



INDIAN INSTITUTE OF SCIENCE

# **STOCHASTIC HYDROLOGY**

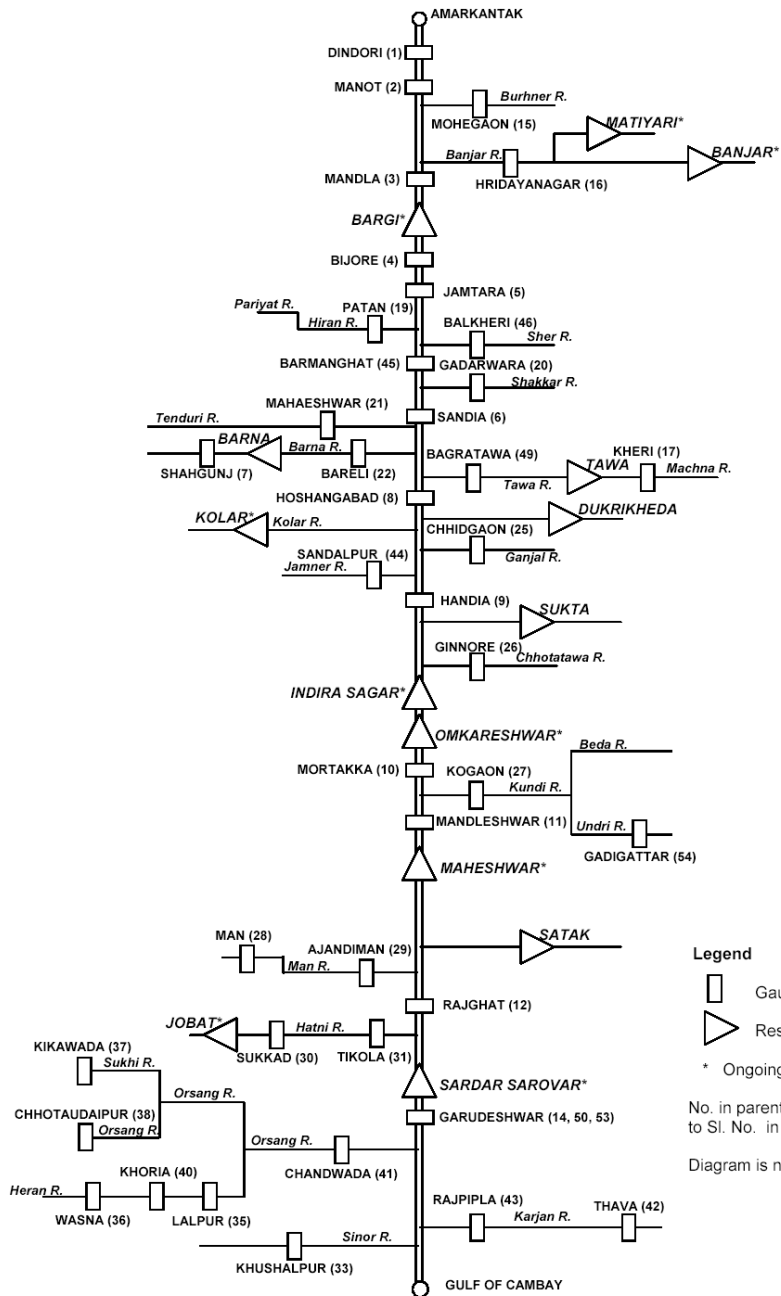
Lecture -38

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Department of Civil Engg., IISc.

# Summary of the previous lecture

- Data consistency checks
  - Double Mass Curve
  - Specific Flow



**Legend**

□ Gauge Site

▷ Reservoir

\* Ongoing

No. in parenthesis refers to Sl. No. in Table 1

Diagram is not to scale

# Data Consistency Checks

- (c) Comparison of specific flows:
- For comparison, gauge sites are put in four different groups based on the range of annual specific flows
  - annual specific flows of a downstream gauge site are compared with the those obtained for the surrounding upstream gauge sites

Annual average specific flow range (MCum/ sq.km)	Gauge site	Annual average specific flow (MCum/ sq.km)
0.2 to 0.3	Kogaon	0.2756
	Ajandiman	0.2559
	Bagratawa	0.2994
0.3 to 0.4	Garudeshwar	0.3670
	Patan	0.3895
	Tikola	0.3974
	Chandwada	0.3044
0.4 to 0.5	Sandia	0.4247
	Handia	0.4684
	Mortakka	0.4659
	Mandleshwar	0.4565
	Rajghat	0.4349
	Mohegaon	0.4758
	Hridayanagar	0.4598
	Maheshwar	0.4984
	Bareli	0.4760
	Ginnore	0.4380
	Sandalpur	0.4096
	Barmanghat	0.4779
	Balkheri	0.4789
Barman	0.4364	
0.5 to 0.7	Dindori	0.5460
	Bijore	0.5984
	Jamtara	0.5350
	Hoshangabad	0.5148
	Gadarwara	0.5749
	Chhidgaon	0.5824
	Manot	0.6519

# Data Consistency Checks

Consistency of Specific Flows in Intervening Catchments:

Let specific flows at stations A and B be  $S_A$  and  $S_B$ ,  
catchment areas  $C_A$  and  $C_B$  resp.

$$\text{Flow at A} = C_A S_A$$

$$\text{Flow at B} = C_B S_B$$



flow from intervening catchment bet. A and B =  $C_B S_B - C_A S_A$

$$\text{specific flow in the intervening catchment} = \frac{(C_B S_B - C_A S_A)}{(C_B - C_A)}$$

## Intervening Catchment Specific Flow Comparisons

S.No.	Description	Gauge site	Annual average sp. flow (MCum/sq.km)	Catchment area (sq.km)	Remarks
1	<b><u>Dindori-Manot</u></b>				Either contributions from controlled flows, or a higher rainfall in the intervening catchment; Otherwise inconsistency is indicated.
	Upstream site	Dindori	0.5460	2,292.00	
	Downstream site	Manot	0.6519	4,667.00	
	Intervening Catchment = (Manot-Dindori)		0.7541	2,375.00	
2	<b><u>Manot-Bijore</u></b>				
	Upstream site	Manot	0.6519	4,667.00	
	Downstream site	Bijore	0.5984	14,561.00	
	Intervening Catchment = (Bijore-Manot)		0.5732	9,894.00	
3	<b><u>Bijore-Jamtara</u></b>				Either significant utilisation or lower rainfall in the catchment above Jamtara, or both. Otherwise, inconsistency is indicated
	Upstream site	Bijore	0.5984	14,561.00	
	Downstream site	Jamtara	0.5350	17,157.00	
	Intervening Catchment = (Jamtara-Bijore)		0.1794	2,596.00	

# Data Consistency Checks

## Observations :

- The specific flow in the intermediate catchment between Dindori and Manot is 0.7541, compared to 0.546 at Dindori.
- This can happen if rainfall between Dindori and Manot is much larger than that above Dindori, or there is a contribution from controlled flows in the intervening catchment (or a combination of both).
- Otherwise, inconsistency is indicated

# Data Consistency Checks

Reservoir inflow:

- Reservoirs considered in simulation studies

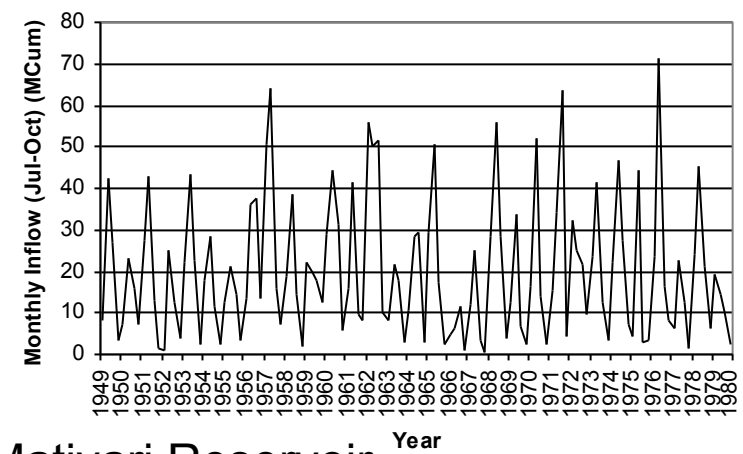
S.No.	Reservoir	Data used (period)	Type of data
1	Banjar	1981-2002	Daily
2	Matiyari	1949-1979	Monthly
3	Bargi	1948-1978	Monthly
4	Dukrikheda	1990-2004	Daily
5	Barna	1977-2002	Monthly
6	Tawa	1948-1993	Monthly
7	Kolar	1991-2000	Monthly
8	Sukta	1989-2003	Daily
9	Indira sagar	1988-2002	Monthly
10	Omkareshwar	-	-
11	Maheshwar	1950-1977	Monthly
12	Satak	-	-
13	Jobat	1961-1980	Monthly
14	Sardar sarovar	Flows at Garudeshwar will be used	Daily



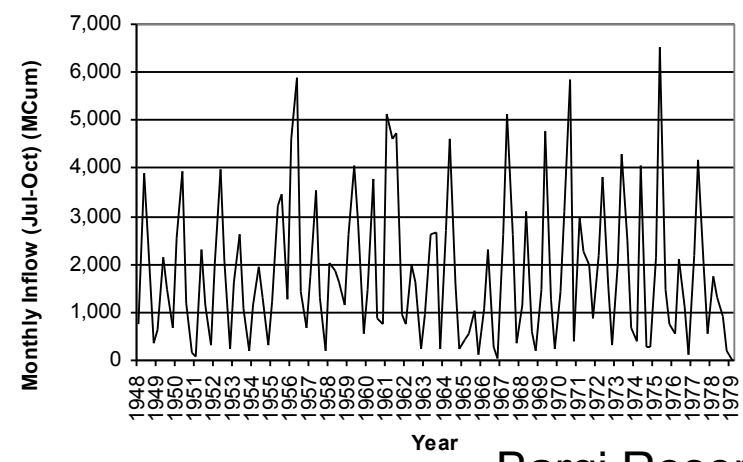
## Statistics of Annual Inflows

S.No	Gauge site	Data used (Period)	Duration (years)	Annual Average (MCum)	Maximum (MCum)	Minimum (MCum)	Standard deviation (MCum)	Coeff. of variation (%)
1	Matiyari*	1949-1979	31	80.43	168.17	23.43	32.25	40.10
2	Bargi*	1948-1978	31	7,392.65	15,430.00	2,152.00	2,957.96	40.01
3	Barna#	1977-2002	26	500.12	1,208.03	67.14	269.11	53.81
4	Tawa	1948-1993	46	3,768.41	9,444.75	1,787.68	1,721.83	45.69
5	Kolar	1991-2000	10	219.09	470.17	78.34	119.71	54.64
6	Sukta	1989-2003	15	71.03	98.81	32.95	22.82	32.13
7	Indira Sagar <sup>+</sup>	1988-2002	15	10,594.85	23,737.80	4,036.20	5,854.42	55.26
8	Maheshwar	1950-1977	28	27,822.55	56,125.10	11,298.90	9,454.72	33.98
9	Jobat#	1961-1980	20	299.49	807.10	39.20	203.16	67.84
10	Sardar sarovar	Flows at Garudeshwar will be used	-	-	-	-	-	-

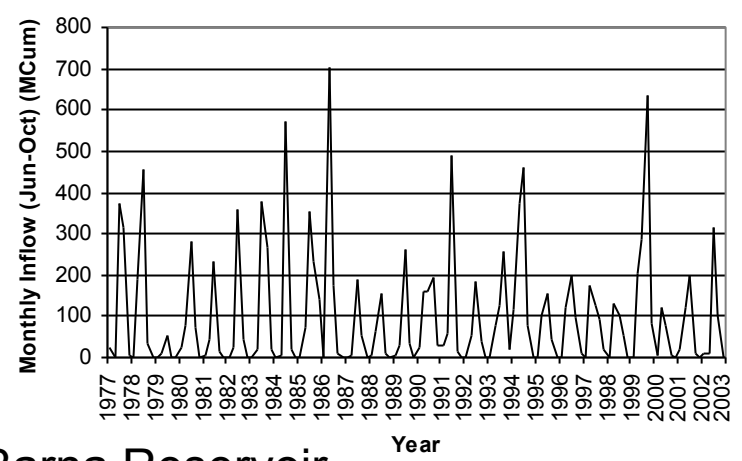
# Monthly Inflows



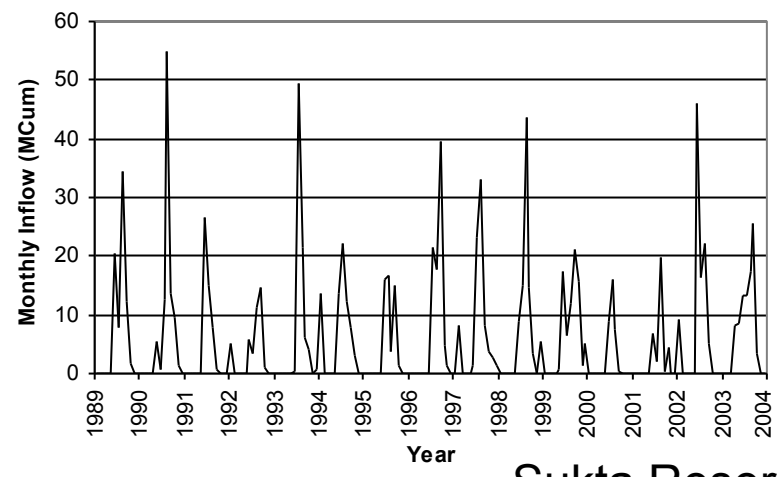
Matiyari Reservoir



Bargi Reservoir



Barna Reservoir



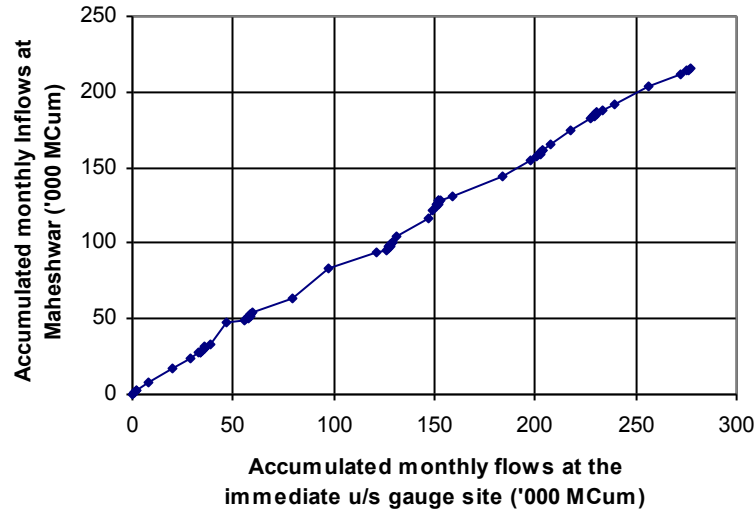
Sukta Reservoir

# Data Consistency Checks

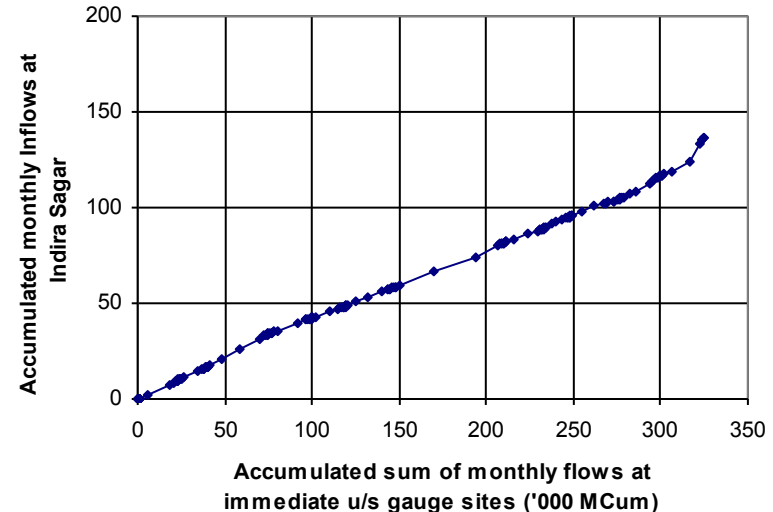
## Consistency of Reservoir Inflow Data :

- Similar to the gauge discharge data, consistency checks are performed for the reservoir inflow data.
- Double mass curves for inflows are prepared
- The double mass curves do not indicate any obvious inconsistency in the data

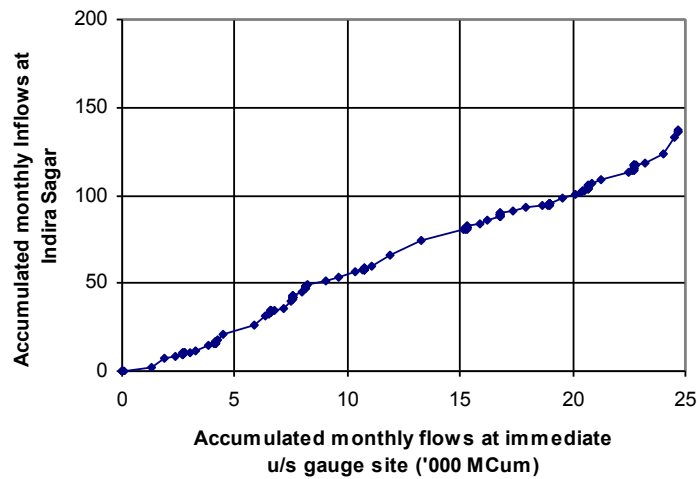
# Double mass curves for inflows



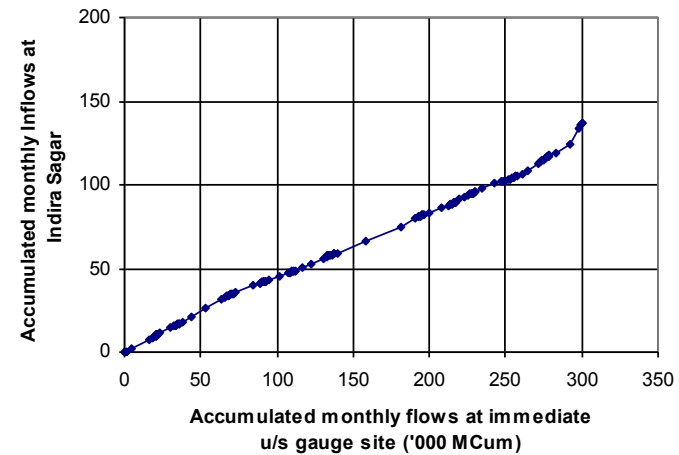
Maheshwar – u/s gauges



Indira Sagar – u/s gauges



Indira Sagar – Ginnore

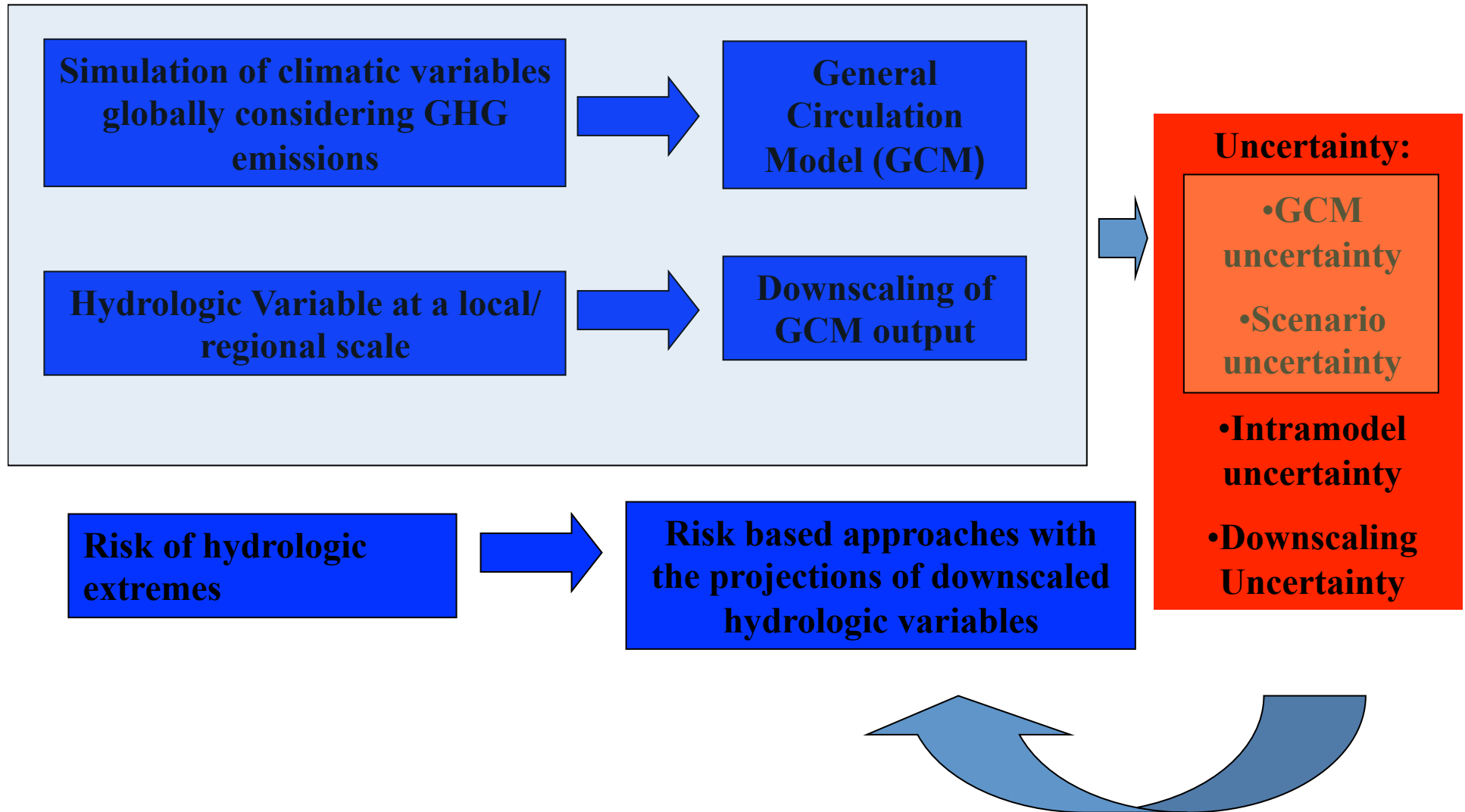


Indira Sagar – Handia

**Recent Applications of Stochastic Hydrology**

**Hydrologic Impacts of Climate Change:  
Quantification of Uncertainty**

# Assessment of Climate Change Impacts





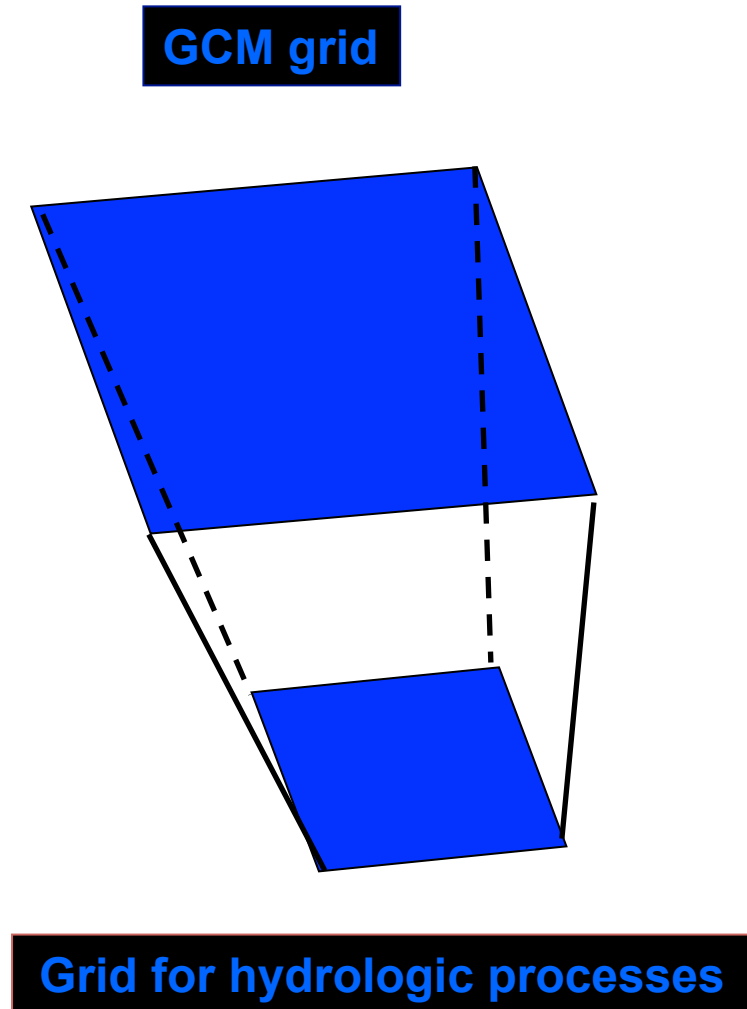
# Downscaling

**Downscaling:** to model the hydrologic variables (e.g., precipitation) at a smaller scale based on large scale GCM outputs.

**Statistical Downscaling:** produces future scenarios based on statistical relationship between large scale climate features and hydrologic variables like precipitation.

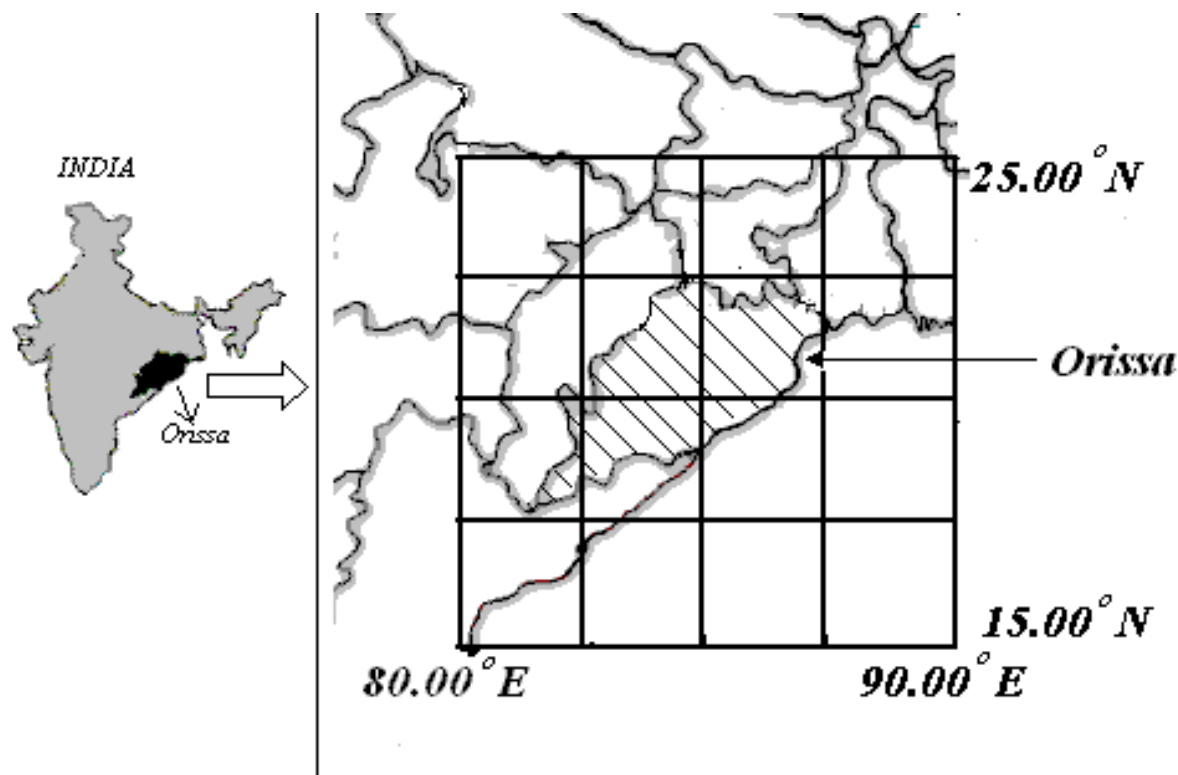
**Assumption-** Statistical relationship hold good in future for changed climate scenario.

**Advantage-** computationally simple and easily adjusted to new areas.





# Case-study Area: Orissa Meteorological Subdivision



- Coastal Area
- Increase of **hydrologic extremes** in recent past
- Increase in temperature: **1.1°C/century**, whereas in average increase in India: 0.4°C/century.

Ref : Subimal Ghosh and P.P. Mujumdar (2006) "Future Rainfall Scenario over Orissa with GCM Projections by Statistical Downscaling" *Current Science*, 90(3), Feb 10, 2006, pp. 396-404. (Pub : Indian Academy of Sciences, Bangalore)

# Regression with Cluster Membership

## ➤ Regression Equation:

- Without Fuzzy Clustering

$$RAIN_t = C + \sum_{k=1}^K \gamma_k \times pc_{kt} \quad \mathbf{R} \text{ value obtained: } 0.802$$

- With Cluster Membership

$$RAIN_t = C + \sum_{i=1}^{I-1} \beta_i \times \mu_{it} + \sum_{k=1}^K \gamma_k \times pc_{kt} + \sum_{i=1}^{I-1} \sum_{k=1}^K \rho_{ik} \times \mu_{it} \times pc_{kt}$$

**R** value obtained: 0.861

# Results of the Regression Model

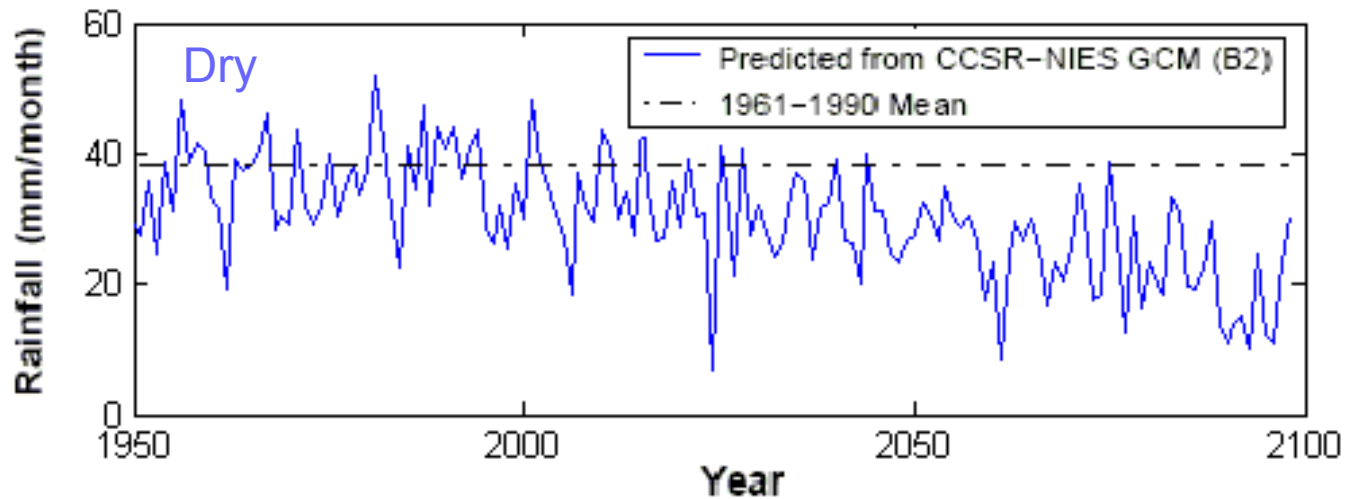
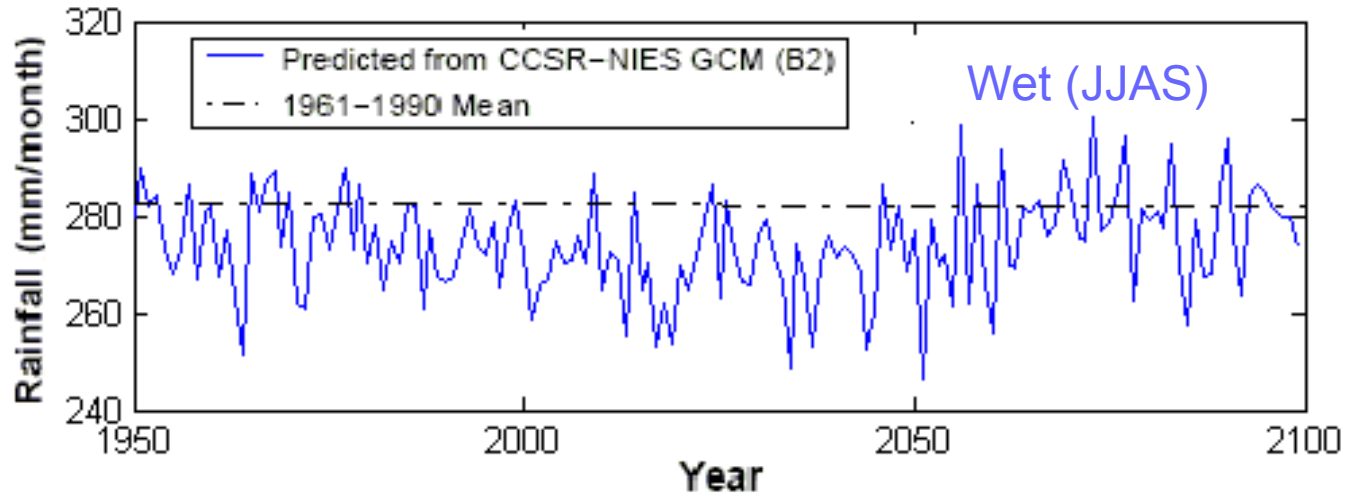
- **Multicollinearity, heteroscedasticity, normality of residuals, and autocorrelations of the residuals are tested.**
- **Long term mean and median**

Period	Obs. Mean	Pred. Mean	Obs. Median	Pred. Median
Wet (JJAS)	281.4 mm/month	281.3 mm/month	281.9 mm/month	283.3 mm/month
Dry	74.9 mm/month	74.3 mm/month	73.8 mm/month	73.6 mm/month

● **Nash – Sutcliffe Coefficient (E): 0.83**

$$E = 1 - \frac{\sum_t (P_{ot} - P_{pt})^2}{\sum_t (P_{ot} - \bar{P})^2}$$

# Prediction with CCSR/NIES GCM and B2 Scenario

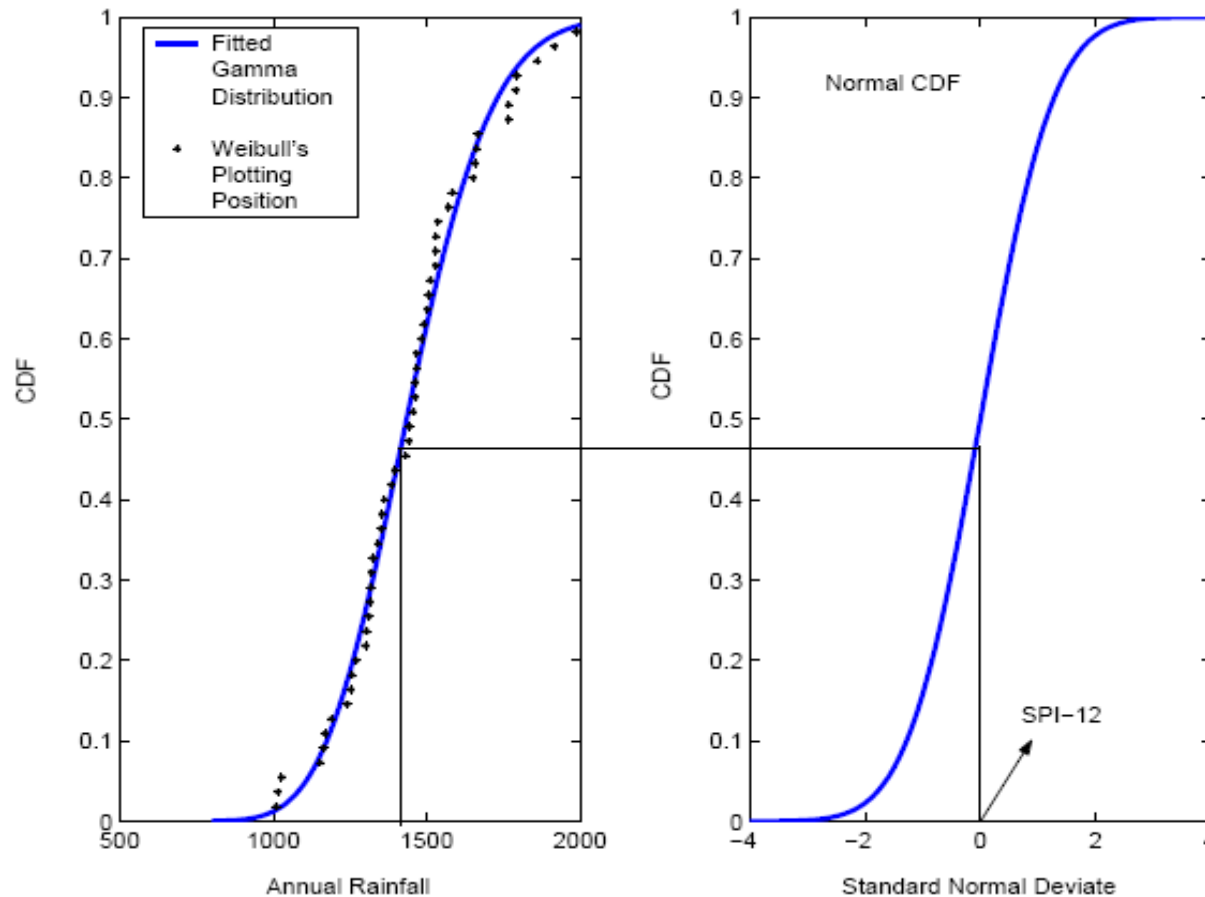


# Drought Assessment: Drought Indices

- Drought analysis is performed with drought indices
- A drought indicator, briefly defined, is a variable to identify and assess drought conditions (Steinemann, 2003)
- Different drought indices:
  - **Standardized Precipitation Index (SPI)** : Developed by McKee et. al (1993). Input data required: precipitation
  - **Palmer Drought Severity Index (PDSI)** : Developed by Palmer (1965). Input data required: precipitation, temperature data and local Available Water Content (AWC) of the soil
  - **Bhalme-Mooley Drought Index (BMDI)**: Monthly index. Input data required : monthly precipitation
  - **Effective Drought Index (EDI)**: Calculated in daily time step. Input data required : precipitation
- Index used in the present analysis: **SPI-12** (Annual SPI)

# SPI-12: Equiprobability Transformation

Corresponding to the CDF of the rainfall the standard normal deviate (mean 0 and variance 1) is termed as SPI.



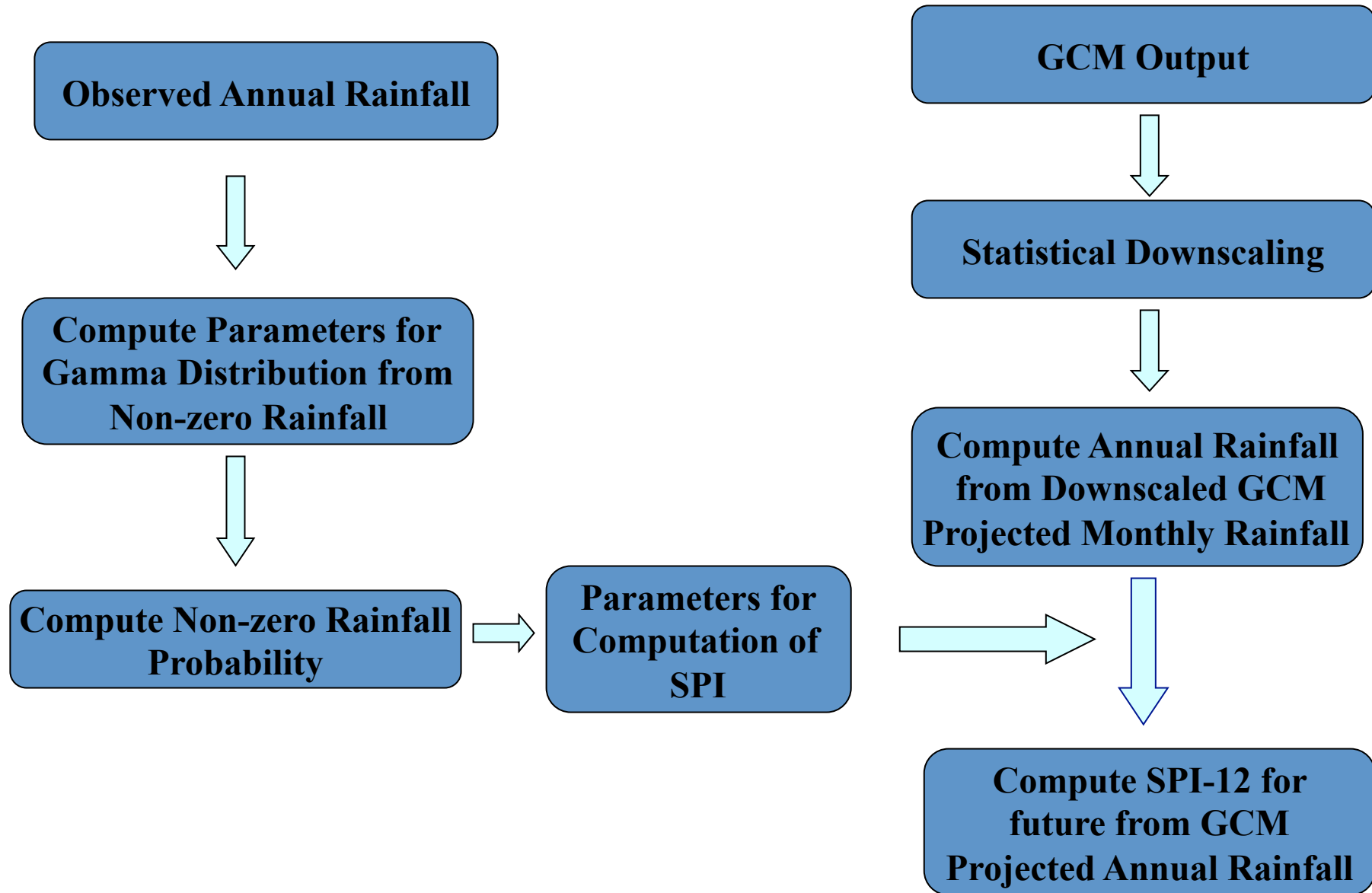
# Classification of Drought based on SPI (McKee et al., 1993)

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SPI Values	Drought Category
0 to -0.99	Near Normal
-1.00 to -1.49	Mild to Moderate Drought
-1.50 to -1.99	Severe Drought
-2.00 or less	Extreme Drought

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# SPI-12 Computation

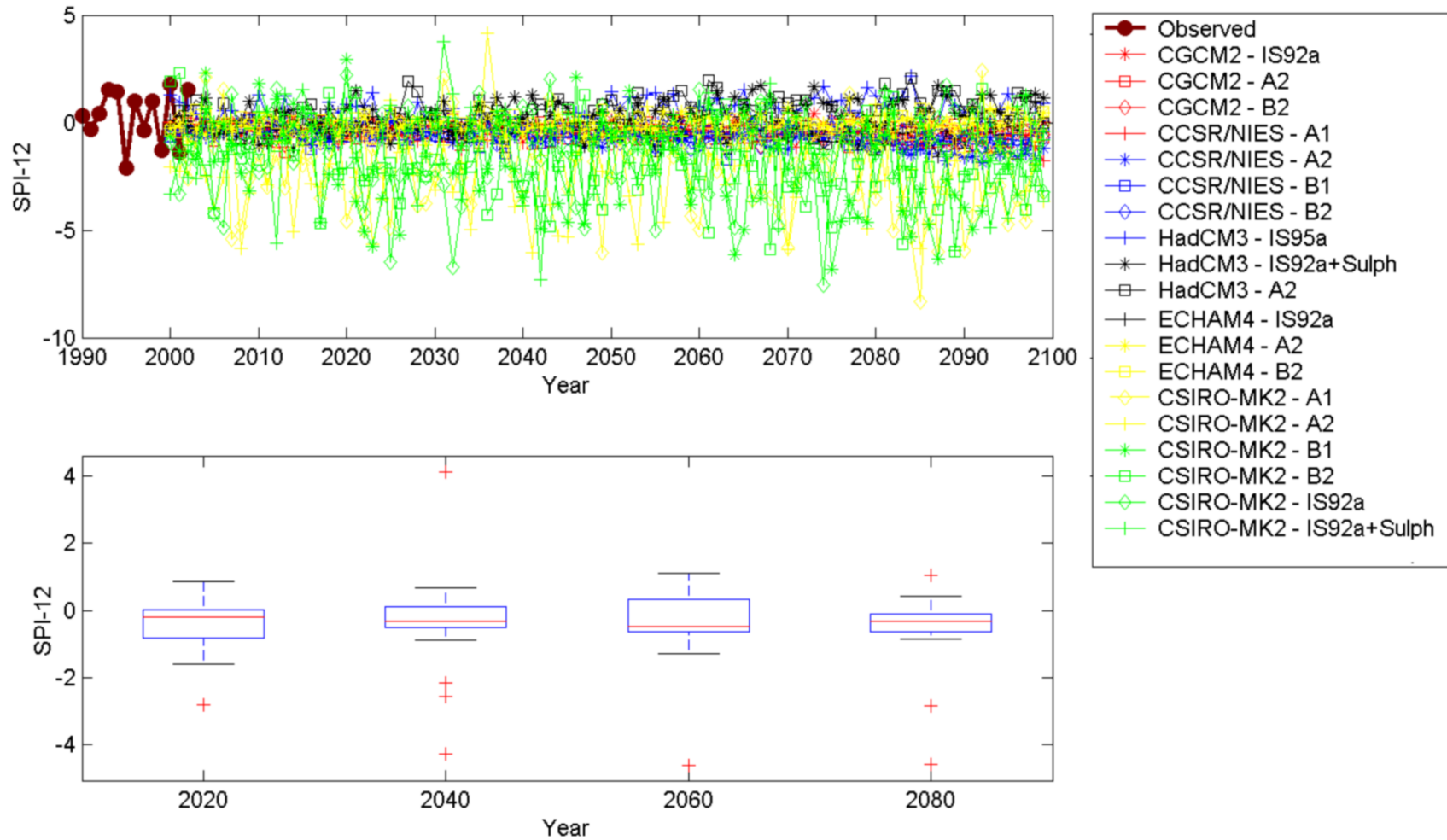




# GCMs and Scenarios Used

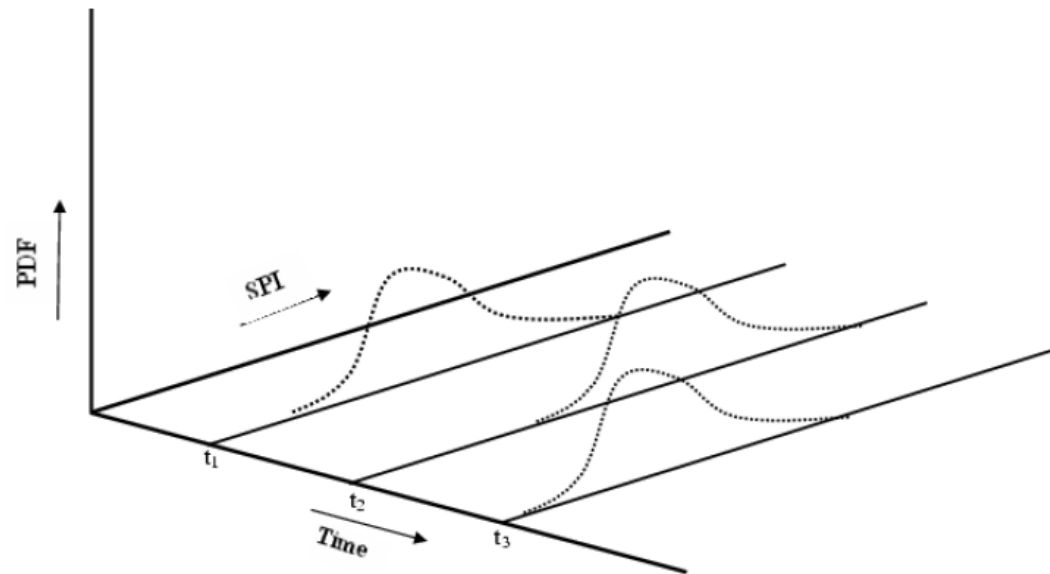
GCM	Organization	Scenarios Available
CCSR/NIES Coupled GCM	Center for Climate Research Studies (CCSR) and National Institute for Environmental Studies (NIES), Japan	A1, A2, B1, B2
Second Generation Coupled Global Climate Model (CGCM2)	Canadian Center for Climate Modelling and Analysis, Canada	IS92a, A2, B2
HadCM3	Hadley Centre for Climate Prediction and Research (HCCPR), UK	IS95a, (GHG+ Ozone+Sulphate), A2
ECHAM4/OPYC3	Max Planck Institute für Meteorologie, Germany.	IS92a, A2, B2
CSIRO-MK2	Australia's Commonwealth Scientific and Industrial Research Organisation (CSIRO)	(IS92a+Sulph), IS92a, A1, A2, B1, B2

# Projections of SPI-12



# Probability Density Function of SPI-12

- All the **scenarios** are **equally possible** (IPCC report, 2001)
- Outputs of all the **GCM models** are **equally accurate**.
- Time series generated by **each GCM model** for each of the scenario is considered as **a realization**.
- All the generated time series together are considered as **stochastic process**.
- At **each time step** there is a **marginal pdf of SPI-12**.



# Determination of pdf at each time step

- Assumption of Normal Distribution
- Kernel Density Estimation Method

# Assumption of Normal Distribution

At each time step (year) the SPI values are assumed to follow **normal distribution**

**CDF values** are estimated based on normal distribution, and probability of predicted droughts are estimated.

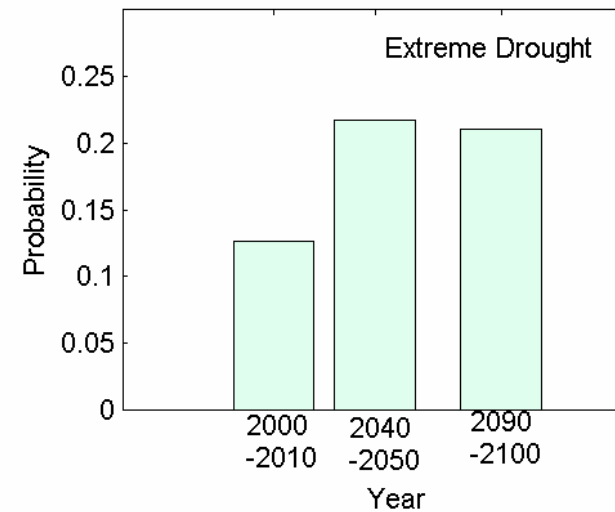
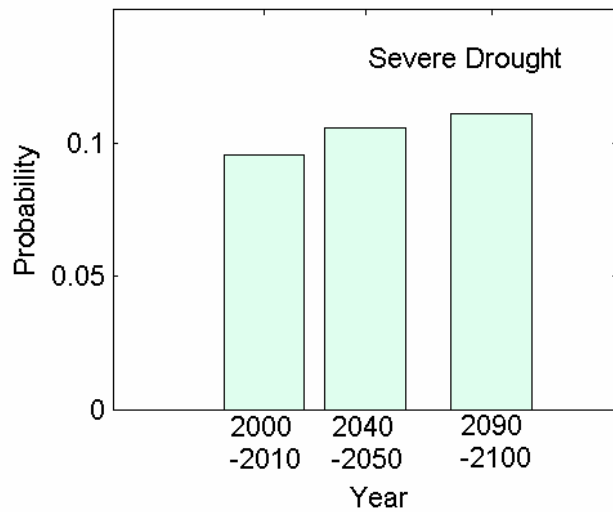
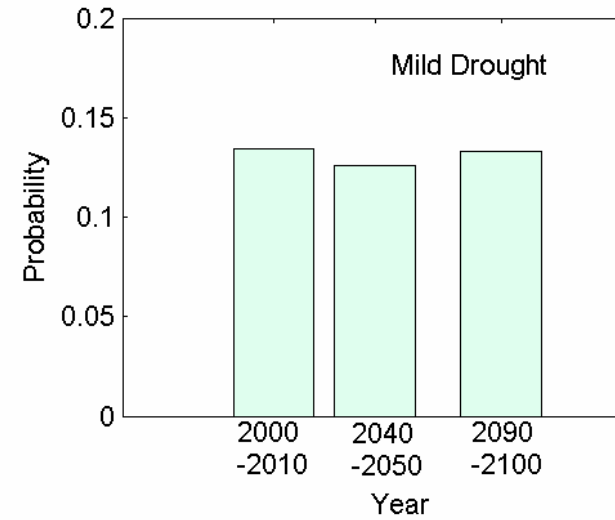
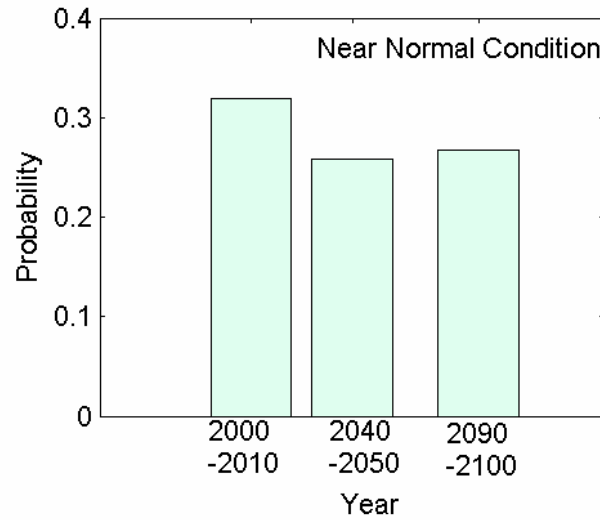
$$P(\text{ Extreme Drought } ) = F_{\text{SPI}}(-2)$$

$$P(\text{ Severe Drought } ) = F_{\text{SPI}}(-1.5) - F_{\text{SPI}}(-2)$$

$$P(\text{ Mild Drought } ) = F_{\text{SPI}}(-1.0) - F_{\text{SPI}}(-1.5)$$

$$P(\text{ Near Normal } ) = F_{\text{SPI}}(0) - F_{\text{SPI}}(-1.0)$$

# Results: Probability of Drought



# Kernel Density Estimation

- *Basic Equation*

$$\hat{f}(x) = (nh)^{-1} \sum_{l=1}^n K((x - X_l)/h)$$

$\hat{f}(x)$  - kernel density estimator of a pdf at  $x$

$n$  - number of observations

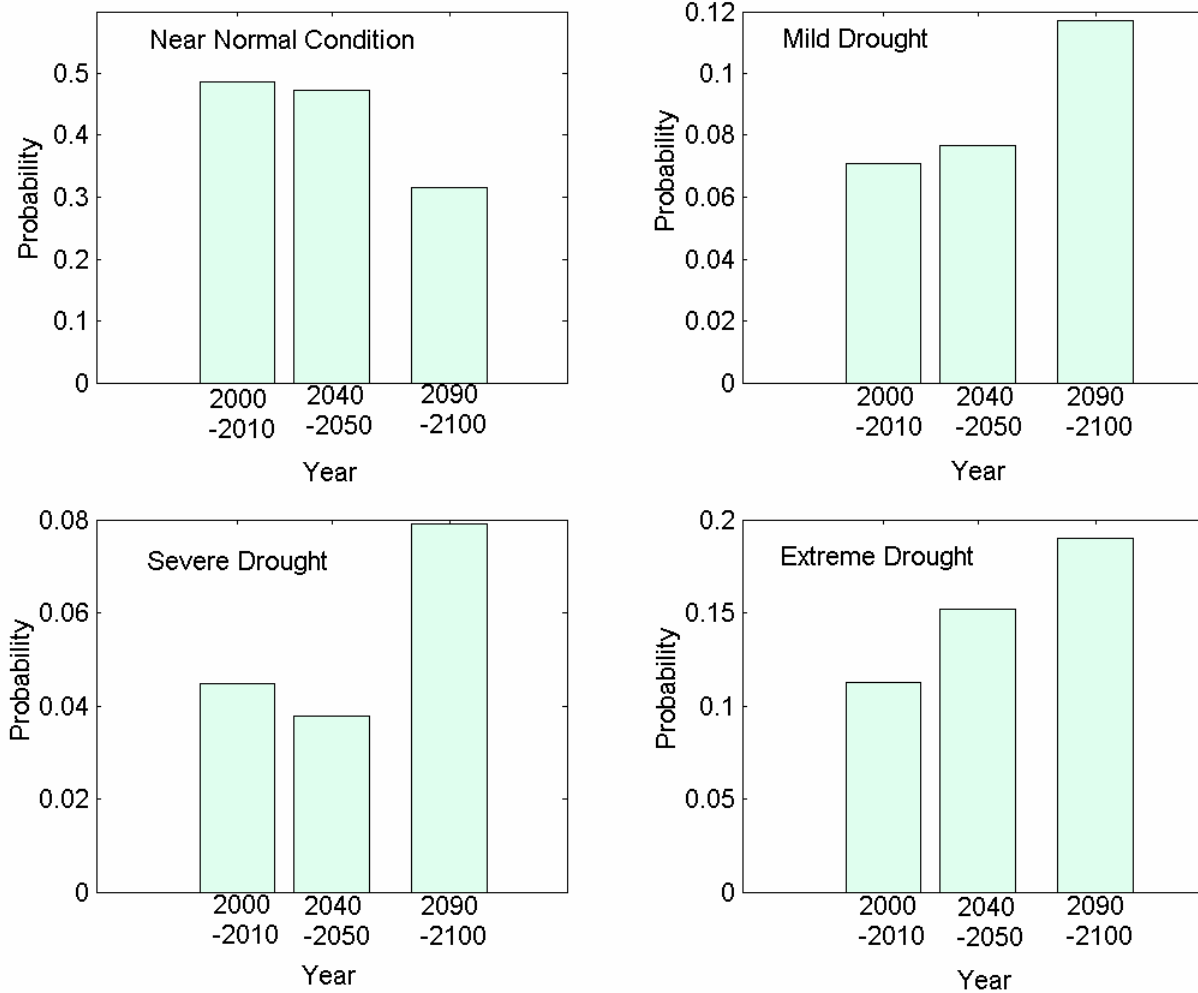
$h$  - smoothing parameter known as bandwidth

Selection of bandwidth - an important step in kernel estimation method.

Conventional Method  
(Silverman, 1986):

$$\left\{ \begin{array}{l} h_0 = (1.587)\sigma n^{-1/3} \\ \sigma = \min \left\{ S, \frac{IQR}{1.349} \right\} \end{array} \right.$$

# Kernel Density Estimation: Results





# Kernel Density Estimation: Drawbacks

- A large sample can give a better estimate of kernel density estimator. In the present analysis, the *sample size is small* with only the downscaled SPI of the available GCM output, which may not lead to accurate results
- The bandwidth is estimated by assuming the *actual density as normal, which may not be the actual case*. In such cases the estimate may be inaccurate.