INDIAN INSTITUTE OF SCIENCE

Water Resources Systems:

Modeling Techniques and Analysis

Lecture - 14

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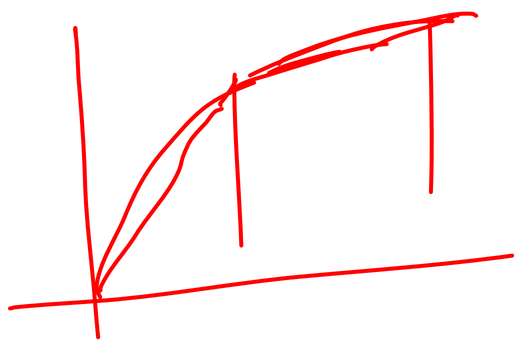
Summary of the previous lecture

- Dual problem
    - Formulation of dual problem
    - Dual problem solution from solution of the primal problem
- Sensitivity analysis
    - Change in the RHS of constraints

Piecewise Linearization



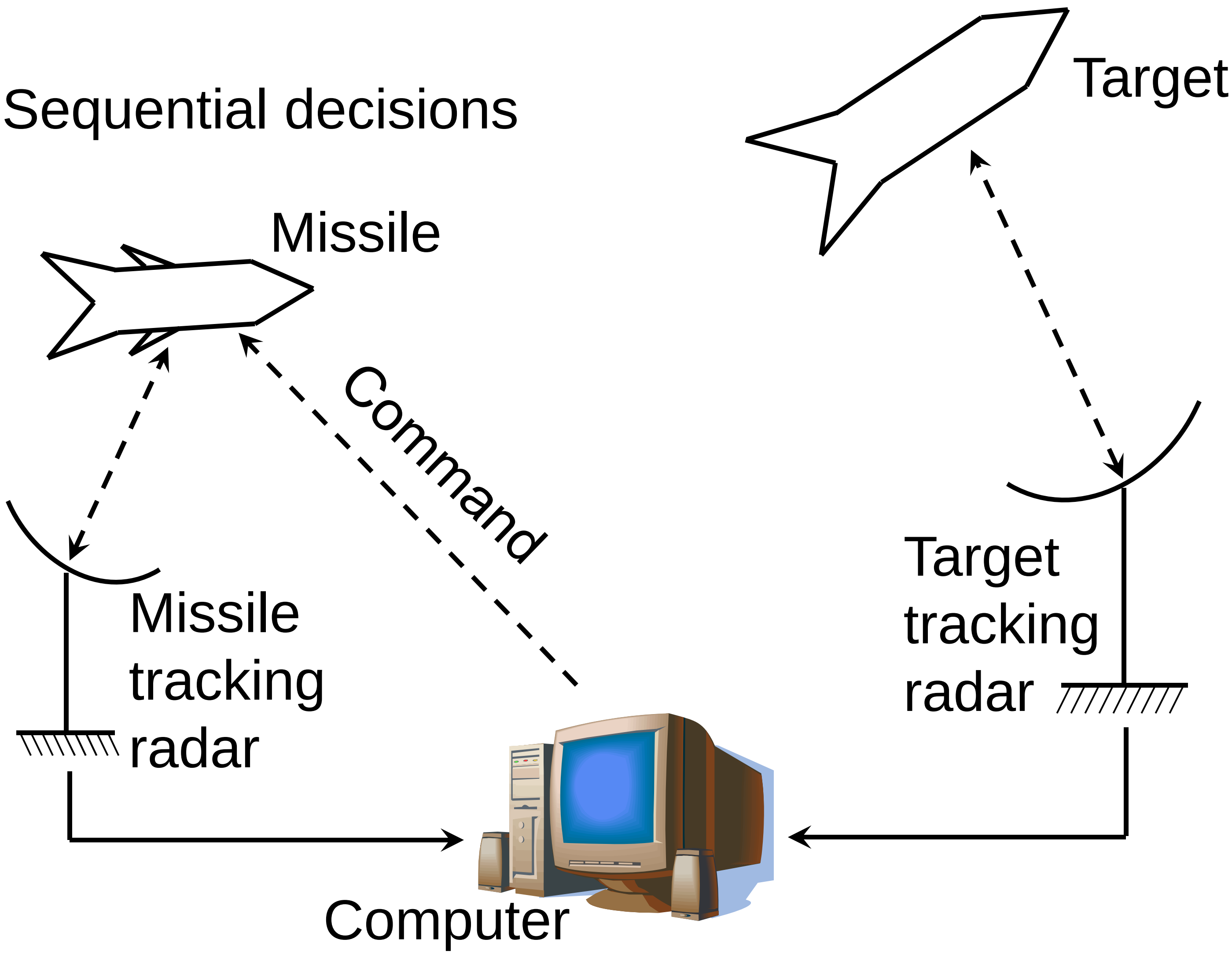
DYNAMIC PROGRAMMING

Dynamic Programming

- Dynamic programming (DP) is ideally suited for sequential decision problems.
- DP is a mathematical technique well suited for the optimization of multistage decision problems.
- Developed by Richard Bellman in the early 1950's.
- Applications :
  - Reservoir operation, water allocation, capacity expansion, irrigation scheduling, water quality control, shortest route problems etc.

An Example of Multi-stage Decisions

- Sequential decisions



Ref: S.S.Rao (1996), Engineering optimization, Theory and practice, John Wiley & Sons, Inc.

Dynamic Programming

- State of the system
  - Current position, speed and orientation of Missile and Target.

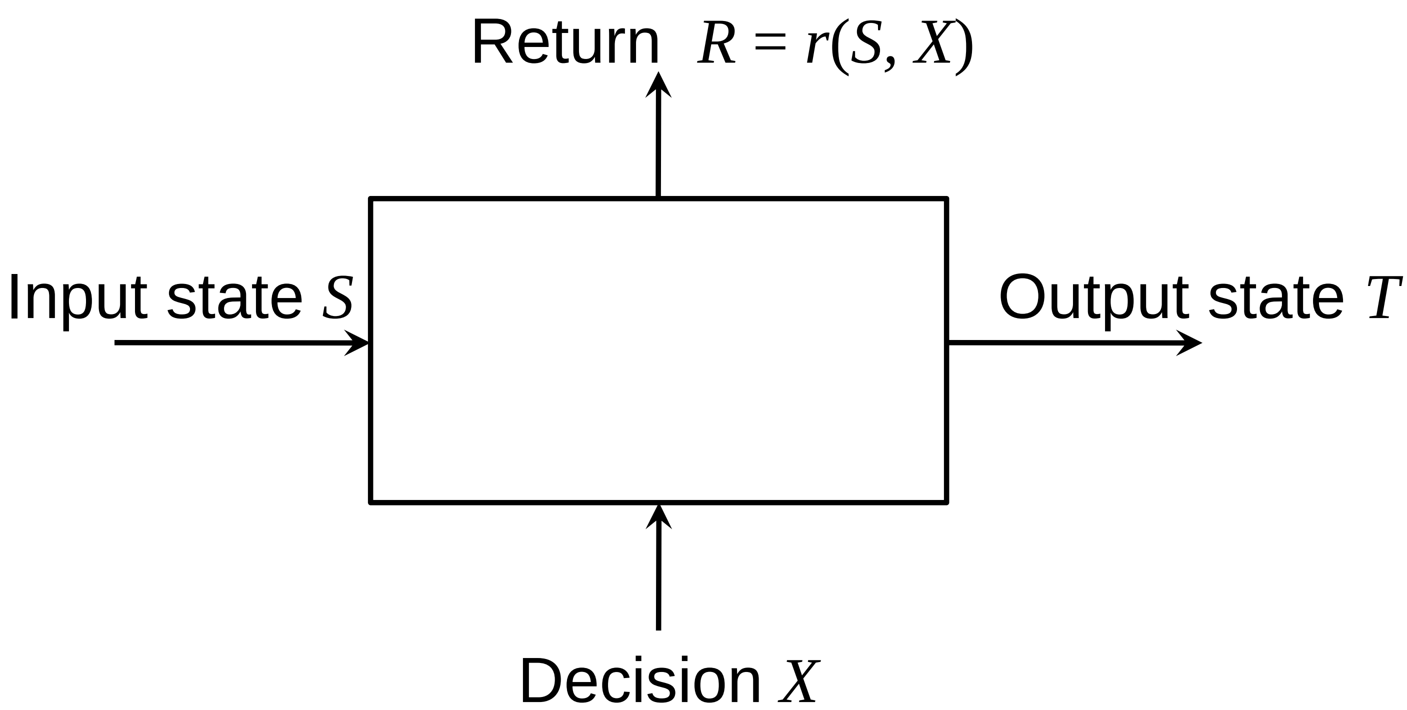
- Decision: Speed and orientation for the Missile during next time interval.

- Objective: To hit the Target in minimum time.

Dynamic Programming

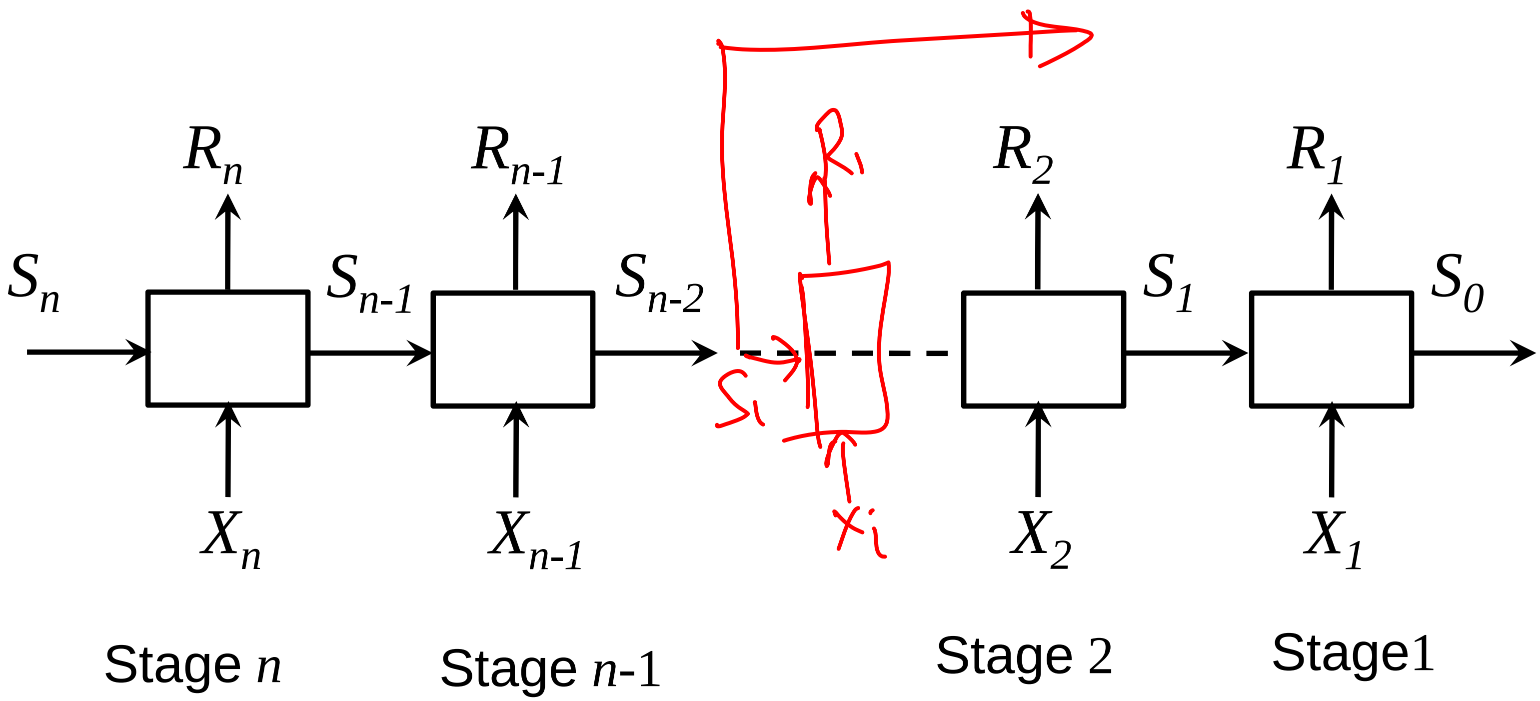
Representation of DP problem:

- Single stage decision problem



Dynamic Programming

- Serial multi-stage decision problem
  - Output from one state is input to the next



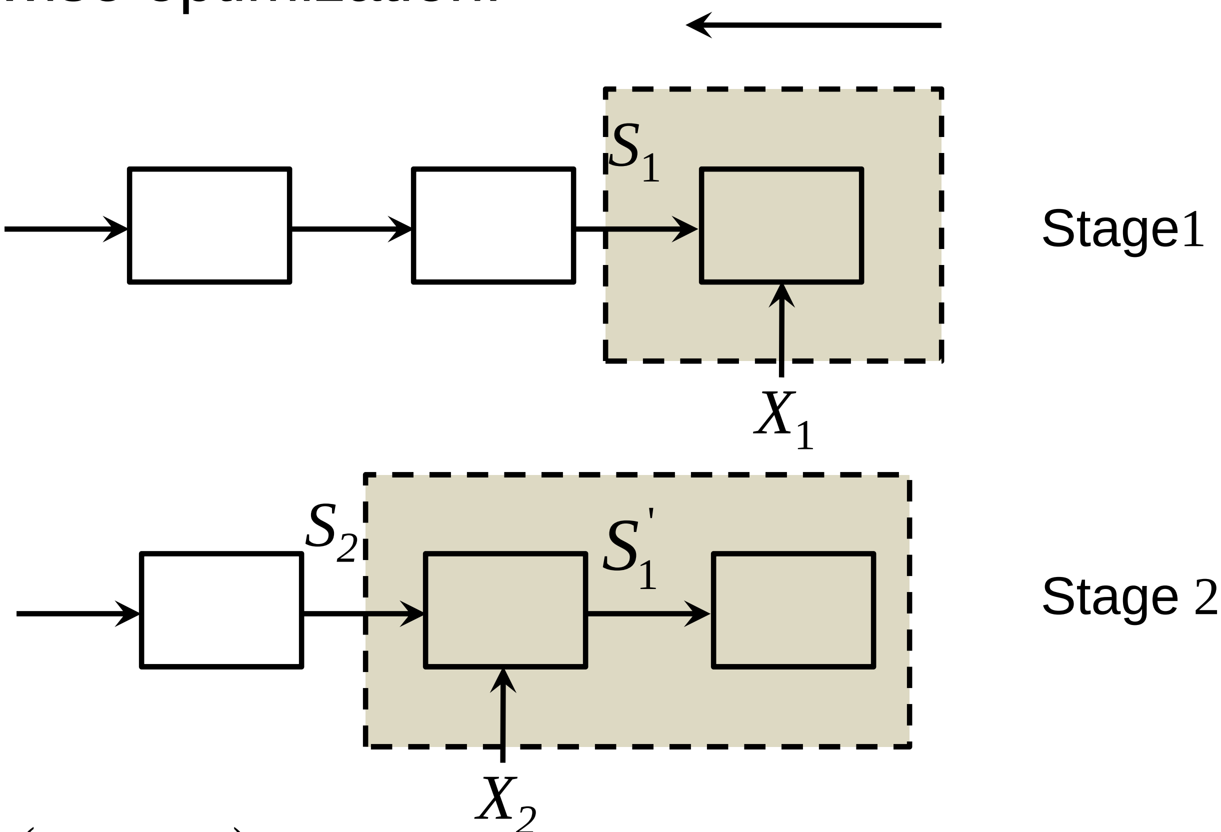
Dynamic Programming

Bellman's principle of optimality:

- "Given the current state of system, the optimal policy (sequence of decisions) for the remaining stages is independent of the policy adopted in the previous stages".
- The principle implies that, given the state S_i of the system at a stage i, one must proceed optimally till the last stage, irrespective of how one arrived at the state S_i.

Dynamic Programming

Stage-wise optimization:



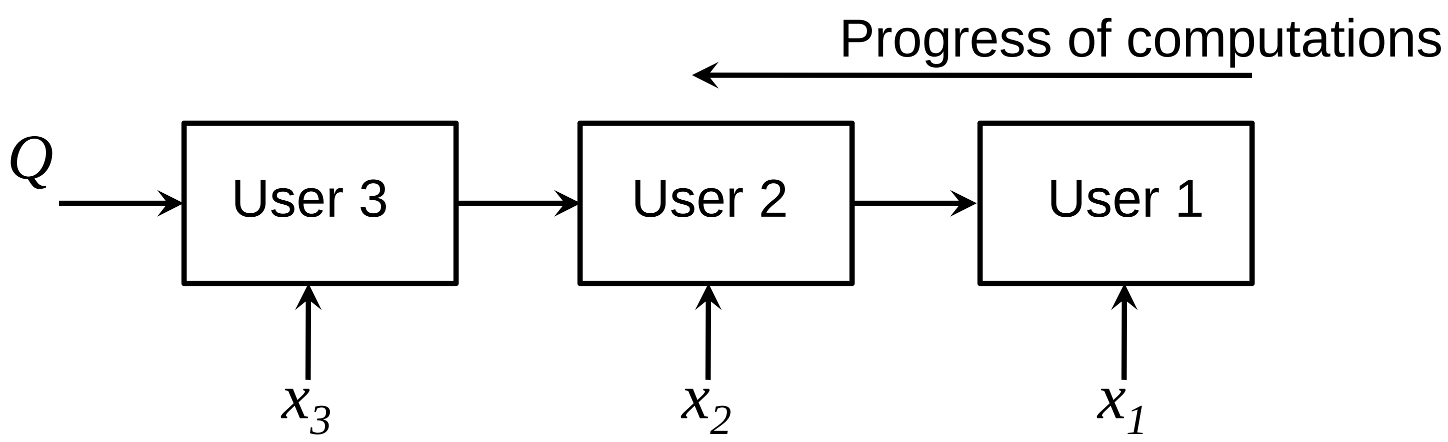
$S_1^{'} = T\left(S_2, X_2\right)$ …. State transformation: A function of state variable S_2 and decision X_2 .

Dynamic Programming

Water allocation problem:

- A total of 6 units of water is to be allocated optimally to three users, User 1, User 2 and User 3.

- The allocation is made in discrete steps of one unit ranging from 0 to 6.



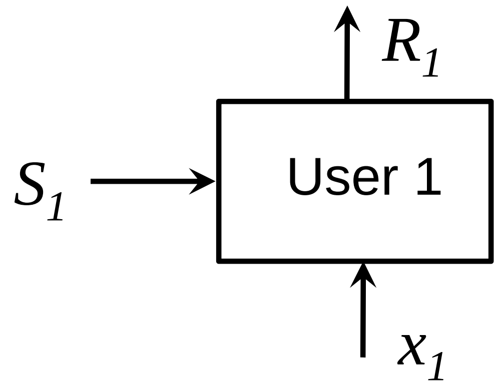
Dynamic Programming

- The returns obtained from the users for a given allocation are as follows

Amount of water allocated X	Return from		
	User 3 $R_3(x)$	User 2 $R_2(x)$	User 1 $R_1(x)$
0	0	0	0
1	5	5	7
2	8	6	12
3	9	3	15
4	8	-4	16
5	5	-15	15
6	0	-30	12

Dynamic Programming

Stage 1:



$$f_1^*(S_1) = \text{Max}\left[R_1(x_1)\right]$$

$$0 \le x_1 \le S_1$$

$$0 \le S_1 \le Q$$

Total amount available (=6)

S_i : Amount of Water available to be allocated to all users included in that stage.

S_1 : Amount of water available for allocation to User 1

x_1 : Amount of water allocated to User 1

x_1^* : Allocation to User 1, that results in $f_1^*(S_1)$

$f_1^*(S_1)$: Maximum return due to allocation ~~of S_1~~

Dynamic Programming

S_1	x_1	$R_1(x_1)$	$f_1^*(S_1) = \text{Max}\left[R_1(x_1)\right]$	x_1^*
0	0	0	0	0
1	0	0	7	1
	1	7		
2	0	0	12	2
	1	7		
	2	12		
3	0	0	15	3
	1	7		
	2	12		
	3	15		

Contd.

Contd.

S_1	x_1	$R_1(x_1)$	$f_1^*(S_1) = \text{Max}\left[R_1(x_1)\right]$	x_1^*
4	0 1 2 3 4	0 7 12 15 16	16	4
5	0 1 2 3 4 5	0 7 12 15 16 15	16	4
6	0 1 2 3 4 5 6	0 7 12 15 16 15 12	16	4