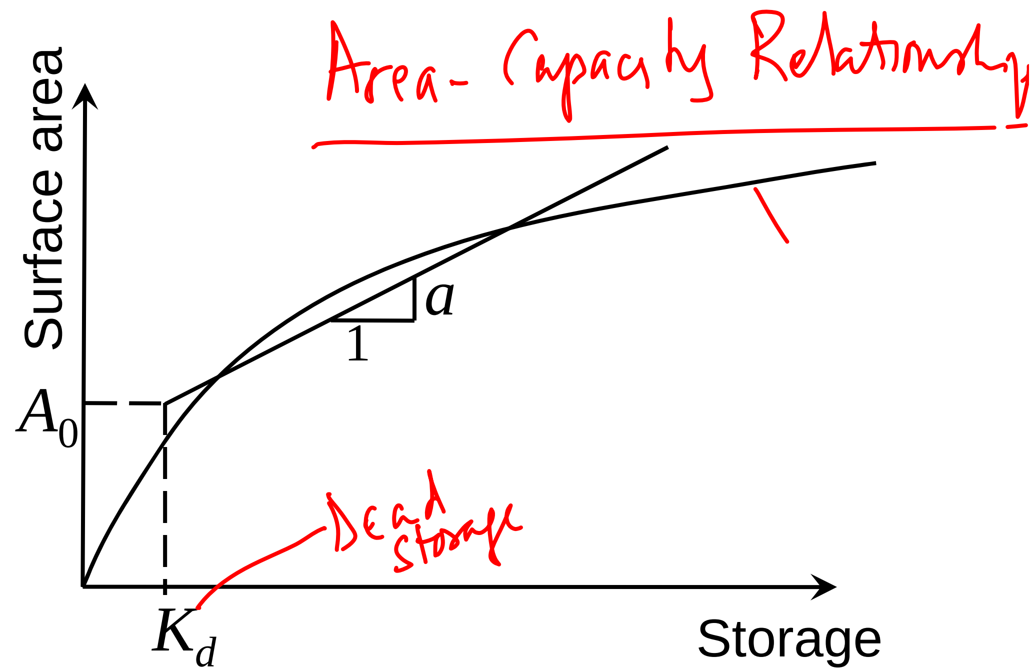
Reservoir Capacity Using LP

Continuity, with evaporation loss accounted

K_d : dead storage

A_0 : Surface area at dead storage

a : area per unit active storage above A_0.



Total evaporation in period t is given by

$$E_t = A_0 e_t + a\left(\frac{S_t + S_{t+1}}{2}\right)e_t$$