



INDIAN INSTITUTE OF SCIENCE

STOCHASTIC HYDROLOGY

Lecture -39

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Summary of the previous lecture

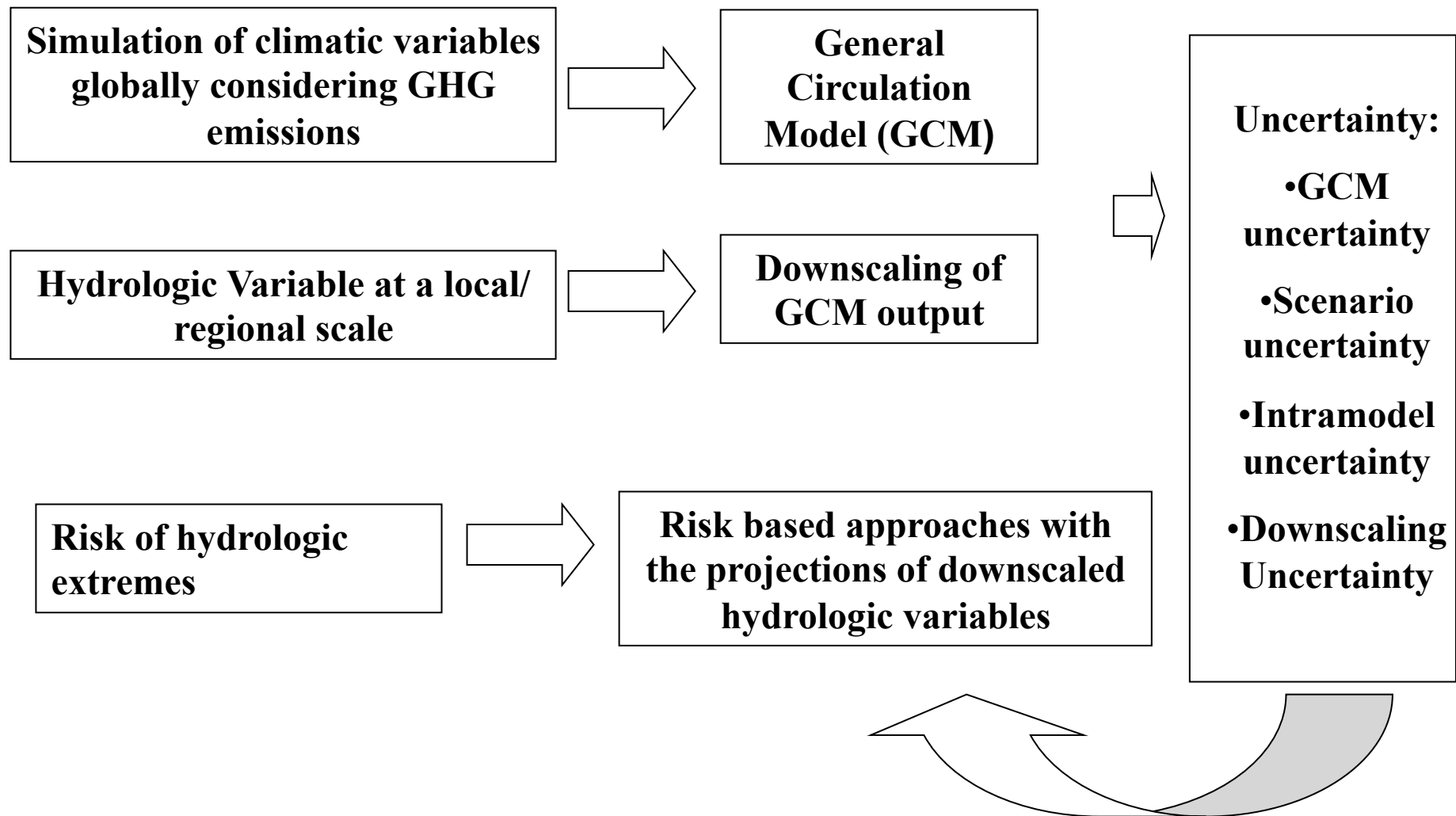
- Data consistency checks
 - Specific Flow: Case study
- Data representation through box plots
- Normalisation of flow data

Recent Applications of Stochastic Hydrology

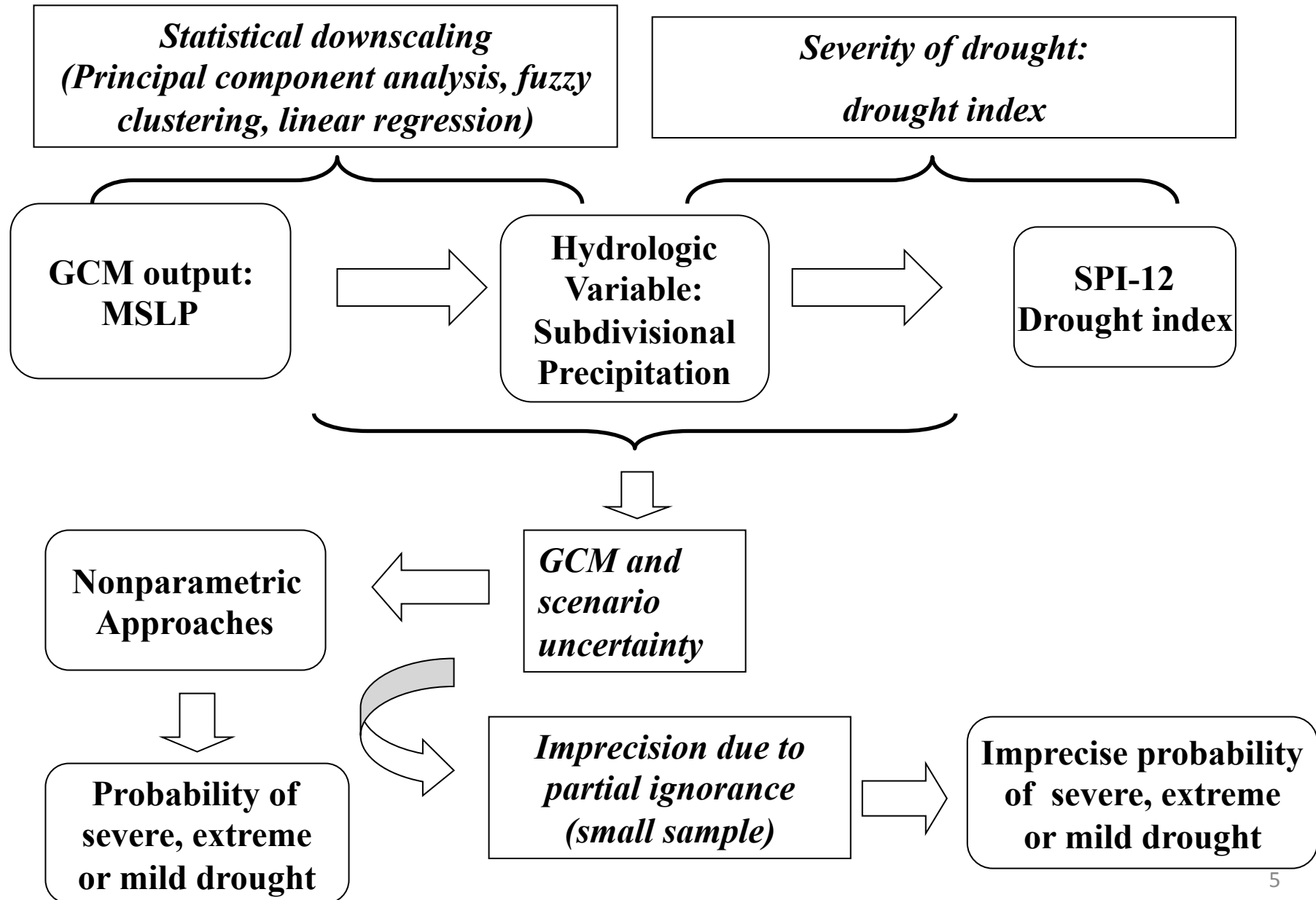
Hydrologic Impacts of Climate Change: Quantification of Uncertainty

Acknowledgment : Slides in this lecture are taken with permission from PhD Thesis Presentation of Subimal Ghosh, IISc. Bangalore

Assessment of Climate Change Impacts



Uncertainty Modeling : Probabilistic Approach



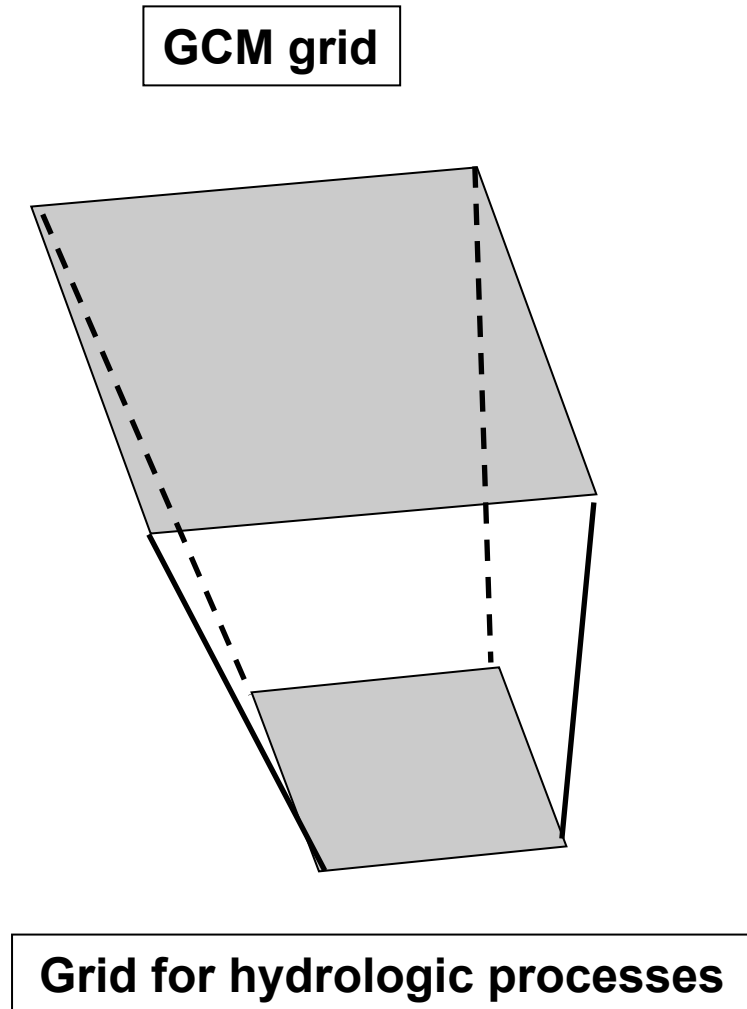
Downscaling

Downscaling: to model the hydrologic variables (e.g., precipitation) at a smaller scale based on large scale GCM outputs.

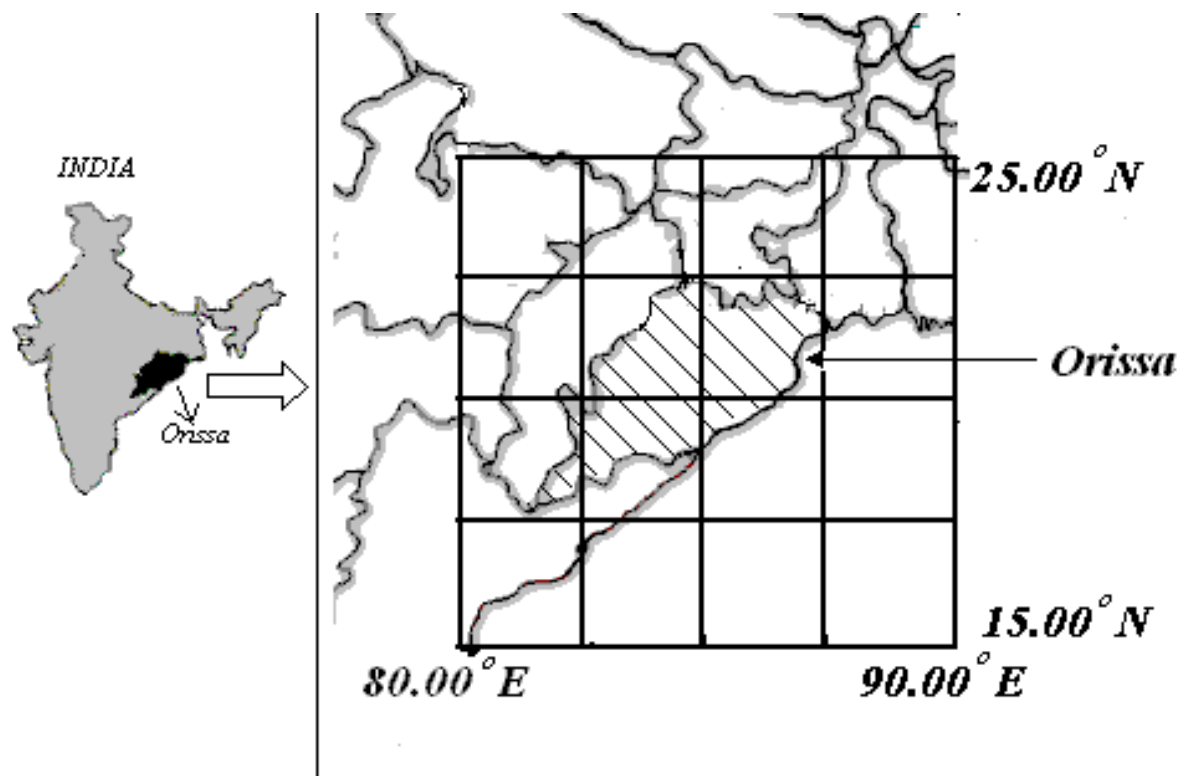
Statistical Downscaling: produces future scenarios based on statistical relationship between large scale climate features and hydrologic variables like precipitation.

Assumption- Statistical relationship hold good in future for changed climate scenario.

Advantage- computationally simple and easily adjusted to new areas.



Case-study Area: Orissa Meteorological Subdivision



- Coastal Area
- Increase of **hydrologic extremes** in recent past
- Increase in temperature: **1.1°C/century**, whereas in average increase in India: 0.4°C/century.

➤ **Rainfall Data Used:**
Monthly data from
1950-2003

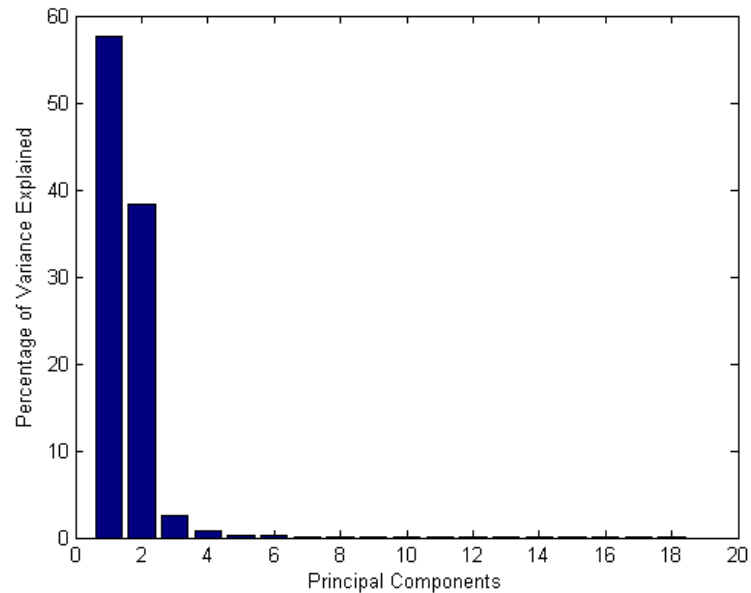
Ref : Subimal Ghosh and P.P. Mujumdar (2006) "Future Rainfall Scenario over Orissa with GCM Projections by Statistical Downscaling" *Current Science*, 90(3), Feb 10, 2006, pp. 396-404. (Pub : Indian Academy of Sciences, Bangalore)

- **GCM Used:** AGCM (CCSR/NIES, Japan)
- **Scenario Used:** B2 (IPCC TAR Scenario)
- **Climate predictor :** MSLP
- **Length of GCM Output Used:** 1950-2100

Principal Component Analysis

The percentage of total variance w_k explained by the k^{th} principal component is given by:

$$w_k = \frac{\lambda_k}{\sum_{m=1}^M \lambda_m} \times 100$$



Number of Principal Components used=3

Regression with PCs

➤ **Regression Equation:**

$$RAIN_t = C + \sum_{k=1}^K \gamma_k \times pc_{kt}$$

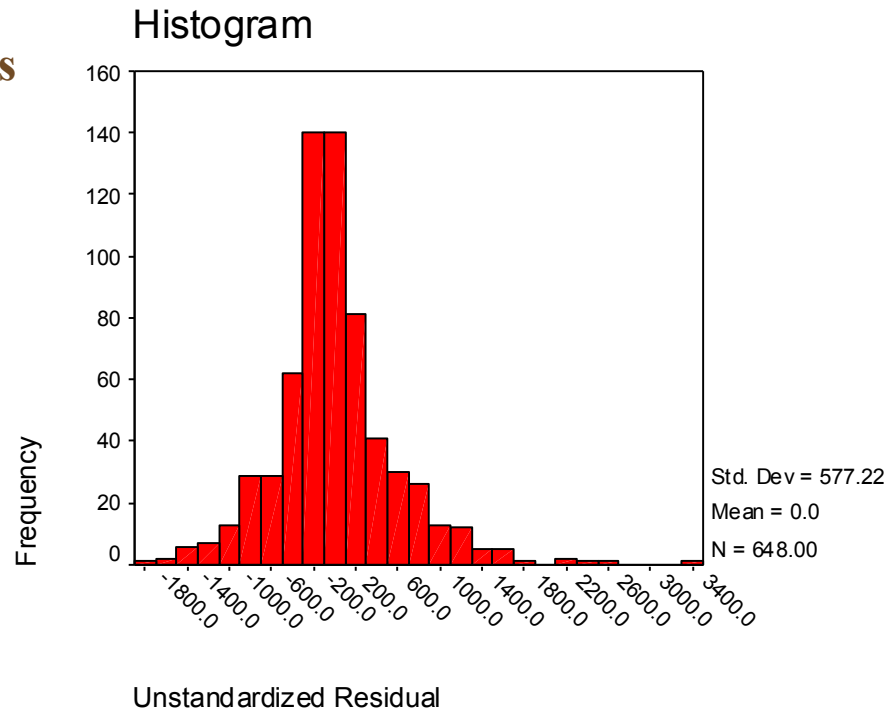
$$RAIN_t = 172.227 + 144.930 \times pc_{1t} - 493.944 \times pc_{2t} - 480.663 \times pc_{3t}$$

Correlation Coefficient (R) = 0.789

Regression with Principal Components

$$RAIN_t = 172.227 + 144.930 \times pc_{1t} - 493.944 \times pc_{2t} - 480.663 \times pc_{3t}$$

Normality of Residuals



Results of the Regression Model

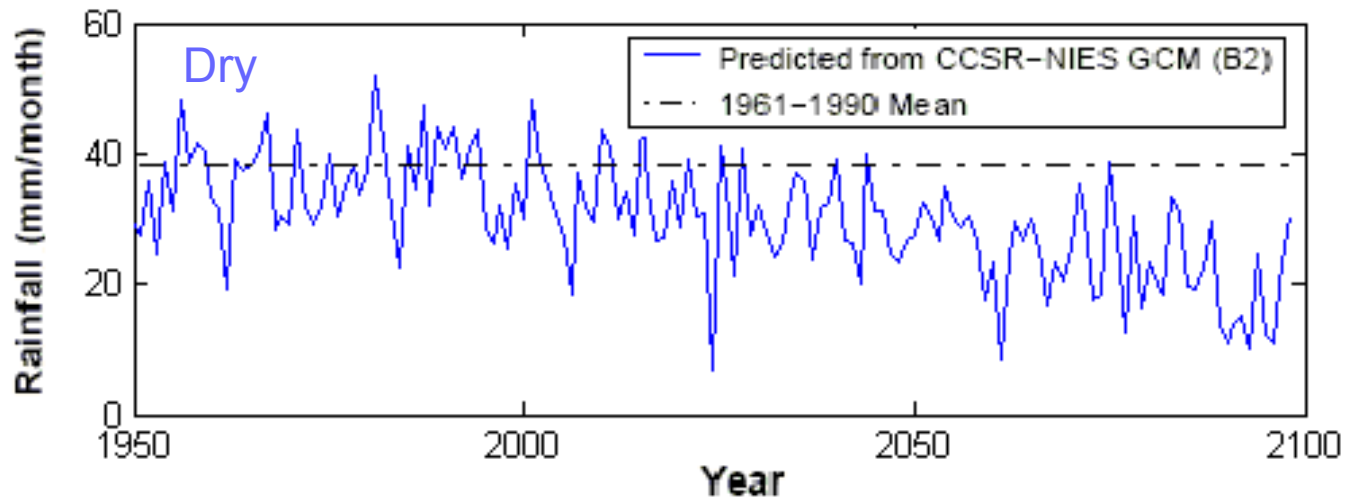
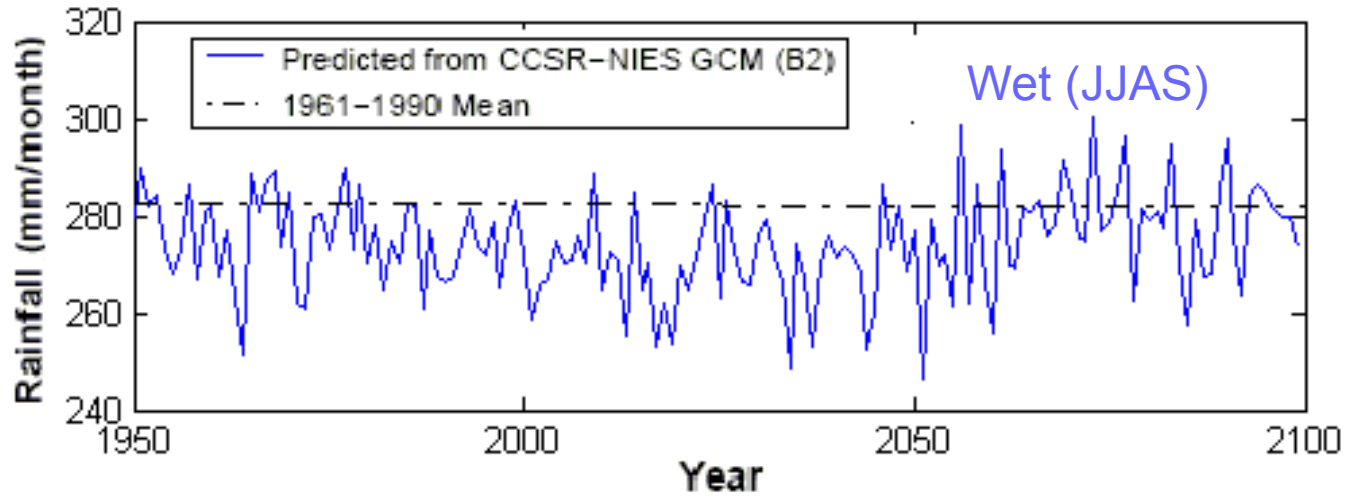
- Long term mean and median (training of model)

Period	Obs. Mean	Pred. Mean	Obs. Median	Pred. Median
Wet (JJAS)	281.4 mm/month	281.3 mm/month	281.9 mm/month	283.3 mm/month
Dry	74.9 mm/month	74.3 mm/month	73.8 mm/month	73.6 mm/month

● **Nash – Sutcliffe Coefficient (E): 0.83**

$$E = 1 - \frac{\sum_t (P_{ot} - P_{pt})^2}{\sum_t (P_{ot} - \bar{P})^2}$$

Prediction with CCSR/NIES GCM and B2 Scenario

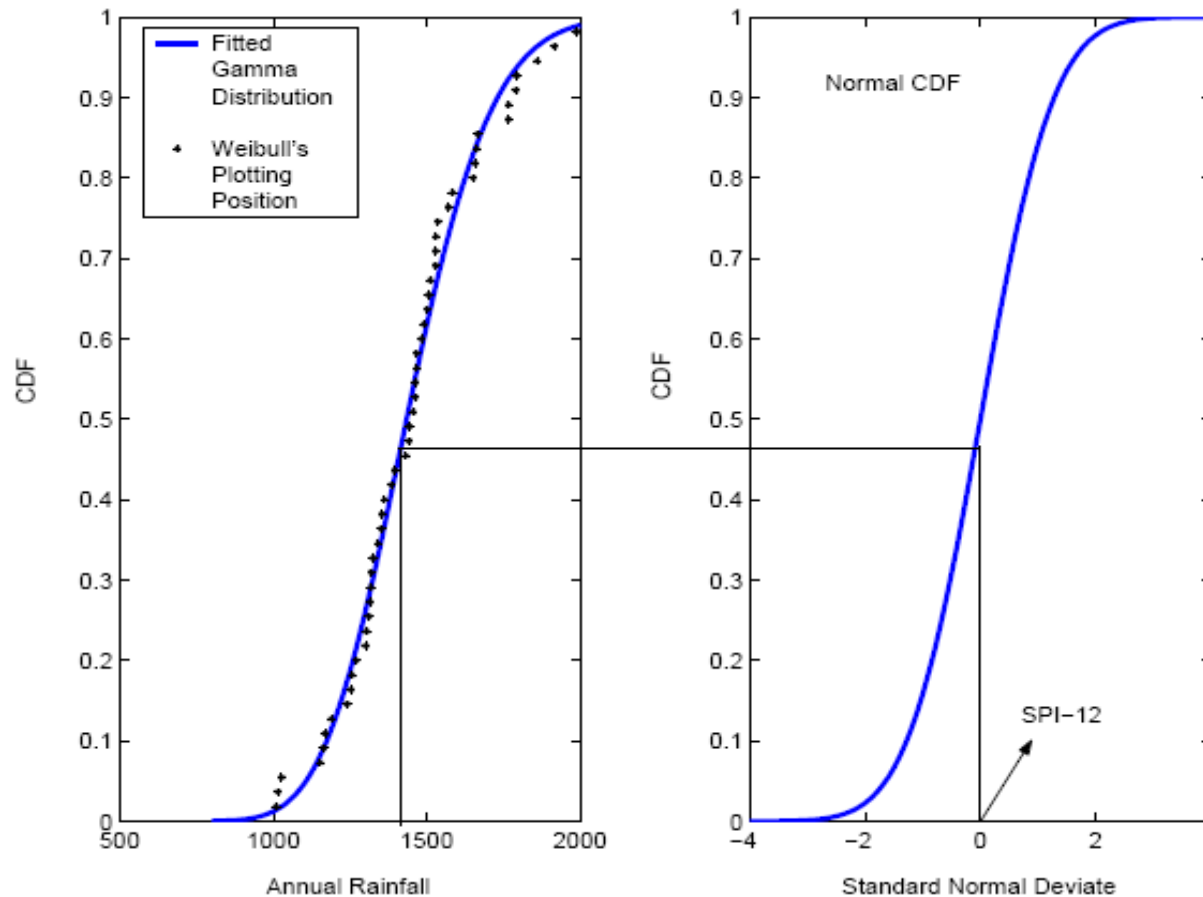


Drought Assessment: Drought Indices

- Drought analysis is performed with drought indices
- A drought indicator, briefly defined, is a variable to identify and assess drought conditions (Steinemann, 2003)
- Different drought indices:
 - **Standardized Precipitation Index (SPI)** : Developed by McKee et. al (1993). Input data required: precipitation
 - **Palmer Drought Severity Index (PDSI)** : Developed by Palmer (1965). Input data required: precipitation, temperature data and local Available Water Content (AWC) of the soil
 - **Bhalme-Mooley Drought Index (BMDI)**: Monthly index. Input data required : monthly precipitation
 - **Effective Drought Index (EDI)**: Calculated in daily time step. Input data required : precipitation
- Index used in the present analysis: **SPI-12** (Annual SPI)

SPI-12: Equiprobability Transformation

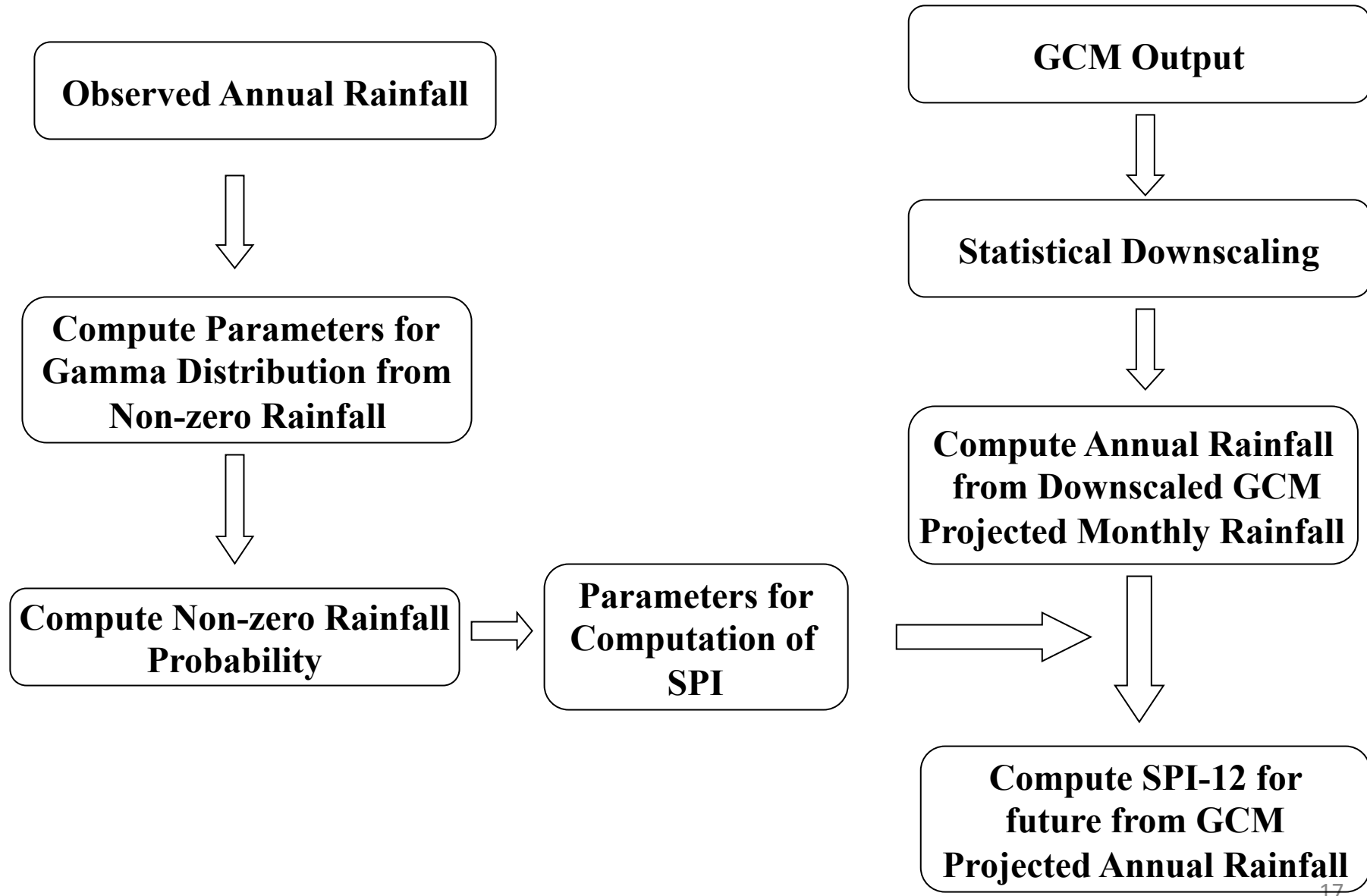
Corresponding to the CDF of the rainfall the standard normal deviate (mean 0 and variance 1) is termed as SPI.



Classification of Drought based on SPI (McKee et al., 1993)

SPI Values	Drought Category
0 to -0.99	Near Normal
-1.00 to -1.49	Mild to Moderate Drought
-1.50 to -1.99	Severe Drought
-2.00 or less	Extreme Drought

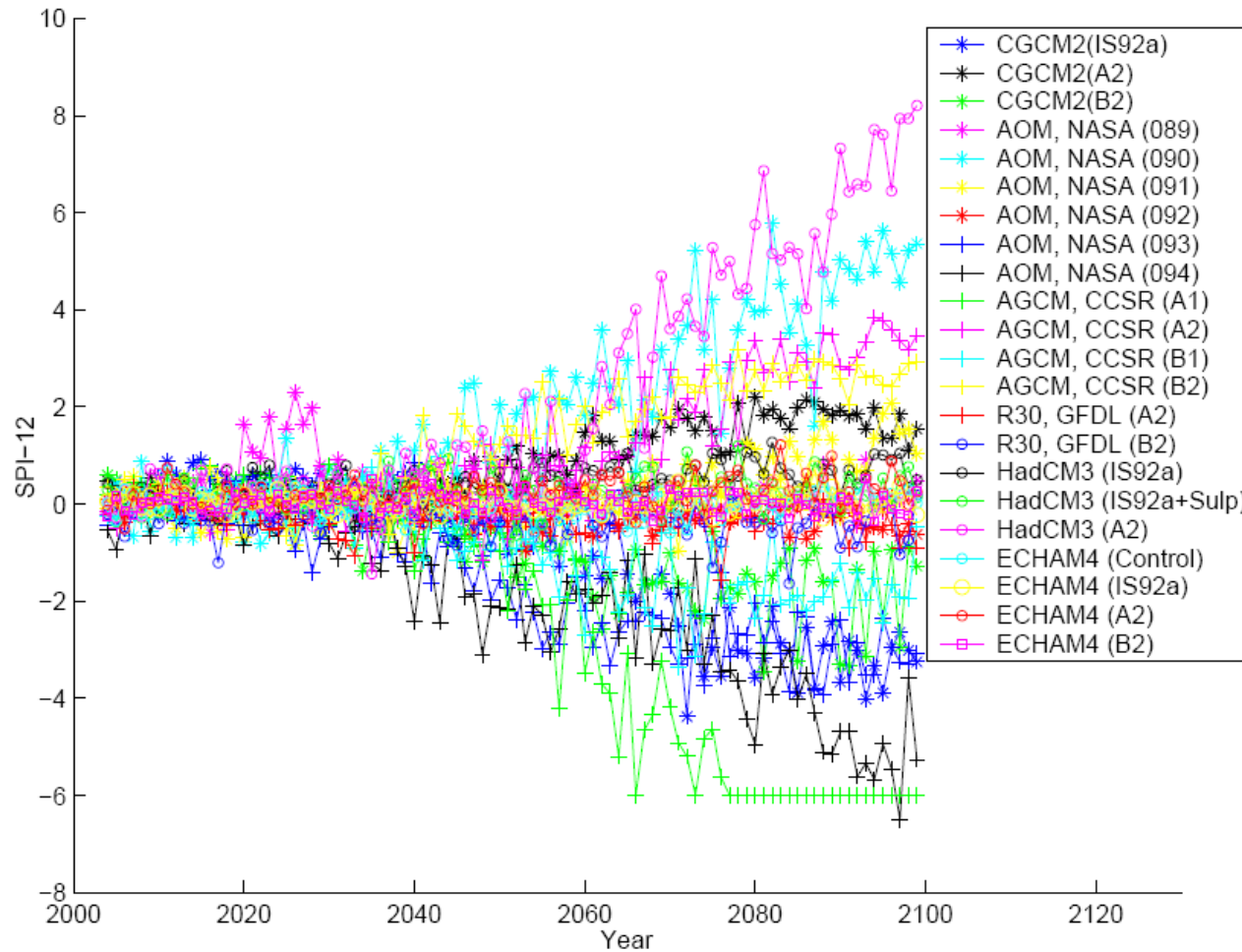
SPI-12 Computation



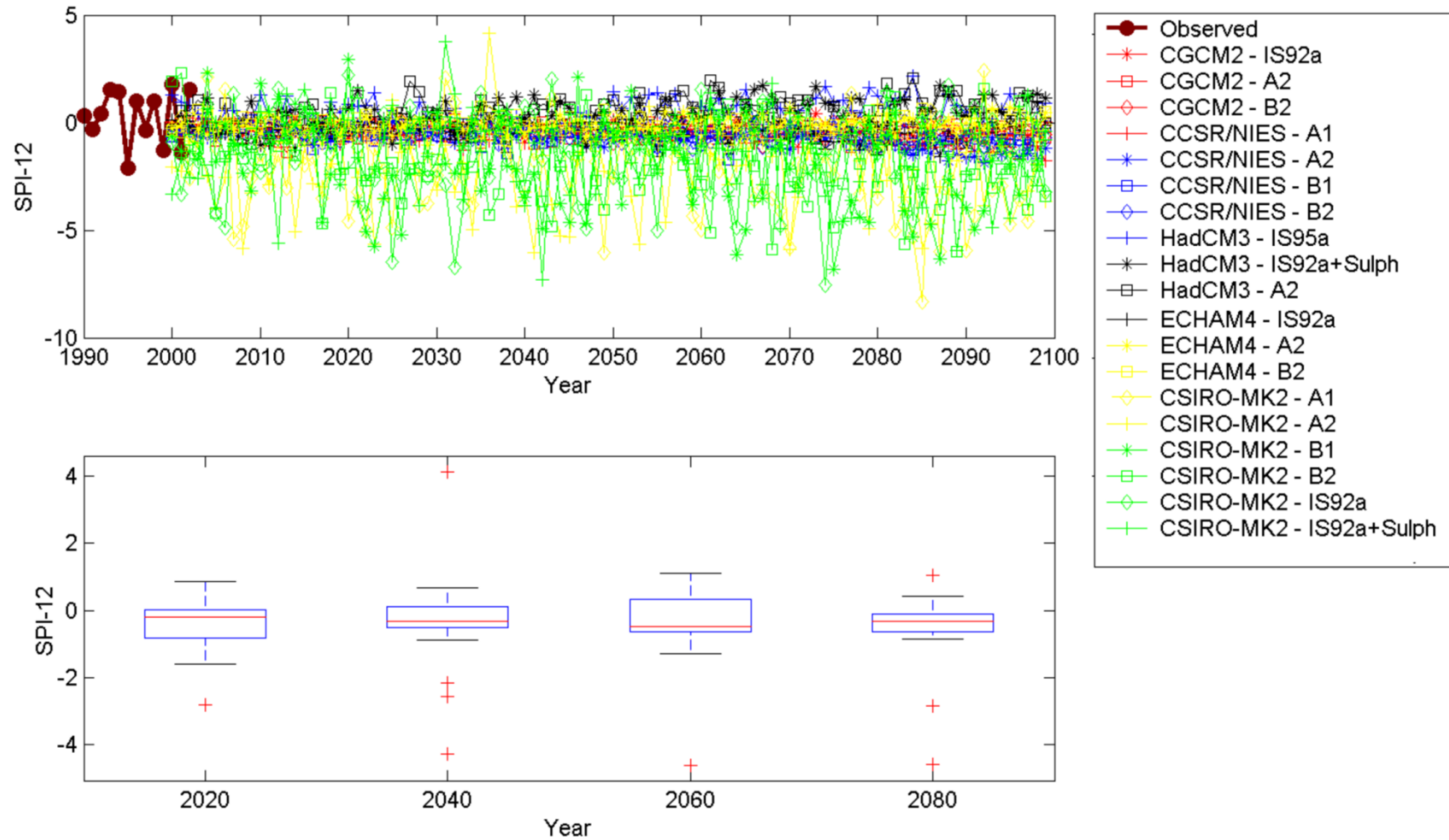
GCMs and Scenarios Used

GCM	Organization	Scenarios Available
CCSR/NIES Coupled GCM	Center for Climate Research Studies (CCSR) and National Institute for Environmental Studies (NIES), Japan	A1, A2, B1, B2
Second Generation Coupled Global Climate Model (CGCM2)	Canadian Center for Climate Modelling and Analysis, Canada	IS92a, A2, B2
HadCM3	Hadley Centre for Climate Prediction and Research (HCCPR), UK	IS95a, (GHG+ Ozone+Sulphate), A2
ECHAM4/OPYC3	Max Planck Institute für Meteorologie, Germany.	IS92a, A2, B2
CSIRO-MK2	Australia's Commonwealth Scientific and Industrial Research Organisation (CSIRO)	(IS92a+Sulph), IS92a, A1, A2, B1, B2

Projections of SPI-12

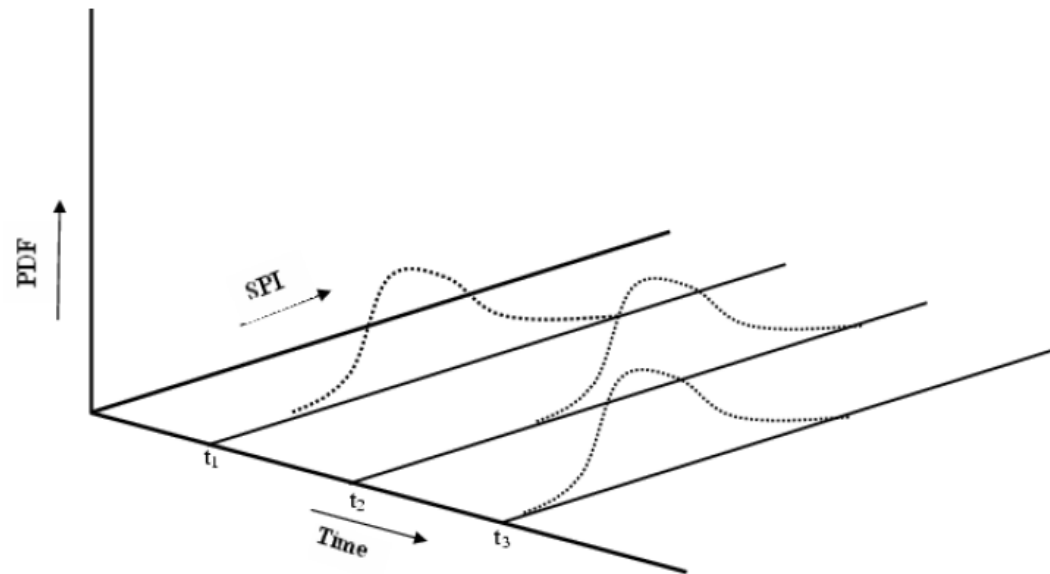


Projections of SPI-12



Probability Density Function of SPI-12

- All the **scenarios** are **equally possible** (IPCC report, 2001)
- Outputs of all the **GCM models** are **equally accurate**.
- Time series generated by **each GCM model** for each of the scenario is considered as **a realization**.
- All the generated time series together are considered as **stochastic process**.
- At **each time step** there is a **marginal pdf of SPI-12**.



Determination of pdf at each time step

- Assumption of Normal Distribution
- Kernel Density Estimation Method

Assumption of Normal Distribution

At each time step (year) the SPI values are assumed to follow **normal distribution**

CDF values are estimated based on normal distribution, and probability of predicted droughts are estimated.

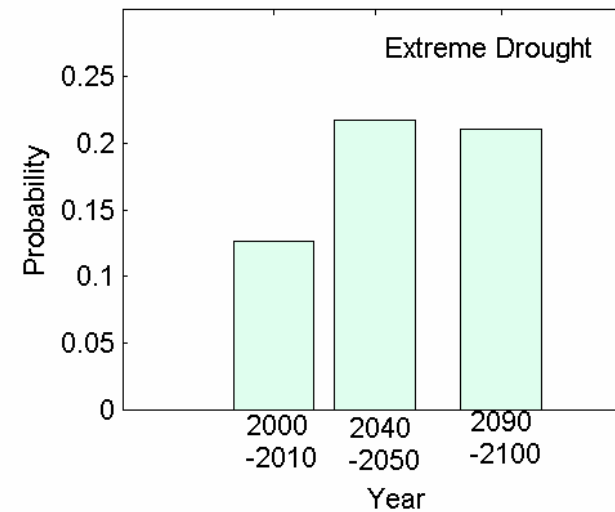
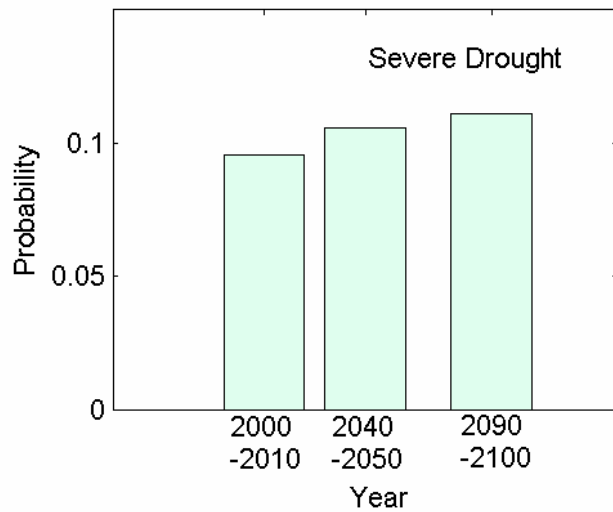
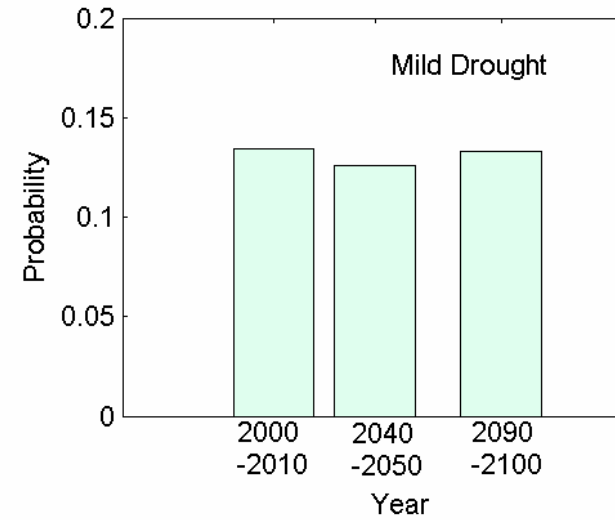
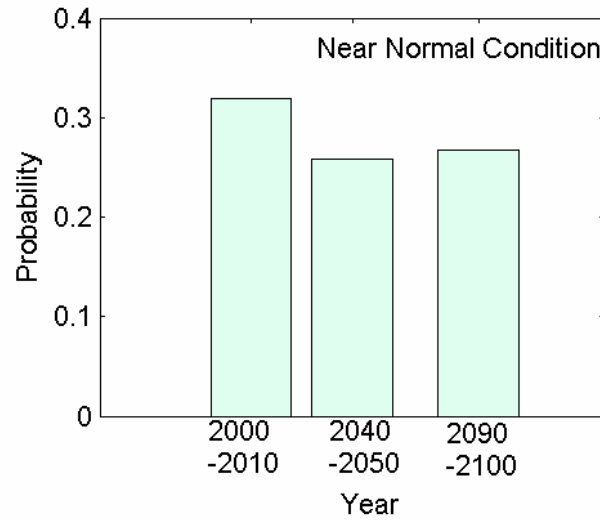
$$P(\text{ Extreme Drought }) = F_{\text{SPI}}(-2)$$

$$P(\text{ Severe Drought }) = F_{\text{SPI}}(-1.5) - F_{\text{SPI}}(-2)$$

$$P(\text{ Mild Drought }) = F_{\text{SPI}}(-1.0) - F_{\text{SPI}}(-1.5)$$

$$P(\text{ Near Normal }) = F_{\text{SPI}}(0) - F_{\text{SPI}}(-1.0)$$

Results: Probability of Drought



Kernel Density Estimation

- *Basic Equation*

$$\hat{f}(x) = (nh)^{-1} \sum_{l=1}^n K((x - X_l)/h)$$

$\hat{f}(x)$ - kernel density estimator of a pdf at x

n - number of observations

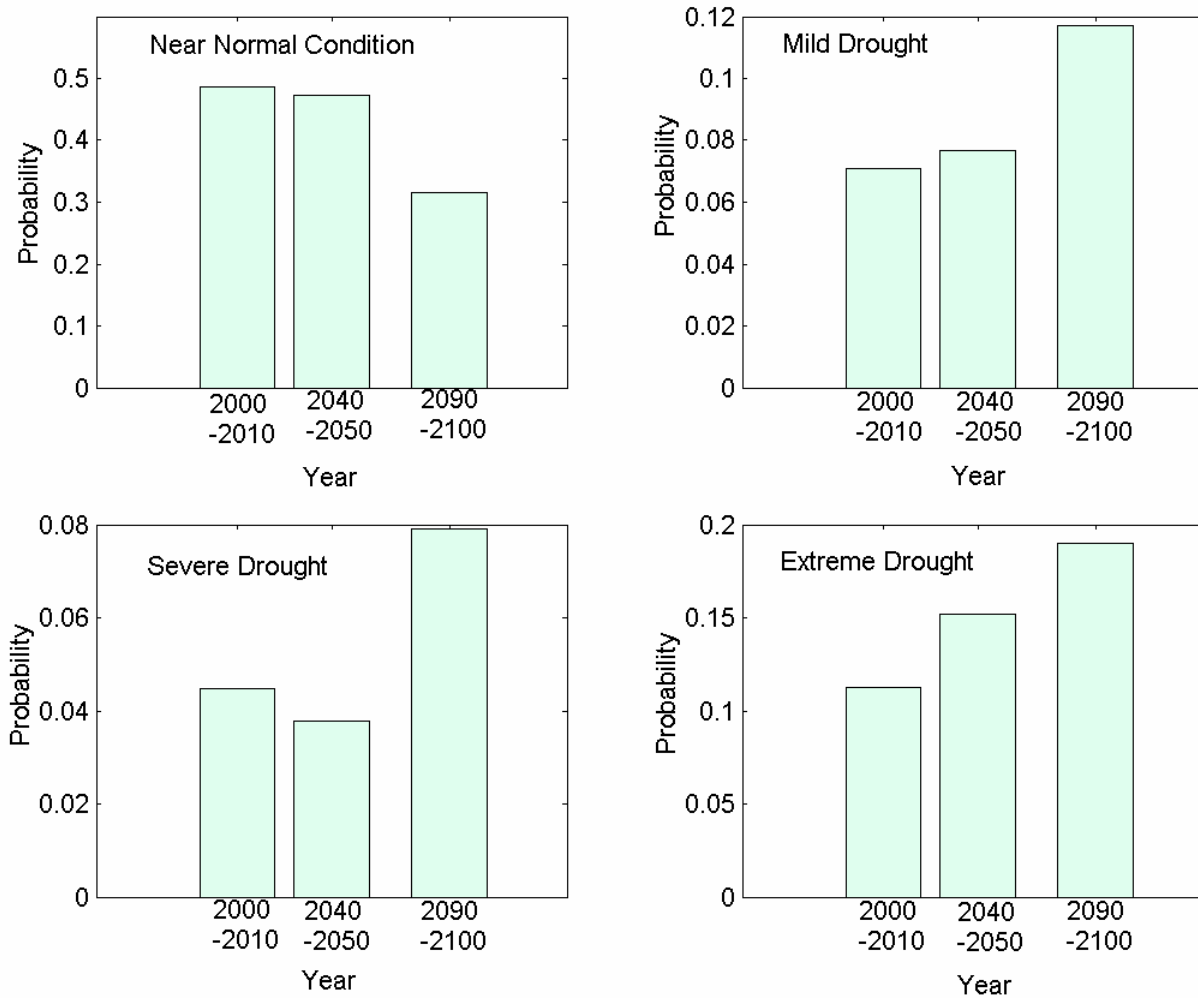
h - smoothing parameter known as bandwidth

Selection of bandwidth - an important step in kernel estimation method.

Conventional Method
(Silverman, 1986):

$$\left\{ \begin{array}{l} h_0 = (1.587)\sigma n^{-1/3} \\ \sigma = \min \left\{ S, \frac{IQR}{1.349} \right\} \end{array} \right.$$

Kernel Density Estimation: Results



Kernel Density Estimation: Drawbacks

- A large sample can give a better estimate of kernel density estimator. In the present analysis, the *sample size is small* with only the downscaled SPI of the available GCM output, which may not lead to accurate results
- The bandwidth is estimated by assuming the *actual density as normal, which may not be the actual case*. In such cases the estimate may be inaccurate.