Stochastic Structural Dynamics

Lecture-34

Probabilistic methods in earthquake engineering-3

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Examples of stochastic models for earthquake ground motions

- Single component: stationary & nonstationary models
- Multi-component and spatially varying load models
- Gaussian and Poisson pulse process models

Main concerns

- frequency content
- transient nature and duration
- time dependent frequency content
- multi-component nature
- spatial variability
- translations and rotations
- models for displacement and velocity components
- seismological considerations

Spectral representation of an evolutionary random process

$$X(t) = \int_{-\infty}^{\infty} a(t,\omega)\exp(i\omega t)\,dZ(\omega)$$

$a(t,\omega) =$ deterministic function (in general, complex valued)

$Z(\omega) =$ orthogonal increment random process (complex valued)

with $\langle dZ(\omega)\rangle = 0$ & $\langle dZ(\omega_1)\,dZ^*(\omega_2)\rangle = \delta(\omega_1 - \omega_2)\,d\Psi(\omega)$

$$\langle X(t_1)X^*(t_2)\rangle = \int_{-\infty}^{\infty} a(t_1,\omega)\,a^*(t_2,\omega)\exp\left[i\omega(t_1 - t_2)\right]d\Psi(\omega)$$

$$\sigma_X^2(t) = \int_{-\infty}^{\infty} |a(t,\omega)|^2\,d\Psi(\omega)$$

If $d\Psi(\omega) = \Phi(\omega)\,d\omega,$ we get $\sigma_X^2(t) = \int_{-\infty}^{\infty} |a(t,\omega)|^2\,\Phi(\omega)\,d\omega$

We interpret $S_{XX}(\omega) = |a(t,\omega)|^2\,\Phi(\omega)$ as the nonstationary (evolutionary) PSD function of $X(t)$.

Filtered Poisson Process models for earthquake ground motions

Rationale

During earthquakes slips occur along fault lines in an intermittent manner. This sends out a train of stress waves in the earth crust. This eventually results in ground shaking.

Recall

$$X(t) = \sum_{j=1}^{N(T)} Y_j w(t, \tau_j); 0 < t \le T$$

$N(T)$ = counting process, Poisson; arrival rate $= \lambda(t)$

τ_{j} = arrival times; random

$w(t, \tau_j)$ = Deterministic pulse shape $\left(= 0 \forall \mathrm{t} \le \tau_j\right)$.

Y_j = random magnitude of the j-th pulse.

$$m_X(t) = m_Y \int_0^t w(t,\tau)\lambda(\tau)\,d\tau,$$

$$C_{XX}(t_1,t_2) = E(Y^2) \int_0^{\min(t_1,t_2)} w(t_1,\tau)\,w(t_2,\tau)\,\lambda(\tau)\,d\tau,$$

$$\sigma_X^2(t) = E(Y^2)\int_0^t w^2(t,\tau)\,\lambda(\tau)\,d\tau$$

Reference

Y K Lin and G C Cai, 1995, McGraw Hill, NY.

Let $X(t) = \sum_{j=1}^{N(T)} Y_j w(t - \tau_j); 0 < t \le T$

$$m_X(t) = m_Y \int_0^t w(t-\tau)\lambda(\tau)\,d\tau,$$

$$C_{XX}(t_1,t_2) = E(Y^2) \int_0^{\min(t_1,t_2)} w(t_1-\tau)\,w(t_2-\tau)\,\lambda(\tau)\,d\tau,$$

Let $\lambda(\tau) = 0 \,\forall\, \tau < 0$. Since $\mathrm{w}(t-\tau) = 0 \,\forall\, t-\tau < 0$ we can write

$$C_{XX}(t_1,t_2) = E(Y^2) \int_{-\infty}^{\infty} w(t_1-\tau)\,w(t_2-\tau)\,\lambda(\tau)\,d\tau$$

We introduce

$$b(t,\omega)=\int_{-\infty}^{\infty} w(u)\sqrt{\lambda(t-u)}\exp(-i\omega u)\,du$$

so that

$$w(u)\sqrt{\lambda(t-u)}=\frac{1}{2\pi}\int_{-\infty}^{\infty} b(t,\omega)\exp(i\omega u)\,d\omega$$

Let $t\text{ - }u=\tau \Rightarrow$

$$w(t-\tau)\sqrt{\lambda(\tau)}=\frac{1}{2\pi}\int_{-\infty}^{\infty} b(t,\omega)\exp\left[i\omega(t-\tau)\right]d\omega$$

LHS is real $\Rightarrow$

$$w(t-\tau)\sqrt{\lambda(\tau)}=\frac{1}{2\pi}\int_{-\infty}^{\infty} b^{*}(t,\omega)\exp\left[-i\omega(t-\tau)\right]d\omega$$

Substitute

$$w\left(t_1-\tau\right)\sqrt{\lambda\left(\tau\right)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}b\left(t_1,\omega\right)\exp\left[i\omega\left(t_1-\tau\right)\right]d\omega$$

$$w\left(t_2-\tau\right)\sqrt{\lambda\left(\tau\right)}=\frac{1}{2\pi}\int_{-\infty}^{\infty}b^*\left(t_2,\omega\right)\exp\left[-i\omega\left(t_2-\tau\right)\right]d\omega$$

into $C_{XX}\left(t_1,t_2\right)=E\left(Y^2\right)\int_{-\infty}^{\infty}w\left(t_1-\tau\right)w\left(t_2-\tau\right)\lambda\left(\tau\right)d\tau$

and noting that $\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[-i\left(\omega_2-\omega_1\right)\tau\right]d\tau=\delta\left(\omega_2-\omega_1\right)\Rightarrow$

$$C_{XX}\left(t_1,t_2\right)=\frac{E\left(Y^2\right)}{2\pi}\int_{-\infty}^{\infty}b\left(t_1,\omega\right)b^*\left(t_2,\omega\right)\exp\left[-i\omega\left(t_2-t_1\right)\right]d\omega$$

$$C_{XX}\left(t_1,t_2\right)=\frac{E\left(Y^2\right)}{2\pi}\int_{-\infty}^{\infty} b\left(t_1,\omega\right)b^*\left(t_2,\omega\right)\exp\left[-i\omega\left(t_2-t_1\right)\right]d\omega$$

$$\Rightarrow \sigma_X^2\left(t\right)=\frac{E\left(Y^2\right)}{2\pi}\int_{-\infty}^{\infty}\left|b\left(t,\omega\right)\right|^2 d\omega$$

$$\Rightarrow S_{XX}\left(t,\omega\right)=\frac{E\left(Y^2\right)}{2\pi}\left|b\left(t,\omega\right)\right|^2$$

with

$$b\left(t,\omega\right)=\int_{-\infty}^{\infty} w\left(u\right)\sqrt{\lambda\left(t-u\right)}\exp\left(-i\omega u\right)du$$

Selection of the shape of the pulse

Model -1

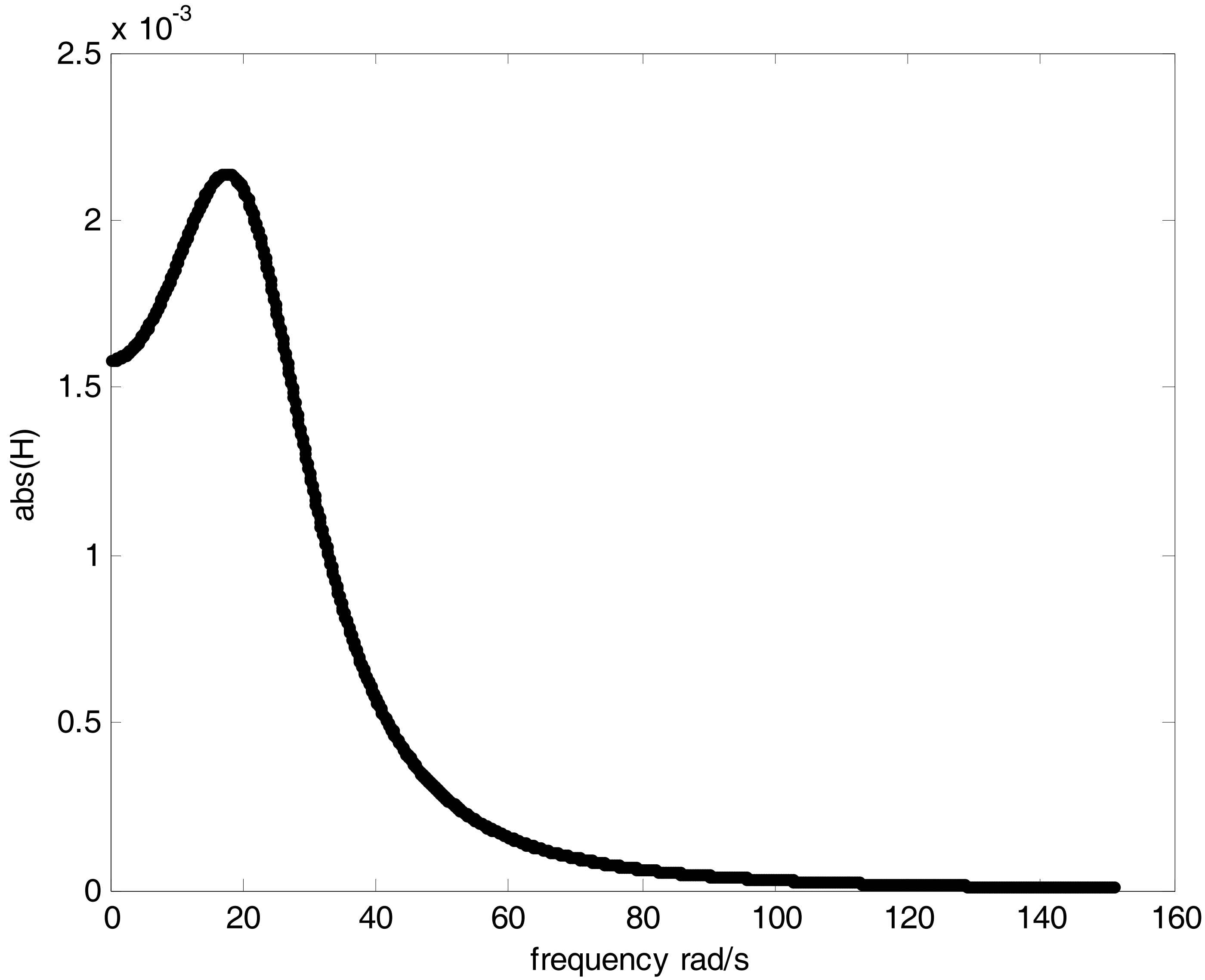
As in Kanai Tajimi model, the soil layer is modeled as an elastic half-space which can be represented as a sdof system.

$$\ddot{u} + 2\eta_g\omega_g\dot{u} + \omega_g^2 u = 2\eta_g\omega_g\dot{R} + \omega_g^2 R$$

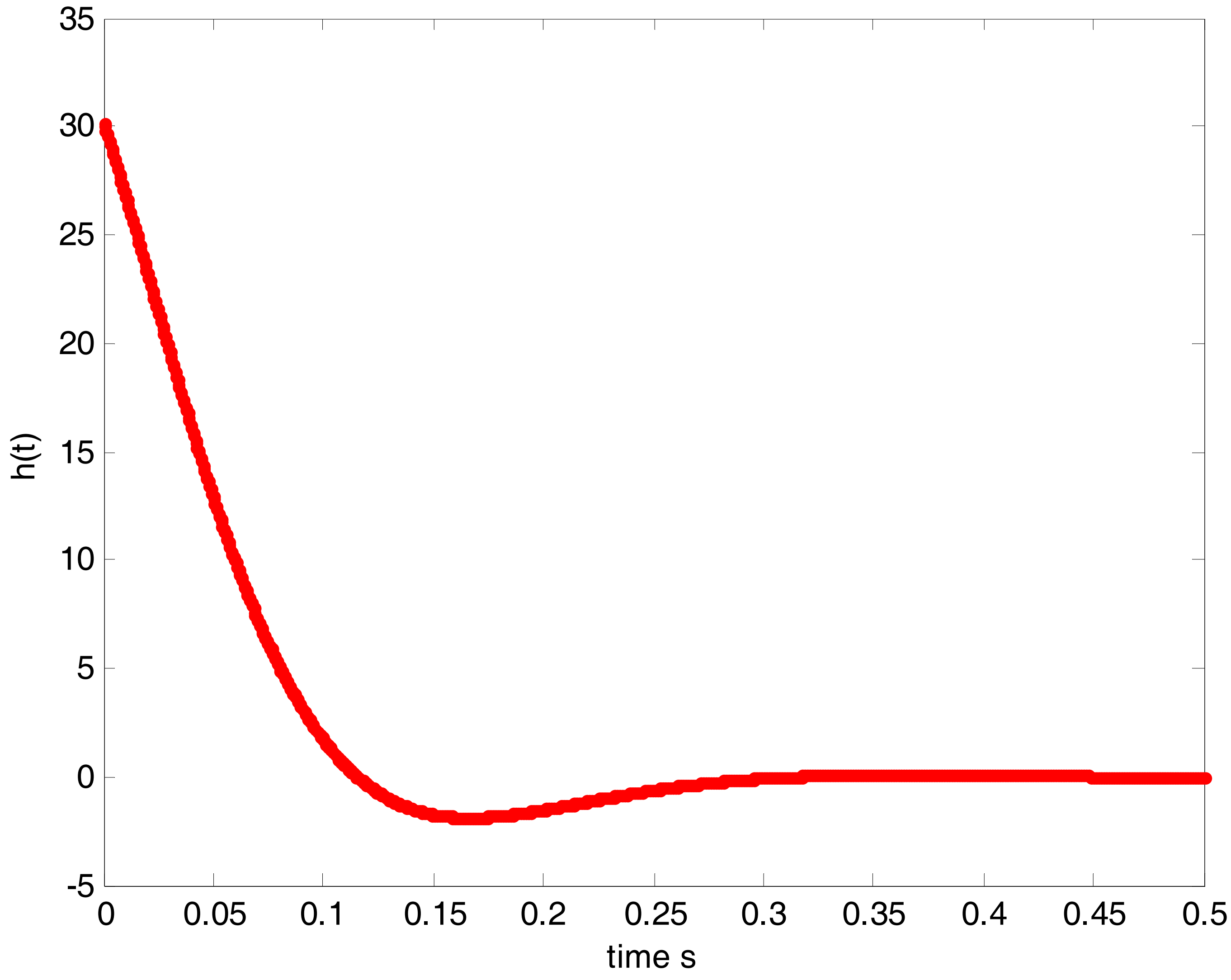
$$H_1(\omega) = \frac{\omega_g^2 + i2\eta_g\omega_g\omega}{\left(\omega_g^2 - \omega^2\right)^2 + \left(2\eta_g\omega_g\omega\right)^2}$$

$$h_1(t) = \omega_g \exp\left(-\eta_g\omega_g t\right)\left\{\frac{1-2\eta_g^2}{\sqrt{1-\eta_g^2}}\sin\omega_{gd}t + 2\eta_g\cos\omega_{gd}t\right\}; t > 0$$

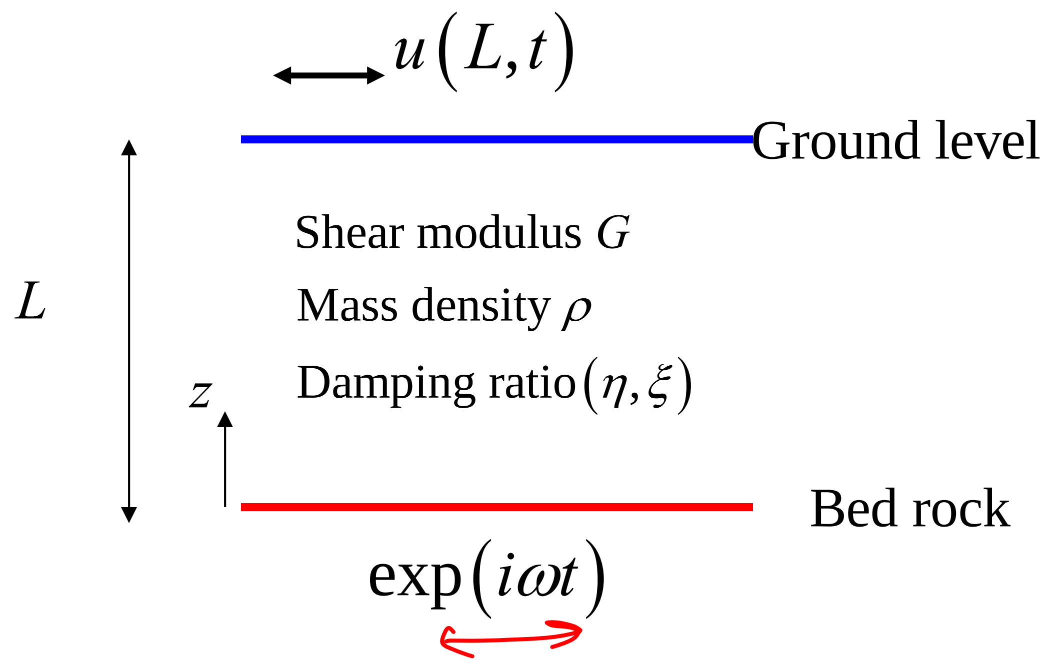
$$G(t) = \sum_{j=1}^{N(T)} Y_j h_1\left(t - \tau_j\right)$$



x 10^-3
2.5
2
1.5
1
0.5
0
abs(H)
0
20
40
60
80
100
120
140
160
frequency rad/s


h(t)
time s
35
30
25
20
15
10
5
0
-5
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5

Seismic wave amplification through soil layers



$$\rho \frac{\partial^2 u}{\partial t^2} = G \frac{\partial^2 u}{\partial z^2} + \eta \frac{\partial^3 u}{\partial z^2 \partial t}$$

$$u(0,t) = \exp(i\omega t); \frac{\partial u}{\partial z}(L,t) = 0$$

$$u(z,t) = \phi(z)\exp(i\omega t)$$

$$\Rightarrow \quad -\rho\omega^2\phi\exp(i\omega t) = G\phi''\exp(i\omega t) + i\eta\omega\phi''\exp(i\omega t)$$

$$\Rightarrow \quad \phi''(G + i\eta\omega) + \rho\omega^2\phi = 0$$

$$\Rightarrow \quad \phi'' + \lambda^2\phi = 0; \quad \lambda^2 = \frac{\rho\omega^2}{(G + i\eta\omega)}$$

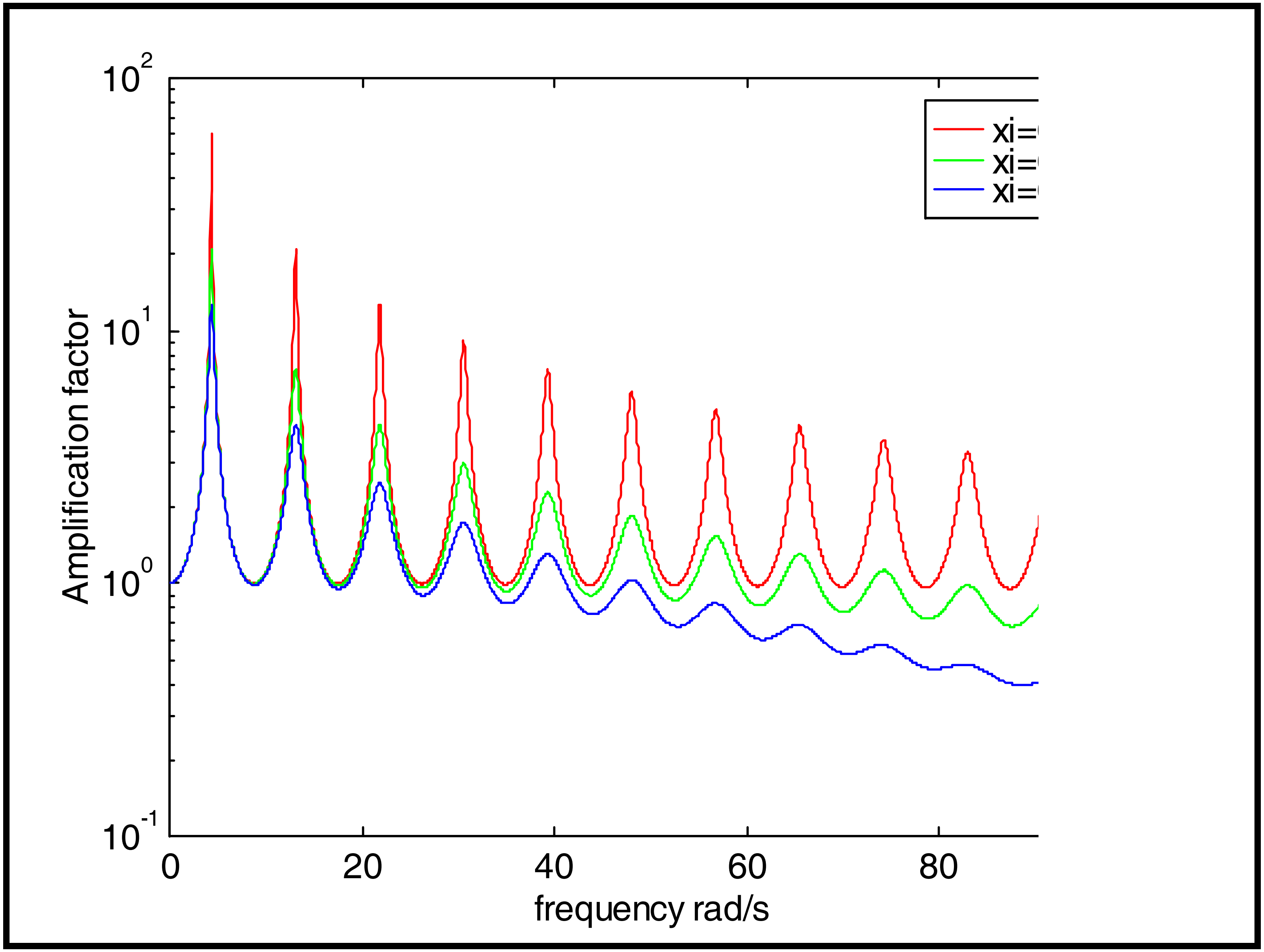
$$\phi(z) = A\cos\lambda z + B\sin\lambda z$$

$$\phi(0) = 1 \quad \phi'(L) = 0$$

$$\Rightarrow \quad \phi(x) = \cos\lambda z + \tan\lambda L\sin\lambda z$$

$$\phi(L) = \frac{1}{\cos \lambda L} = \frac{1}{\cos\left(\frac{\omega L}{\nu^*}\right)}$$

$$\nu^* = \sqrt{\frac{G^*}{\rho}} = \sqrt{\frac{G(1+i\omega\eta)}{\rho}} = \sqrt{\frac{G(1+2i\xi)}{\rho}}$$



xi=
xi=
xi=
Amplification factor
frequency rad/s

Selection of the shape of the pulse

Model -2

Soil layer modeled as a shear beam with hysteretic damping

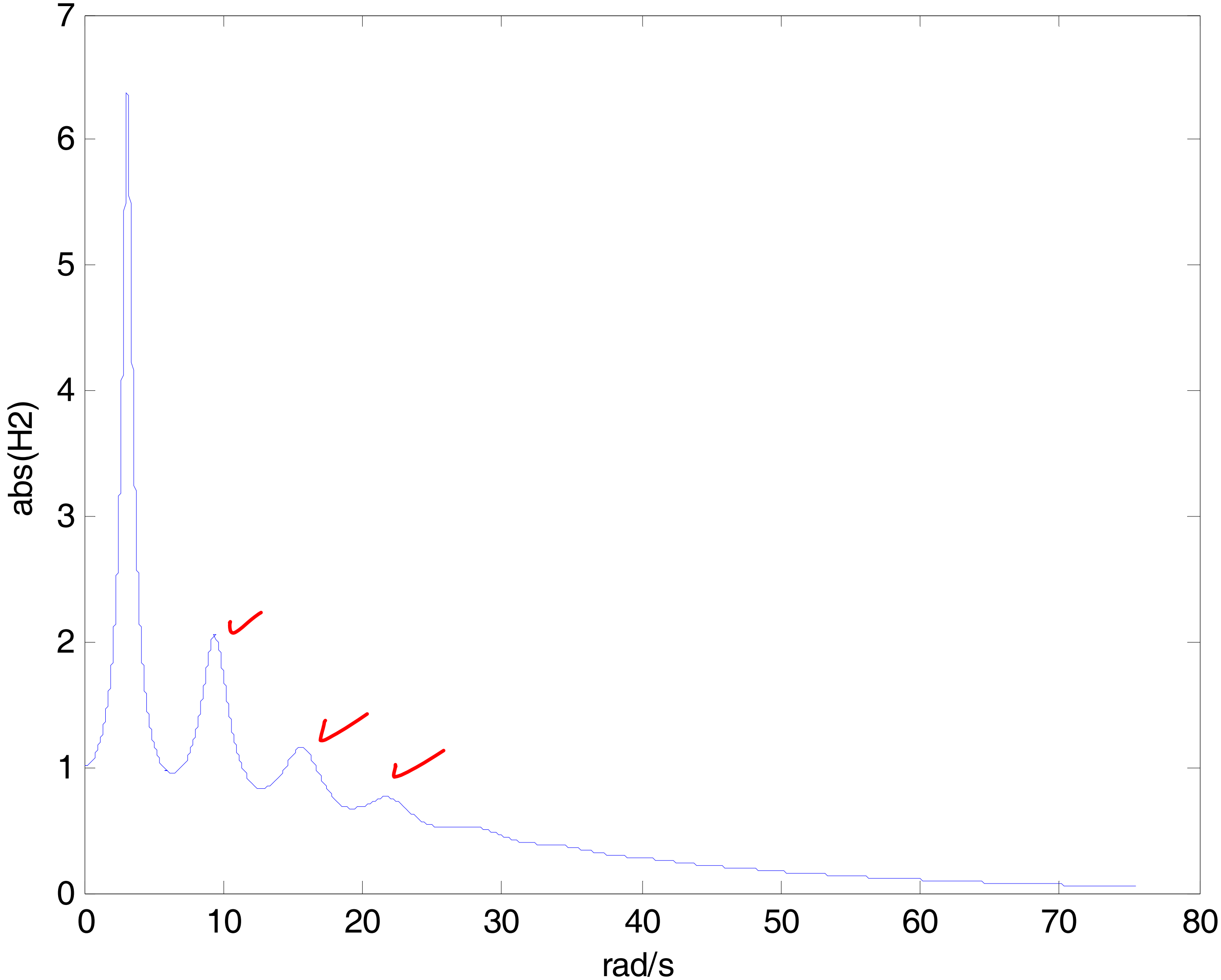
$$\frac{\partial^2 w}{\partial t^2} - \beta^2 \frac{\partial^2 w}{\partial y^2} = 0$$

$$H_2(\omega) = \left[\cos\left\{ \frac{\omega l}{\beta(1 + i\gamma \operatorname{sgn} \omega)} \right\} \right]^{-1}$$

$$h_2(t) = \frac{2\beta}{l} \sum_{n=0}^{\infty} (-1)^n \exp\left[-\left(n + \frac{1}{2}\right) \frac{\pi\beta\gamma}{l} t \right]$$

$$\left\{ \gamma \cos\left[\left(n + \frac{1}{2}\right) \frac{\pi\beta\gamma}{l} t \right] + \sin\left[\left(n + \frac{1}{2}\right) \frac{\pi\beta\gamma}{l} t \right] \right\} t > 0$$

$$G(t) = \sum_{j=1}^{N(T)} Y_j h_2(t - \tau_j)$$



7
6
5
4
3
2
1
0
abs(H2)
0
10
20
30
40
50
60
70
80
rad/s

Selection of the shape of the pulse

Model - 3

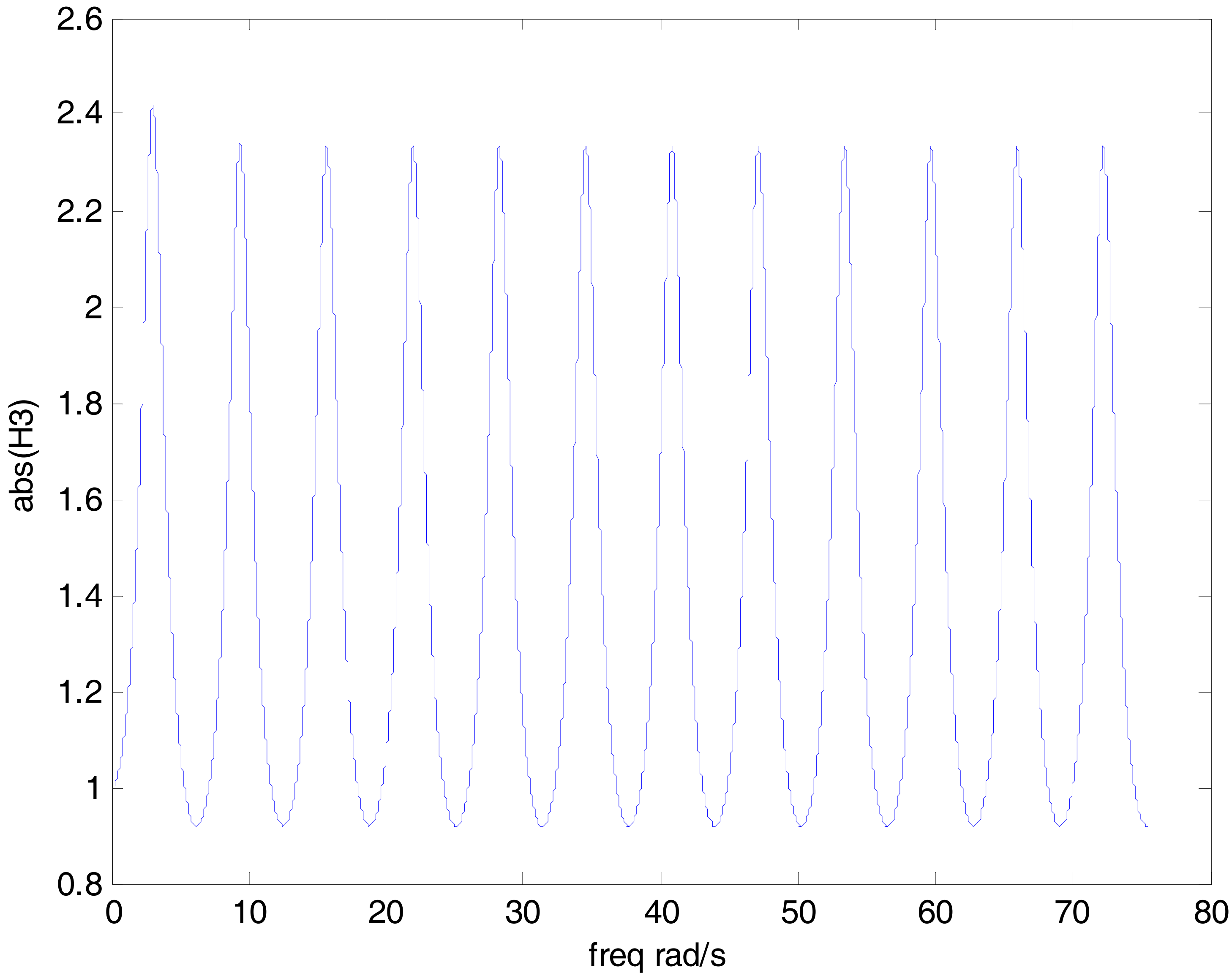
Soil layer modeled as a viscously damped shear beam

$$\frac{\partial^2 w}{\partial t^2} + \frac{1}{\tau_r}\frac{\partial w}{\partial t} - \beta^2 \frac{\partial^2 w}{\partial y^2} = 0$$

$$H_3(\omega) = \left[\cos\left\{\frac{l}{\beta}\sqrt{\omega^2 - i\frac{\omega}{\tau_r}}\right\}\right]^{-1}$$

$$h_3(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_3(\omega)\exp(i\omega t)d\omega;\ t > 0$$

$$G(t) = \sum_{j=1}^{N(T)} Y_j h_3(t - \tau_j)$$



2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
abs(H3)
0
10
20
30
40
50
60
70
80
freq rad/s

Selection of the shape of the pulse

Model -4

Soil layer modeled as a inhomogeneous hysteretically damped shear beam

$$\frac{\partial^2 w}{\partial t^2} - \beta^2 \frac{\partial^2 w}{\partial y^2} - \beta^2 \frac{d\{\ln A(y)\}}{dy} \frac{\partial w}{\partial y} = 0$$

$$H_4(\omega) = \exp(-my)\left[\cos(\delta l) - \frac{m}{\delta}\sin(\delta l)\right]$$

$$\delta = \left[\omega^2 \beta^{-2} (1 + i\gamma \operatorname{sgn} \omega)^{-2} - m^2\right]^{0.5}$$

$$h_4(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H_3(\omega) \exp(i\omega t)\, d\omega; \quad t > 0$$

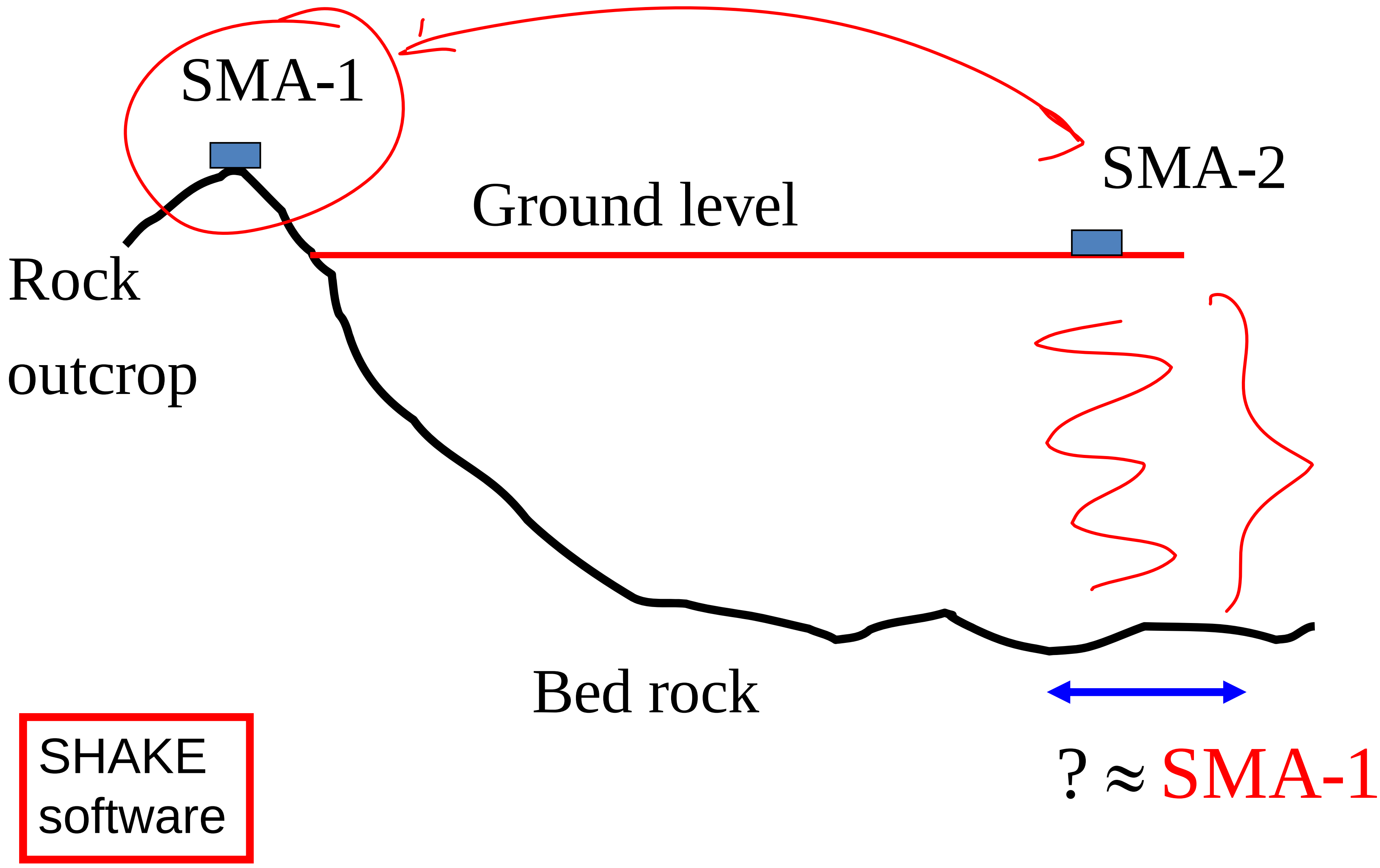
$$G(t) = \sum_{j=1}^{N(T)} Y_j h_4(t - \tau_j)$$

General model

$$G_k(\mathbf{r},t)=\sum_{j=1}^{N(T)} Y_j g_k(\mathbf{r},t;\rho,\upsilon)$$

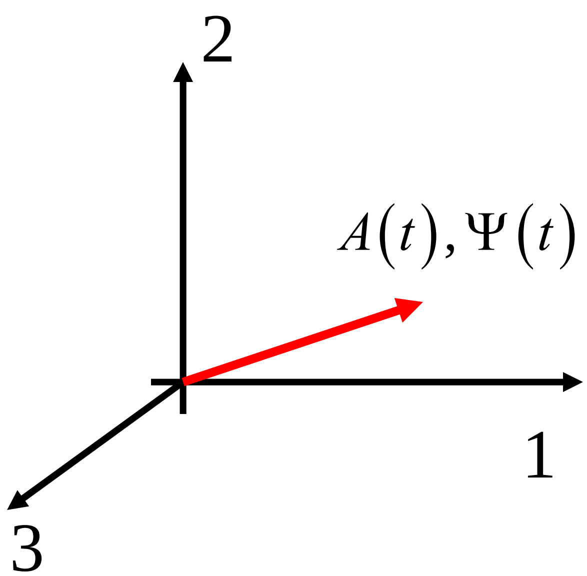
$g_k(\mathbf{r},t;\rho,\upsilon)=$ Green's function which describes the ground acceleration in the $k-$th direction at a site location $\mathbf{r}$ and time t due to an impulsive application of a double couple.

Use elaborate models (3d-layered soil half-space) to estimate the Green's function.


SMA-1
SMA-2
Ground level
Rock
outcrop
Bed rock
SHAKE
software
? ≈ SMA-1

Models for multi-component earthquake ground motions

- Earthquake ground acceleration at any point can be resolved into three components along three orthogonal directions.



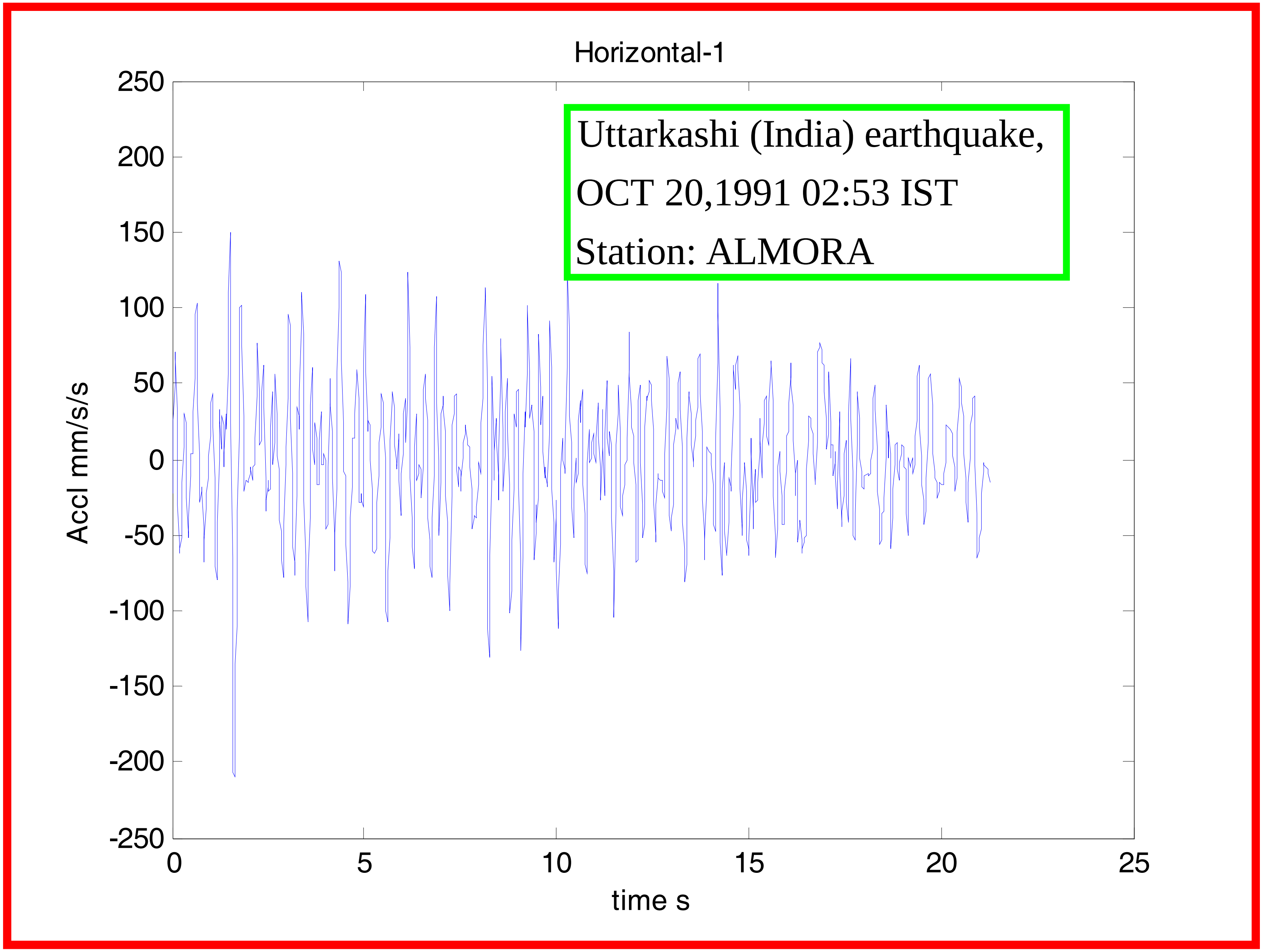
Translation

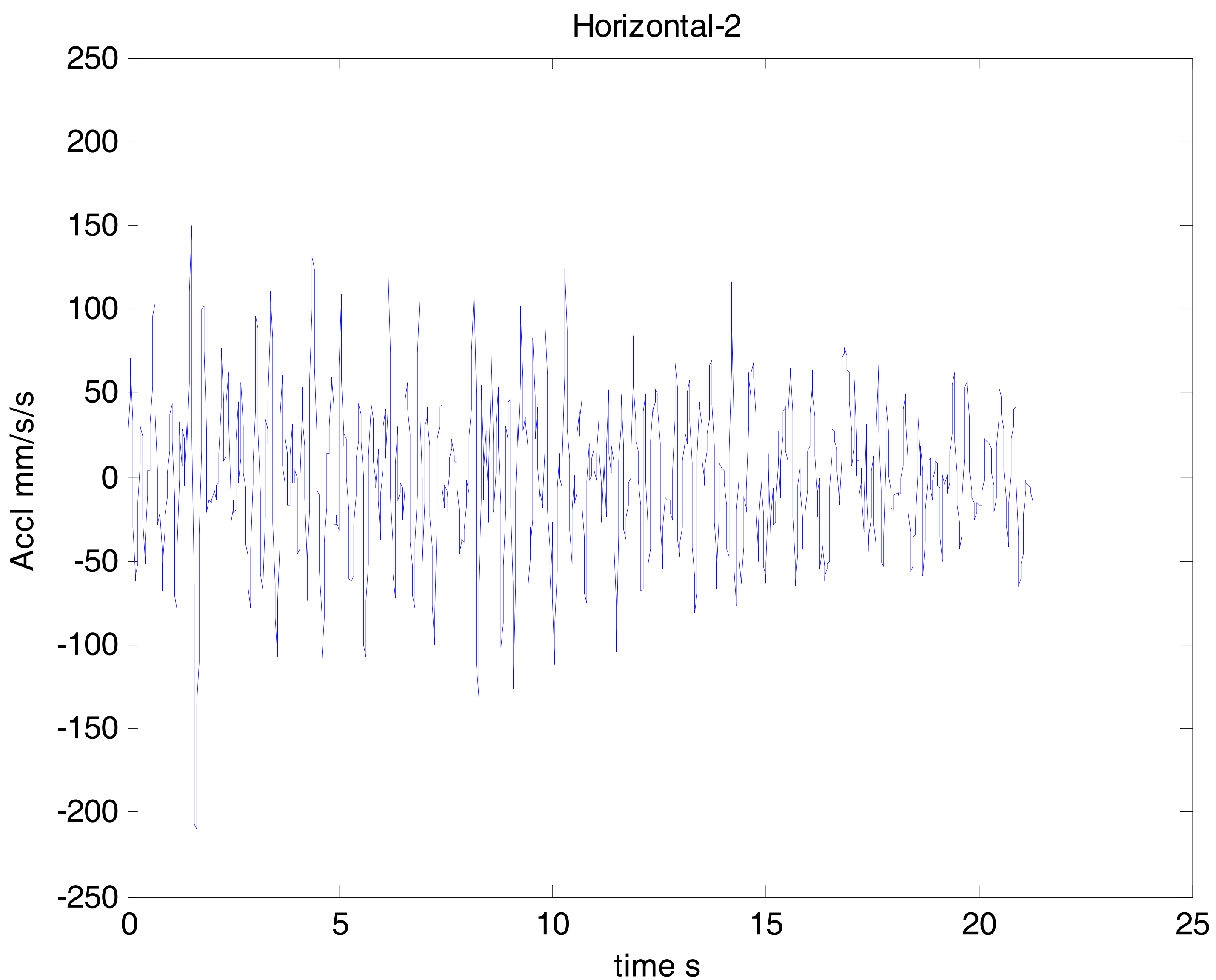
$$A(t) = iX_1(t) + jX_2(t) + kX_3(t)$$

Rotation

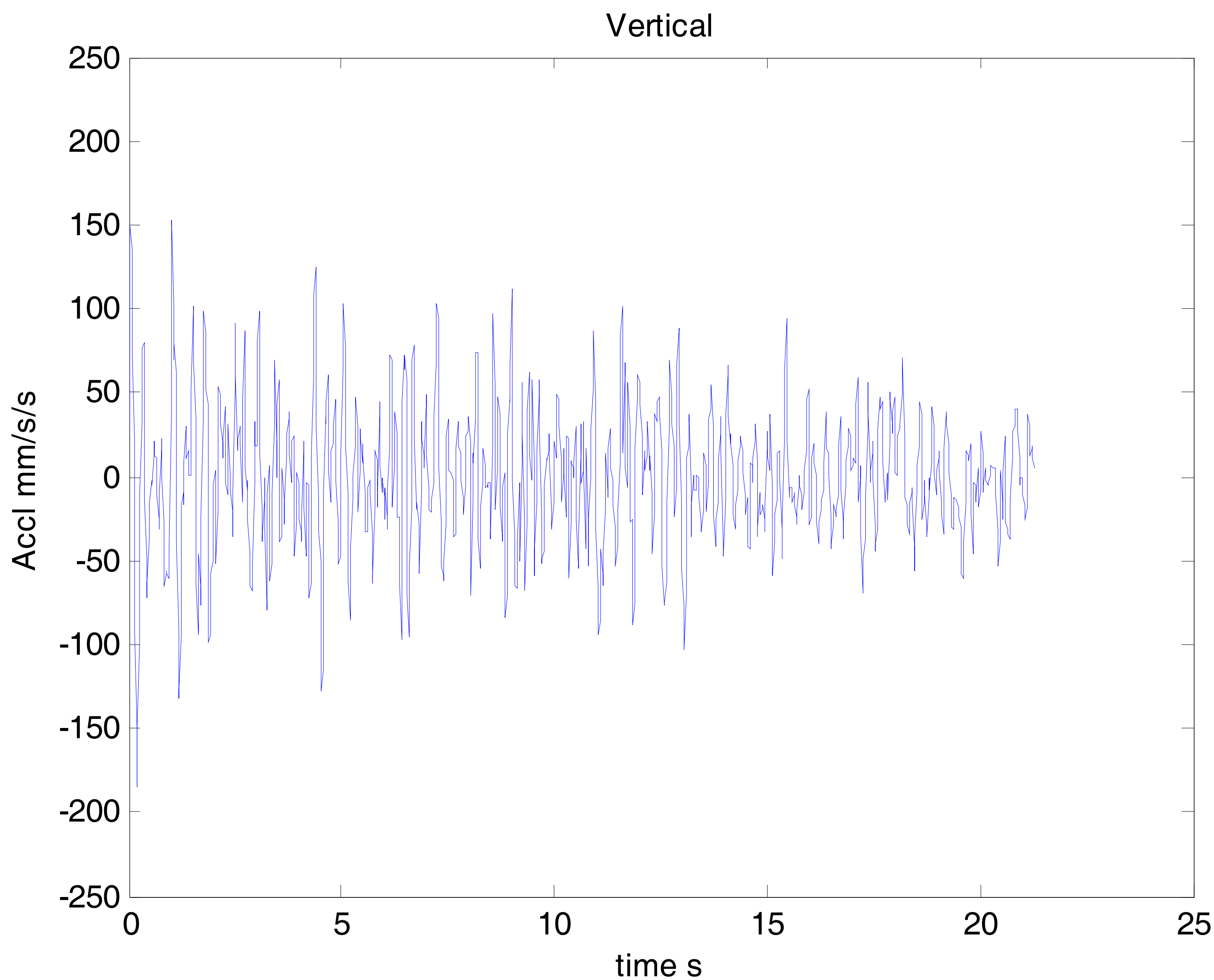
$$\Psi(t) = i\theta_1(t) + j\theta_2(t) + k\theta_3(t)$$

$$X(t) = \begin{Bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{Bmatrix}; \theta = \begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{Bmatrix}$$


Horizontal-1
Uttarkashi (India) earthquake,
OCT 20,1991 02:53 IST
Station: ALMORA
Accl mm/s/s
time s
250
200
150
100
50
0
-50
-100
-150
-200
-250
0
5
10
15
20
25



Horizontal-2
Accl mm/s/s
time s
250
200
150
100
50
0
-50
-100
-150
-200
-250
0
5
10
15
20
25


Vertical
250
200
150
100
50
0
-50
-100
-150
-200
-250
Accl mm/s/s
0
5
10
15
20
25
time s

$$\Sigma(t) = \begin{Bmatrix} X(t) \\ \Psi(t) \end{Bmatrix}_{6\times 1}$$: Treat this as a vector random process

$$\langle \Sigma(t) \rangle = 0$$

Focus attention on translations

$$C_{XX}(t) = \langle X^t(t) X(t) \rangle =$$

$$\begin{bmatrix} \langle X_1^2(t) \rangle & \langle X_1(t) X_2(t) \rangle & \langle X_1(t) X_3(t) \rangle \\ \langle X_1(t) X_2(t) \rangle & \langle X_2^2(t) \rangle & \langle X_2(t) X_3(t) \rangle \\ \langle X_1(t) X_3(t) \rangle & \langle X_2(t) X_3(t) \rangle & \langle X_3^2(t) \rangle \end{bmatrix}$$

$$X_i(t) = e_i(t) S_i(t); i = 1, 2, 3$$

Consider $X(t) = \begin{Bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \end{Bmatrix} = \begin{Bmatrix} e_1(t) S_1(t) \\ e_2(t) S_2(t) \\ e_3(t) S_3(t) \end{Bmatrix}$

where $S(t) = \begin{Bmatrix} S_1(t) \\ S_2(t) \\ S_3(t) \end{Bmatrix}$ is a stationary vector random process

with zero mean and $e_1(t), e_2(t) \,\&\, e_3(t)$ are deterministic

envelope functions. We assume $e_i(t) = e(t); i = 1, 2, 3$

$$\left\langle X(t) X^t(t+\tau) \right\rangle = e(t) e(t+\tau) \left\langle S(t) S^t(t+\tau) \right\rangle$$

$$\Rightarrow R_{XX}(t, t+\tau) = e(t) e(t+\tau) R_{SS}(\tau)$$

$$\Rightarrow R_{XX}(t, t) = e^2(t) R_{SS}(0)$$

Note: $R_{SS}(0)$ is constant since $S(t)$ is stationary.

Also, $R_{SS}(0)$ is symmetric and expected to be fully populated

$$R_{XX}(t,t) = e^2(t)R_{SS}(0)$$

We introduce a trasnformation

$$\tilde{S}(t) = \Phi^t S(t)$$

where Φ is a 3×3 transformaiton matrix.

Clearly, $\langle \tilde{S}(t) \rangle = 0$ and

$$\langle \tilde{S}(t)\tilde{S}^t(t+\tau) \rangle = \langle \Phi^t S(t) S^t(t) \Phi \rangle$$

$$\Rightarrow R_{\tilde{S}\tilde{S}}(\tau) = \Phi^t R_{SS}(\tau)\Phi$$

$$\Rightarrow R_{\tilde{S}\tilde{S}}(0) = \Phi^t R_{SS}(0)\Phi$$

Select Φ such that $\Phi^t R_{SS}(0)\Phi$ is diagonal.

$$\Rightarrow R_{\tilde{S}\tilde{S}}(0) = \mathrm{Diag}\begin{bmatrix} R_{11} & R_{22} & R_{33} \end{bmatrix}$$

$\Rightarrow \Phi$: matirx of eigenvectors of $R_{SS}(0)$.

$$\Rightarrow \bar{X}(t) = e(t)\Phi^t S(t) \Rightarrow R_{\bar{X}\bar{X}}(0) = e^2(t)R_{\tilde{S}\tilde{S}}(0)$$

Simplifying assumptions

• $e_1(t) = e_2(t) = e_3(t) = e(t)$

• $\tilde{S}_1(t), \tilde{S}_2(t),$ and $\tilde{S}_3(t)$: are uncorrelated random processes variables

$\Rightarrow$ The psd matirx of $\tilde{S}(t) = \left\{\tilde{S}_1(t)\ \tilde{S}_2(t)\ \tilde{S}_3(t)\right\}^t$ is diagonal

Note :

$R_{\tilde{S}\tilde{S}}(0)$ is diagonal does not imply that psd matirx of

$\tilde{S}(t) = \left\{\tilde{S}_1(t)\ \tilde{S}_2(t)\ \tilde{S}_3(t)\right\}^t$ is diagonal.

If $\tilde{S}_1(t), \tilde{S}_2(t),$ and $\tilde{S}_3(t)$ are broadbanded, the above assumption can be deemed to be reasonable.

Principal axes of excitations

- Find the direction of coordinate axes in which the $R_{SS}(0)$ matrix becomes diagonal.
- This can be done by solving the eigenvalue problem associated with the matrix $R_{SS}(0)$.
- The major principal direction lies on the horizontal plane in a direction that points towards epicentre from the recording station. The minor direction is in the vertical plane.
- This is an empirically observed feature from recorded data and there exists no "proof" for this.
- Most often structures are designed taking into account only the horizontal components.

- The principal directions for excitations need not coincide with the global coordinate axes used in modeling the structure.

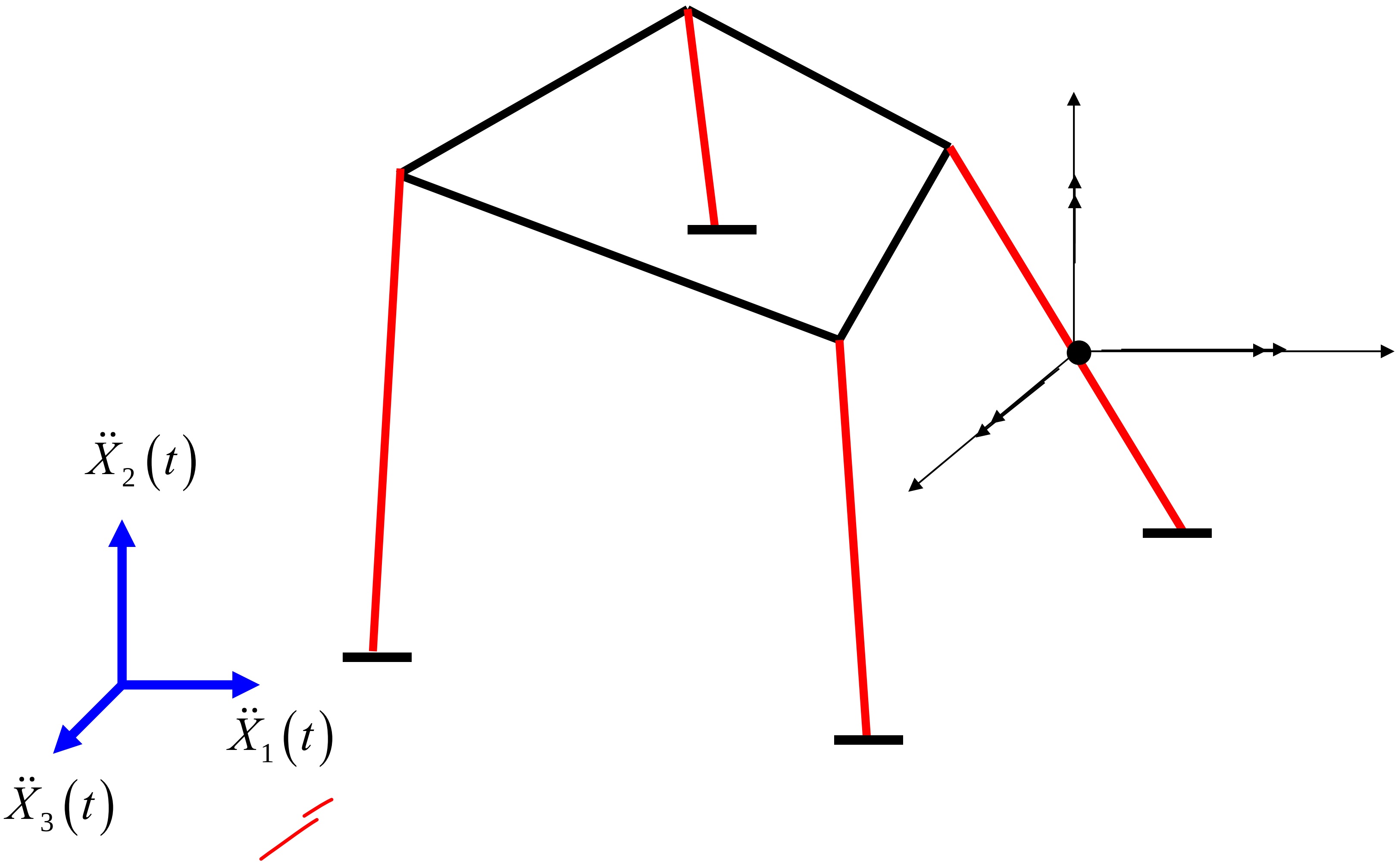
- For a structure that is symmetric in plan, excitation in one of the horizontal directions does not induce stresses in the other orthogonal direction.

- Most structures are irregular in plan and the bending and torsional action could be coupled in the predominant modes of the structural oscillations. The modes could also be closely spaced.

- Under the action of earthquake ground motions the structures undergo significant torsional oscillations. This is one of the most characteristic features of earthquake response of structures.

The structure translates and twists.

Principal axes for excitations exist and the ground motion components are uncorrelated along these axes.



Stationary random vibration analysis and basis for developing modal combination rule

Ref : *W Smeby and A Der Kiureghian*, 1985, *EESD*, 13, 1-12.

$$M\ddot{U} + C\dot{U} + KU = -ML\ddot{X}$$

U = vector of nodal displacements relative to the ground

$$X = \left[\underbrace{X_1(t)\ X_2(t)}_{\text{Horizontal}} \quad \underbrace{X_3(t)}_{\text{Vertical}} \right]$$ = translational components of ground motion

$L = -\left[L_1 \quad L_2 \quad L_3\right]^t$ = influence matrix.

$v(t) = q^t U(t)$ = response quantity of interest.

$$M\ddot{U} + C\dot{U} + KU = -ML\ddot{X}$$

Let $U = \Phi Y$ with

$K\Phi = \Lambda M\Phi,\ \Phi^t M\Phi = I,\ \Phi^t K\Phi = \Lambda, \&\tilde{C} = \Phi^t C\Phi$ diagonal

$$\Rightarrow I\ddot{Y} + \tilde{C}\dot{Y} + \Lambda Y = -\Phi^t ML\ddot{X}(t)$$

$H(\omega) = \left[\Lambda - I\omega^2 + i\omega\tilde{C}\right]^{-1}$: diagonal matrix

$$Y(\omega) = -H(\omega)\Phi^t ML\ddot{X}(\omega)$$

$$G_{YY}(\omega) = H(\omega)\Phi^t MLG_{\ddot{X}\ddot{X}}(\omega)L^t M\Phi H^{*t}(\omega)$$

$$G_{UU}(\omega) = \Phi G_{YY}(\omega)\Phi^t$$

$$v(t) = q^t U$$

$$G_{vv}(\omega) = q^t G_{UU}(\omega)q$$

$$G_{vv}(\omega) = q^t\Phi H(\omega)\Phi^t MLG_{\ddot{X}\ddot{X}}(\omega)L^t M\Phi H^{*t}(\omega)\Phi^t q$$

Generic response quantity: $v(t)=\sum_{k=1}^{3}\sum_{i=1}^{N}\underbrace{\Psi_i^{(k)}}_{\substack{\text{Participation factor}\\ \text{for } i^{\text{th}} \text{ mode and } k^{\text{th}}\\ \text{excitation component}}}\underbrace{Y_i(t)}_{i^{\text{th}} \text{ mode}}$

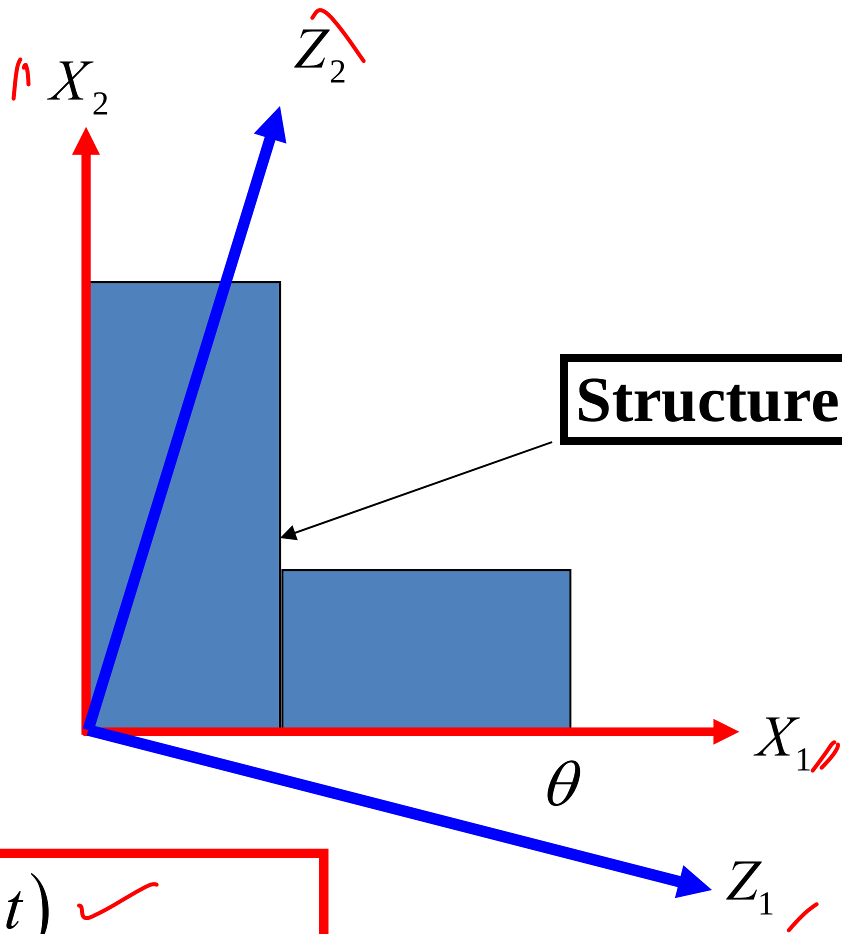
$$G_{vv}(\omega)=\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{i=1}^{N}\sum_{j=1}^{N}\Psi_i^{(k)}\Psi_j^{(l)}H_i(\omega)H_j(-\omega)G_{\ddot{X}_k\ddot{X}_l}(\omega)$$

Let $Z(t)=\begin{bmatrix}Z_1(t) & Z_2(t) & Z_3(t)\end{bmatrix}^t$ be the ground motion components along the principal axes and let

$X(t)=AZ(t)$

$\Rightarrow G_{\ddot{X}\ddot{X}}(\omega)=AG_{\ddot{Z}\ddot{Z}}(\omega)A^t$

where $G_{\ddot{Z}_k\ddot{Z}_l}(\omega)$ is diagonal.



$$X(t) = AZ(t)$$

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X(t)=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}Z(t)$$

$$X_1(t)=Z_1\cos\theta+Z_2\sin\theta\Rightarrow\langle X_1(t)\rangle=0$$

$$X_2(t)=-Z_1\sin\theta+Z_2\cos\theta\Rightarrow\langle X_2(t)\rangle=0$$

$$\sigma_1^2=\mathrm{Var}\left[X_1(t)\right]=\langle Z_1^2\rangle\cos^2\theta+\langle Z_2^2\rangle\sin^2\theta$$

$$\sigma_2^2=\mathrm{Var}\left[X_2(t)\right]=-\langle Z_1^2\rangle\sin^2\theta+\langle Z_2^2\rangle\cos^2\theta$$

$$\sigma_{12}=\langle X_1(t)X_2(t)\rangle=\left\{\langle Z_2^2\rangle-\langle Z_1^2\rangle\right\}\cos\theta\sin\theta$$

$$\rho_{12}=\frac{\sigma_{12}}{\sqrt{\sigma_1^2\sigma_2^2}}=\frac{-(1-\alpha)\sin 2\theta}{\sqrt{(1+\alpha)^2-(1-\alpha)^2\cos^2 2\theta}};\alpha=\frac{\langle Z_2^2\rangle}{\langle Z_1^2\rangle}$$

$$G_{vv}(\omega)=q^t\Phi H(\omega)\Phi^t MLA(\theta)\underbrace{G_{\ddot{Z}\ddot{Z}}(\omega)}_{\text{Diagonal}}A^t(\theta)L^tM\Phi H^{*t}(\omega)\Phi^t q$$

$$\Rightarrow \text{Spectral moments: } \lambda_m=\int_0^\infty \omega^m G_{vv}(\omega)\,d\omega$$

- Leads to peak factors associated with mean and standard deviation of the maximum response over duration τ.
- One can determine the orientation θ for which the response variance reaches its maximum value.
- Alternatively, θ can be treated as a random variable and the expected values of response quantities of interest could be obtained with respect to pdf of θ.

- Forms the basis for development of modal combination rule when the inputs are specfied in terms of a set of response spectra along the principal axes.
- When principal axes of excitation and structure axes coincide, or when excitation intensities along three axes are the same, a combination rule with
  - SRSS for combination over excitation components, and
  - CQC rule for combining over modal contributions

can be obtained.

- More general forms which takes into account the value of θ have also been developed.

Earthquake source mechanism, wave propagation, site amplfication, and ground motion models

Earthquake ground motion=convolution of
the source mechanism with the Green's funciton
representing the wave propagation.

Application of double couples is equivalent to displacement discontinuities due to faulting.

PSD models based on seismological considerations

(Boore and Atkinson, 1986, BSSA)

Fourier amplitude spectrum of ground acceleration

$$A_S(\omega) = C S_1(\omega) S_2(\omega) S_3(\omega)$$

C = scaling factor

$S_1(\omega)$ = source spectrum

$S_2(\omega)$ = amplification factor

$S_3(\omega)$ = attenuation factor

$$C = \frac{R_\phi F P_r}{4\pi\rho\beta^3}\left(\frac{1}{R}\right)$$

$$A_S(\omega) = C S_1(\omega) S_2(\omega) S_3(\omega)$$

$$C = \frac{R_\phi F P_r}{4\pi\rho\beta^3}\left(\frac{1}{R}\right)$$

Factors

R_ϕ : radiation pattern of the seismic wave

F : free surface effect

P_r : partition of energy into horizontal components

ρ : mass density

β : seismic wave velocity

R : hypocentral distance

$$S_1(\omega) = m_0 \frac{\omega^2}{1+\left(\dfrac{\omega}{\omega_c}\right)^2}$$

m_0 = seismic moment

ω_c = the frequency above which the spectral amplitudes of ground displacements begin to fall off; corner frequency (inversely proportional to source radius)

$$f_e = \frac{2\pi}{\omega_c} = 0.49\beta\left(\frac{\Delta\sigma}{m_0}\right)^{\frac{1}{3}}$$

$$m_0 = 10^{(1.5M+9.05)}$$

M = earthquake moment magnitude

$\Delta\sigma$ = stress drop

(All quantities in SI units)

Kanai - Tajimi Model

$$S_2(\omega) = I \frac{\left(\omega_g^4 + 4\eta_g^2\omega_g^2\omega^2\right)}{\left(\omega^2 - \omega_g^2\right)^2 + 4\eta_g^2\omega_g^2\omega^2}$$

Clough and Penzien model

$$S_2(\omega) = I \frac{\left(\omega_g^4 + 4\eta_g^2\omega_g^2\omega^2\right)}{\left(\omega^2 - \omega_g^2\right)^2 + 4\eta_g^2\omega_g^2\omega^2} \underbrace{|H_f(\omega)|^2}_{\text{High pass filter}}$$

$$= I \frac{\left(\omega_g^4 + 4\eta_g^2\omega_g^2\omega^2\right)}{\left(\omega^2 - \omega_g^2\right)^2 + 4\eta_g^2\omega_g^2\omega^2} \underbrace{\frac{\left(\omega / \omega_f\right)^4}{\left[1 - \left(\omega / \omega_f\right)^2\right]^2 + 4\varsigma_f^2\left(\omega / \omega_f\right)^2}}_{\text{High pass filter}}$$

$$S_3(\omega) = \exp\left(-\frac{R\omega}{2Q\beta}\right) f(\omega)$$

Q = quality factor of attenuation

$f(\omega)$ = high cut filter dependent on f_m, a high cut-off frequency

$= \exp(-\theta\omega)$

$$X_g(t) = e(t) F^{-1}(A_s(t))$$

$$S_{gg}(\omega, t) = e^2(t) \frac{1}{2\pi I} \left| A_s(\omega) \right|^2 \qquad I = \int_0^\infty \left| e(t) \right|^2 dt$$

Remark

The model relates the ground acceleration PSD to physical properties of the source and the medium through which the seismic waves travel.

Spatial variability of earthquake ground motions and response of multi - supported structures

•Long span bridges, large dams, pipelines, tunnels,···

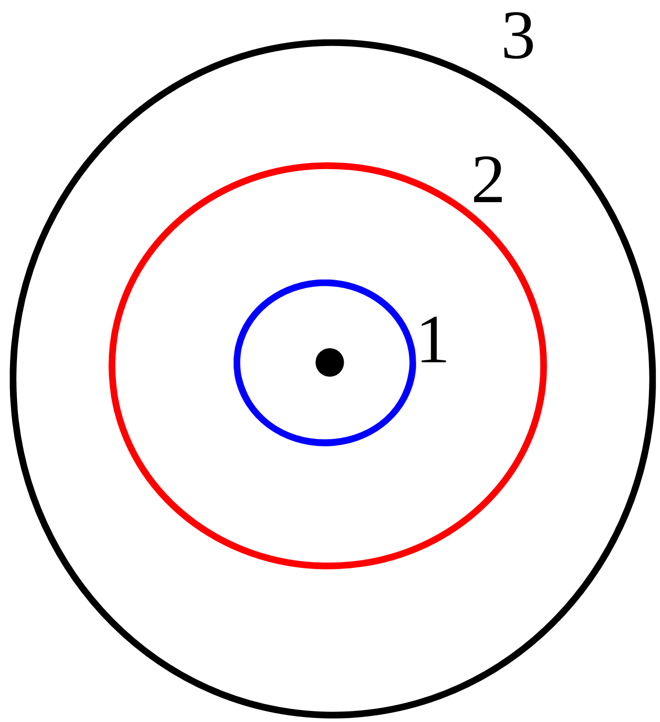
Reference : A Zerva, 2009, Spatial variations of seismic ground motions, CRC Press, Boca Raton

SMART array at Taiwan

Circle	Radius km	SMA-s
0	0	1
1	0.2	4
2	1.0	12
3	2.0	12

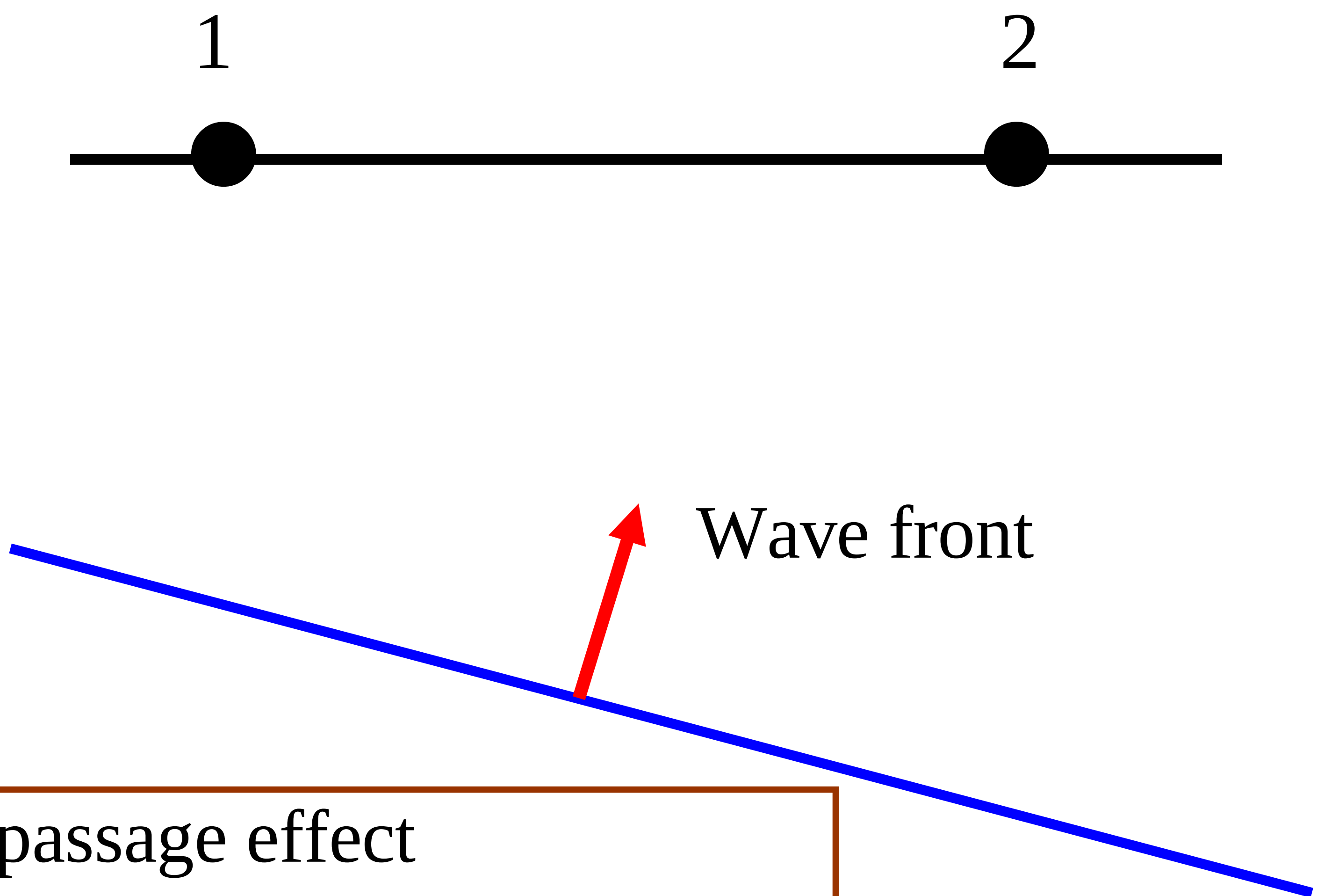
Two more stations at 2.8 and 4.8 km south of the centre.

Tri-axial accelerometers at every station



Why spatial variability occurs?

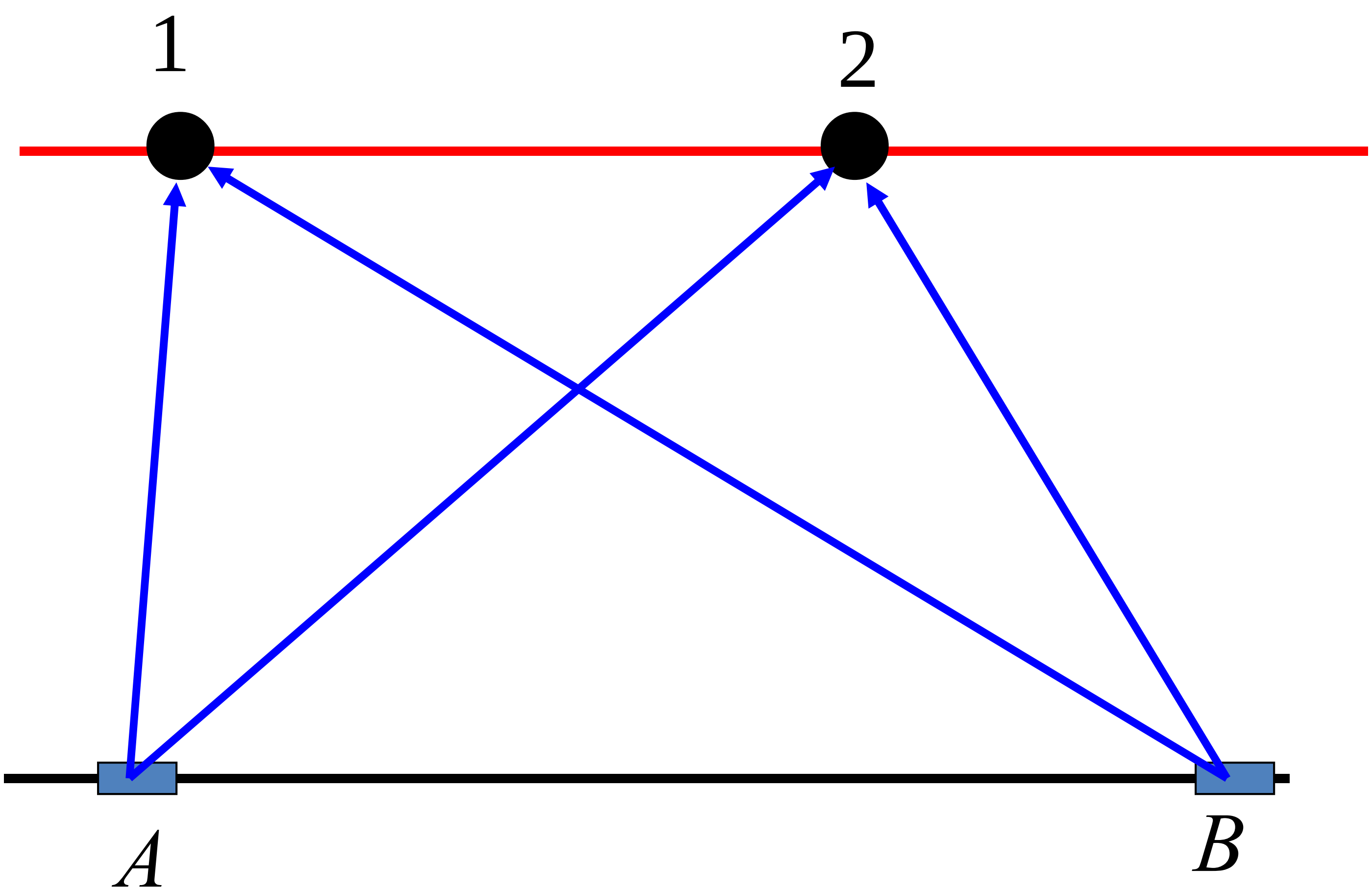
- Wave passage effect
- Extended source effect
- Scattering effect
- Attenuation effect



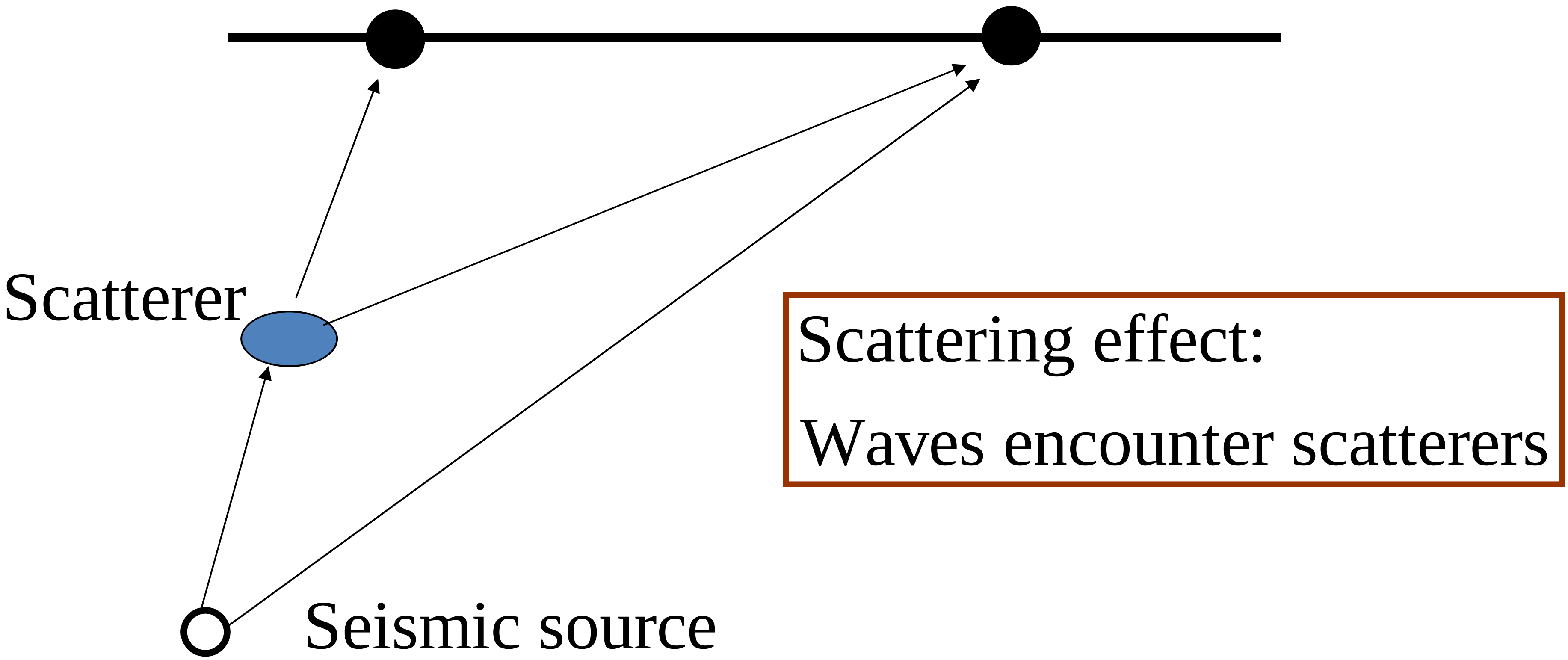
Wave passage effect

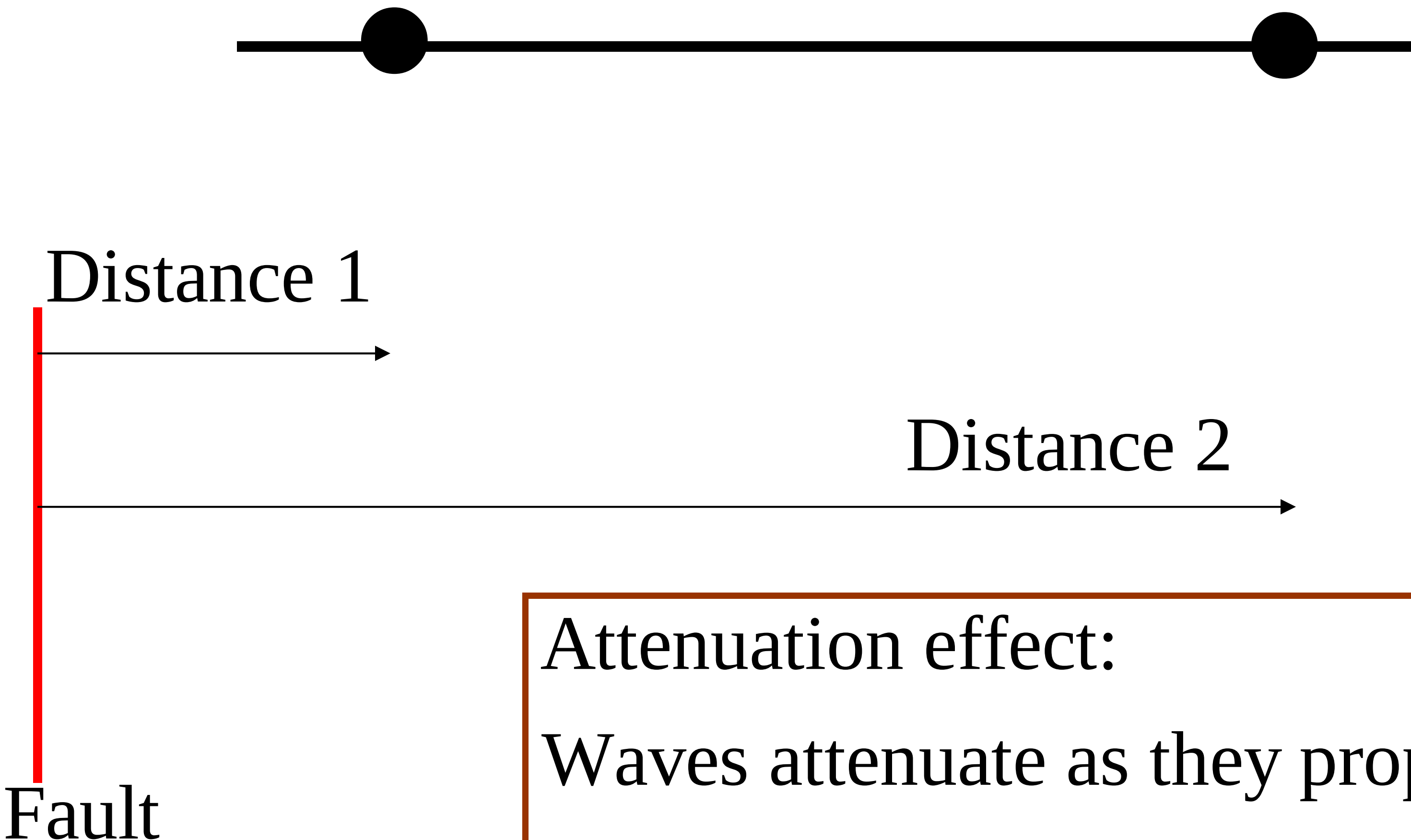
Inclined incidence of plane waves

leading to time delays



Extended source effect: as rupture propagates along an extended fault, it transmits energy that arrives delayed on the ground surface.


Scatterer
Scattering effect:
Waves encounter scatterers
Seismic source



Attenuation effect:

Waves attenuate as they propagate (not very important for engineering structures)

Questions

- What are the phenomenological features associated with response of structures subjected to spatially varying ground motions?
- When it is important to consider them?
- How to model spatially varying ground motions as random processes?
    - Based on data
    - Based on phenomenological considerations
- How to develop modal combination rules when the inputs are specifed in terms of response spectra?