INDIAN INSTITUTE OF SCIENCE

STOCHASTIC HYDROLOGY

Lecture -36

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Summary of the previous lecture

- Multivariate stochastic models
  - Matalas model

$$X_{t+1} = AX_t + B\varepsilon_{t+1}$$

where

X_t and X_{t+1} are p x 1 vectors representing standardized data corresponding to p sites at time steps t and $t+1$ resp

ε_{t+1} is N(0,1); p x 1 vector with ε_{t+1} independent of X_t.

A and B are coefficient matrices of size p x p. B is assumed to be lower triangular matrix

$$A = M_1 M_0^{-1}$$

$$BB' = M_0 - M_1 M_0^{-1} M_1'$$

M_0 is the cross-correlation matrix (size pxp) of lag zero

M_1 is the cross-correlation matrix (size pxp) of lag one

DATA CONSISTENCY CHECKS

Data Consistency Checks

(a) Consistency of flow data at a gauge site with the sum of flows from immediate upstream gauges,

(b) Consistency of flow data with respect to specific flows,

(c) Consistency of flow data with the flow data at an immediate neighboring (upstream) station, and

(d) Consistency of the reservoir inflow data where available, with the data from the surrounding gauge sites

Data Consistency Checks

- Continuous data to be available for checking consistency.
- Missing data to be filled in before checking the consistency.

Data Consistency Checks

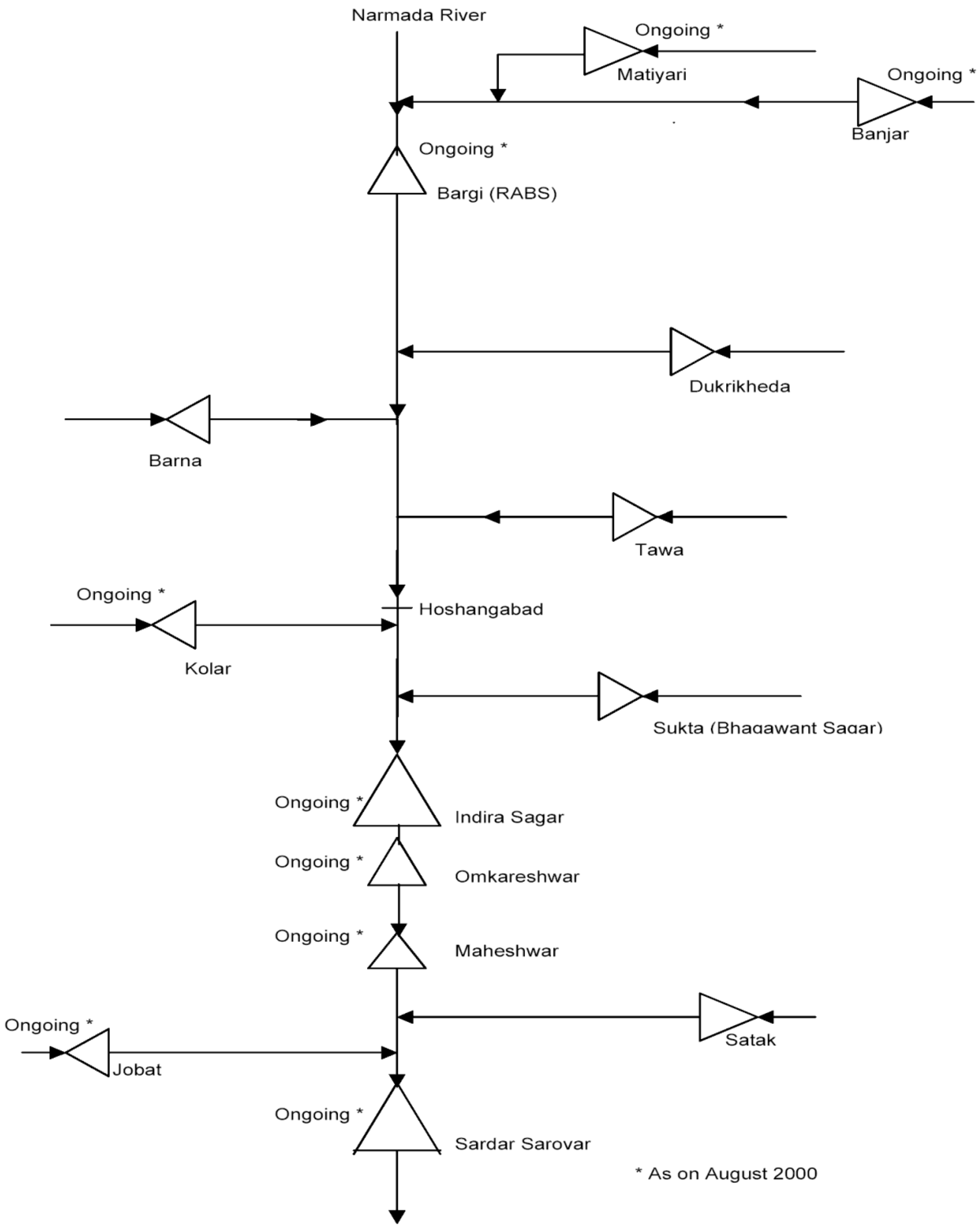
Missing data for an example basin:

- The basin is predominantly monsoon driven.
- Flow patterns in the monsoon and non-monsoon seasons are different.
- Different procedures adopted for filling-up the discharge data for the two seasons

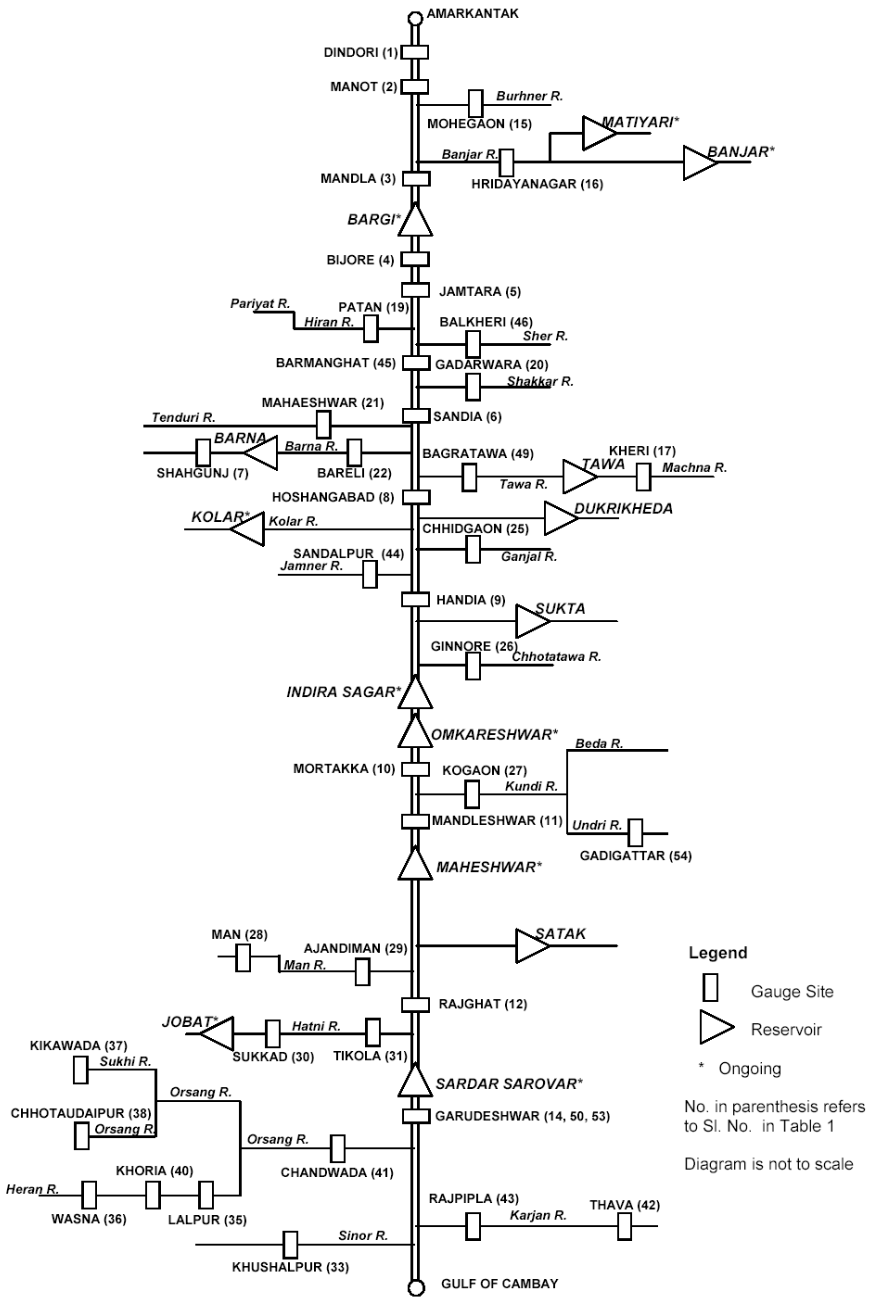
Data Consistency Checks

Data availability for the basin:

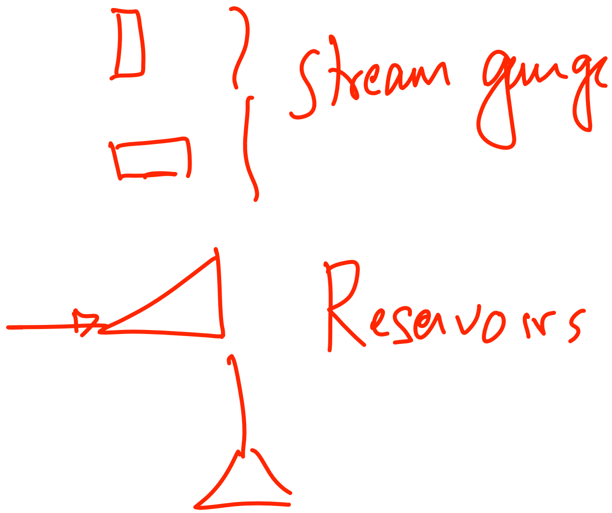
- Daily stream discharge data at 55 gauge sites available.
- Gauge sites at which short length of data (of less than 5 years) is available, are discarded from the analyses.
- 30 gauge sites are selected based on the length of data availability and location.



Narmada River
Ongoing *
Matiyari
Ongoing *
Banjar
Ongoing *
Bargi (RABS)
Dukrikheda
Barna
Tawa
Ongoing *
Hoshangabad
Kolar
Sukta (Bhagawant Sagar)
Ongoing *
Indira Sagar
Ongoing *
Omkareshwar
Ongoing *
Maheshwar
Satak
Ongoing *
Jobat
Ongoing *
Sardar Sarovar
* As on August 2000



Line Diagram of the Basin Showing the Location of Stream Gauge Sites and Reservoirs



Data Consistency Checks

Monsoon Period (Months: May-September):

Number of years for which data is available is greater than or equal to 12

Step-1:

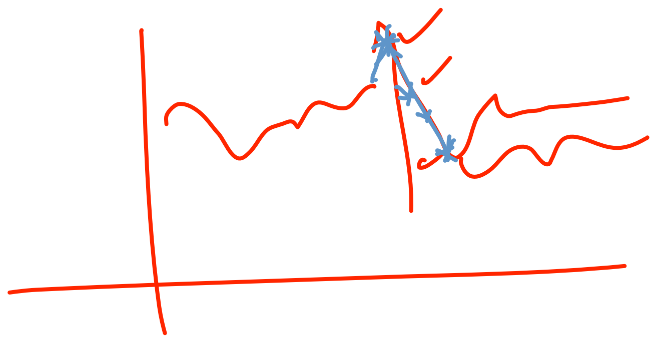
- Replace the missing data by average value for that day, computed from years in which data is available for the particular day.

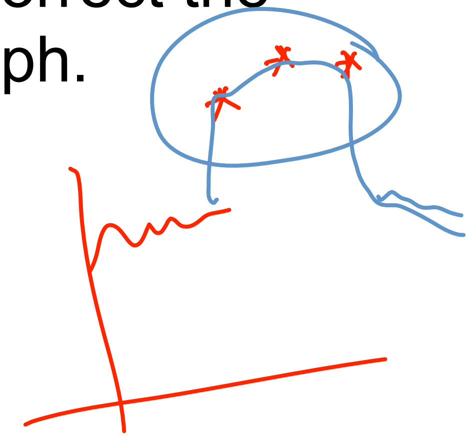
  Ex: Discharge data for June 20^{th} in a year is missing; Compute the average discharge of June 20^{th} from the data of other years for which the discharge data is available for June 20^{th}, and fill up the missing data with this average value

Data Consistency Checks

Step-2:

- Plot the time series of daily flows and check whether there are any significant fluctuations in the hydrograph. If such a case is noticed, correct the filled up data to smoothen the hydrograph.





Data Consistency Checks

Monsoon Period (Months: May-September):

Number of years for which data is in between 5 and 12

- If the missing data is non continuous (i.e., not more than 3 days continuously in a month), plot the daily flow time series for the month and join the curve smoothly to obtain the missing values

Data Consistency Checks

If the missing data is continuous (i.e., more than 3 days continuously in a month)

- Compute the proportion of monthly flow occurring in each day for which the data is missing.

  For example, data for six days, June 20-25, 1998 is missing.

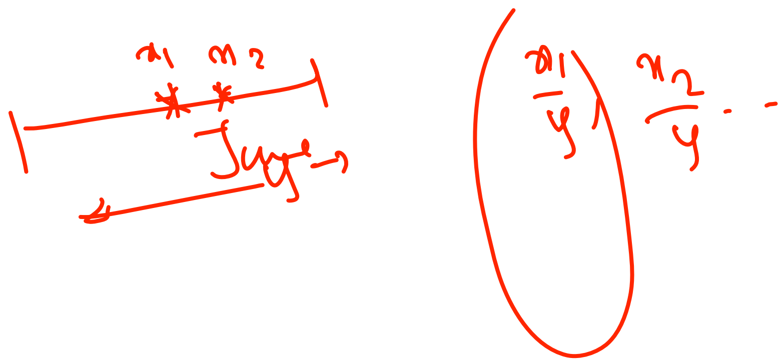
  Data for all days in June is available for 7 years

Data Consistency Checks

From the June 20th data of each of the 7 years, compute the proportion of the June months flow, occurring on June 20th. There will be seven such values.

Similarly compute these 7 proportions for June 21st, .., 25th also

- Compute the average of these proportions for each day of missing data



Data Consistency Checks

- Use this average proportion on the average of the month to fill the data for that day

  In the example above, let the average of June flow computed based on 7 years data = 12.6 cumecs

  Average proportion for June 20th = 0.02

  Then, the data for June 20th 1998 is filled-up as 0.02 * 12.6 = 0.252 cumecs

- Similarly fill-up the data for other days in that month.

Data Consistency Checks

Non-monsoon Period (Months: January-April and October-December):

•Compute the probability that the flow is non-zero for that day. (Clarke, 1973)

Ex: Day - Oct.20th; Number of years of data available = 7

Number of non-zero flows on Oct. 20th in the 6 years (leaving out the year in which the data is being filled) = 2 (i.e., only 2 out of these 6 years have a non-zero flow. Remaining 4 years have a zero flow)

Clarke, R.T. (1973), Mathematical Models in Hydrology, FAO Irrigation and Drainage Paper no. 19, Rome, Italy.

Data Consistency Checks

Probability that a non-zero flow occurs on Oct. 20^{th} = 2/6 = 0.333

- Generate a uniform random number between 0 and 1

e.g., for the Oct. 20^{th} missing data in (a) above, let this random number be 0.248

Data Consistency Checks

- If the random number is less than the probability of non-zero flow for that day then it is assumed that a non-zero flow will occur for that day and the missing data is replaced with the average value for that day; Otherwise, the flow for that day is taken as zero

In the example above, the probability of non-zero flows = 0.333

Random number = 0.248

Since 0.248 < 0.333, non-zero flow occurs.

Number of years in which the flow is non-zero = 2

Average flow on Oct. 20th based on these two years of flow = 2.87 cumecs

Data Consistency Checks

Statistical Analysis of Discharge Data:

- Statistical analysis of discharge data is performed on the data from the selected 30 gauge sites.
- Analyses is conducted over a minimum of 12 years data starting from 1970
- The Garudeshwar data considered is only for 7 years, from 1970-1976

Statistics of Annual Flows

S.No	Gauge site	Data used (Period)	Duration (years)	Average (MCum)	Maximum daily flow (cumec)	Maximum (MCum)	Minimum (MCum)	Standard deviation (MCum)	Coeff. of variation (%)
1	Dindori	1988-1999	12	1,251.50	2,624.00	2,519.00	760.19	438.13	35
2	Manot	1976-1999	24	3,042.13	6,180.00	5,427.17	1,012.56	988.13	32
3	Mandla Town	1977-1980 1993-1995	7	4,588.07	8,409.71	14,336.93	690.92	4,594.22	100
4	Bijore	1988-1999	12	8,713.39	20,349.00	16,473.11	2,870.88	4,168.42	48
5	Jamtara	1971-1999	29	9,179.34	21,355.10	20,985.05	2,371.81	3,706.08	40
6	Sandia	1978-1999	22	14,419.68	18,160.00	37,623.47	6,066.00	6,744.77	47
7	Hoshangabad	1972-1999	28	22,932.10	31,600.00	53,146.10	8,052.22	10,279.85	45
8	Handia	1977-1999	23	25,304.99	26,240.00	60,253.63	11,415.29	10,615.42	42
9	Mortakka	1970-1978 1988-1999	21	31,300.49	59,371.49	62,923.09	16,158.82	13,087.86	42
10	Mandleshwar	1971-1999	29	33,239.03	100,096.80	69,615.62	15,226.93	13,600.32	41
11	Rajghat	1971-1999	29	33,777.94	56,601.30	74,077.80	14,951.77	13,858.23	41
12	Garudeshwar	1971-1975 1980-1999	25	32,254.93	53,749.00	74,077.70	15,513.25	14,016.17	43
13	Mohegaon	1977-1999	23	2,199.34	6,526.90	4,240.24	646.21	791.37	36
14	Hridayanagar	1976-1999	24	1,549.61	3,632.00	4,756.16	544.39	891.54	58
15	Patan	1979-1999	21	1,538.53	1,817.00	3,552.23	497.55	788.92	51

Statistics of Annual Flows (contd.)

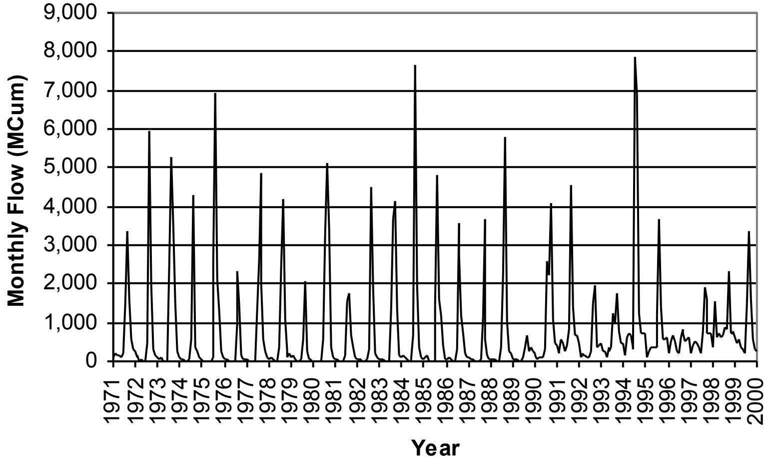
S.No	Gauge site	Data used (Period)	Duration (years)	Average (MCum)	Maximum daily flow (cumec)	Maximum (MCum)	Minimum (MCum)	Standard deviation (MCum)	Coeff. of variation (%)
16	Gadarwara	1977-1999	23	1,304.96	3,080.80	3,016.02	459.23	646.76	50
17	Maheshwar	1985-1993 1996-2000	14	745.17	1,786.00	1,364.76	263.88	299.20	40
18	Bareli	1985-1993 1998-2000	12	756.89	2,964.42	3,749.21	47.10	1,032.01	136
19	Chhidgaon	1976-1999	24	1,007.02	4,460.00	2,187.60	249.97	502.53	50
20	Ginnore	1971-1999	29	2,109.27	12,157.80	4,525.83	553.87	989.50	47
21	Kogaon	1978-1999	22	1,090.14	4,555.00	2,296.16	164.53	660.65	61
22	Ajandiman	1985-1993 1996-2000	14	255.10	1,149.00	625.33	40.74	183.67	72
23	Tikola	1985-1993 1996-1999	13	532.10	1,398.41	1,738.49	61.34	554.75	104
24	Chandwada	1979-1999	21	1,455.58	7,823.80	4,082.33	149.28	1,097.90	75
25	Sandalpur	1987-1993 1996-2000	12	226.10	779.10	423.89	47.77	133.09	59
26	Barmanghat	1988-1999	12	12,642.86	16,283.00	28,749.00	4,052.33	6,120.13	48
27	Balkheri	1977-1999	23	722.18	4,961.10	1,639.75	211.29	350.74	49
28	Barman	1970-1988 1991-1995	24	11,593.41	20,658.20	27,743.94	4,422.63	5,235.99	45
29	Bagratawa	1976-1991	16	1,801.80	9,584.20	5,329.54	61.24	1,383.53	77
30	Gurudeshwar A.M	1970-1976	7	46,756.48	61,000.00	76,885.33	28,104.76	15,774.09	34

Data Consistency Checks

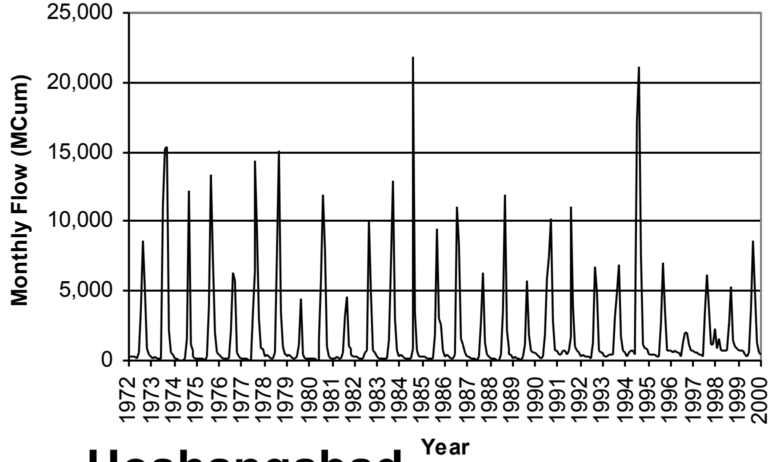
Statistical Analysis of Discharge Data:

- The coefficient variation is more than 100% for the gauge sites, Mandla town, Bareli and Tikola, indicating a high variation of flows
- Similar statistics are computed for monthly flow data
- In non-monsoon months a large variation is indicated
- As the flow magnitudes are small in the non-monsoon months, the large variation is not of much practical significance

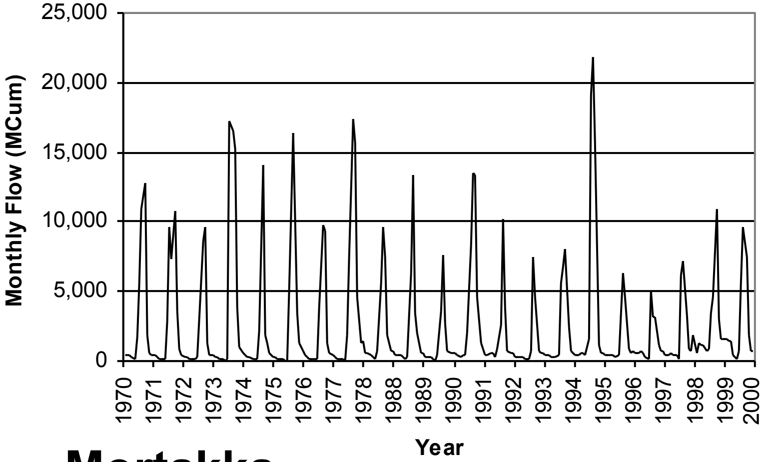
Monthly Flows



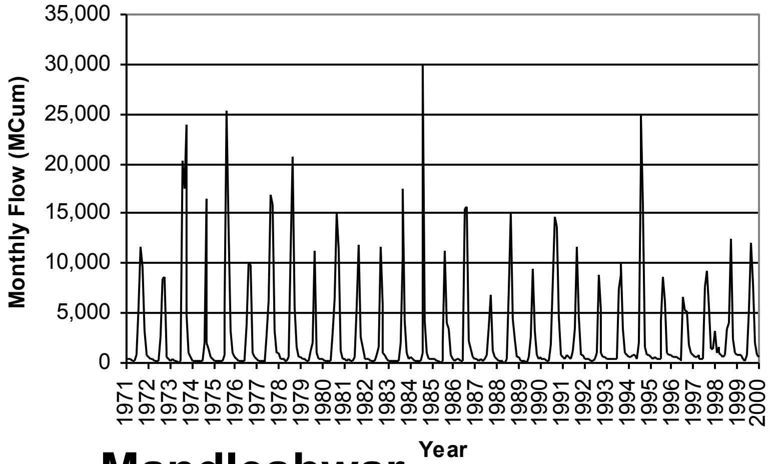
Jamtara



Hoshangabad



Mortakka



Mandleshwar

Data Consistency Checks

Specific flows:

- The specific flow is expressed as flow volume per unit area of the catchment
- Represents the catchment response to precipitation
- If a number of gauge stations are located in the same hydroclimatic region with similar land use patterns, then the specific flows computed with data at the gauge stations must be comparable
- Annual specific flows are computed as the ratio of average annual flow to catchment area

Data Consistency Checks

- This statistic is useful in comparing the runoff per unit area from different sub-catchments within the basin
- The specific flow is computed in MCum/sq.km; a measure of the annual average runoff in meters
- The average annual rainfall in the basin varies from around 700 mm to around 1650 mm

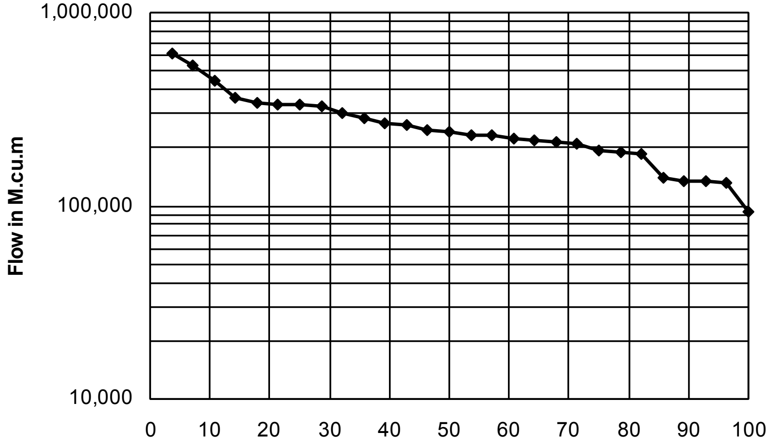
Annual and Seasonal Specific Flows for Gauge Sites

S.No.	Gauge site	Catch-ment area (sq. km)	Data used (period)	Duration (years)	Annual specific flows (MCum per sq. km)	Seasonal specific flows (MCum per sq. km)
1	Dindori	2,292.00	1988-1999	12	0.5460	0.4881
2	Manot*	4,667.00	1976-1999	24	0.6519	0.6113
3	Mandla Town	Not available	1977-1980 1993-1995	7	NA	NA
4	Bijore	14,561.00	1988-1999	12	0.5984	0.4361
5	Jamtara	17,157.00	1971-1999	29	0.5350	0.4569
6	Sandia	33,953.50	1978-1999	22	0.4247	0.3569
7	Hoshangabad	44,543.00	1972-1999	28	0.5148	0.4577
8	Handia	54,027.00	1977-1999	23	0.4684	0.4116
9	Mortakka	67,184.00	1970-1978 1988-1999	21	0.4659	0.4145
10	Mandleshwar	72,809.30	1971-1999	29	0.4565	0.4132
11	Rajghat	77,674.10	1971-1999	29	0.4349	0.3943
12	Garudeshwar	87,892.00	1971-1975 1980-1999	25	0.3670	0.3278
13	Mohegaon	4,622.00	1977-1999	23	0.4758	0.4506
14	Hridayanagar	3,370.00	1976-1999	24	0.4598	0.4411
15	Patan	3,950.00	1979-1999	21	0.3895	0.3563

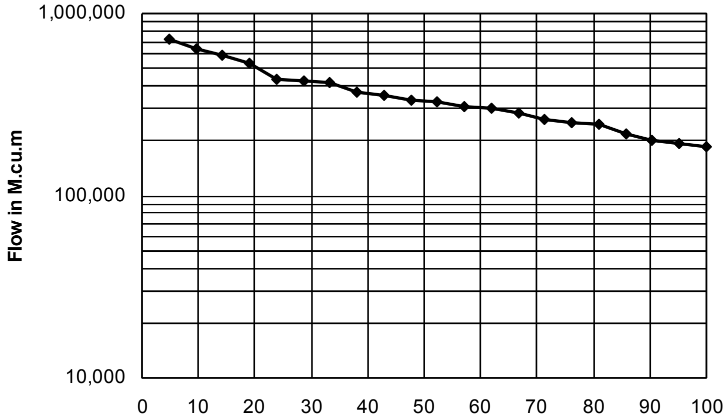
Annual and Seasonal Specific Flows for Gauge Sites

S.No.	Gauge site	Catch-ment area (sq. km)	Data used (period)	Duration (years)	Annual specific flows (MCum per sq. km)	Seasonal specific flows (MCum per sq. km)
16	Gadarwara	2,270.00	1977-1999	23	0.5749	0.5373
17	Maheshwar	1,495.00	1985-1993 1996-2000	14	0.4984	0.4269
18	Bareli	1,590.00	1985-1993 1998-2000	12	0.4760	0.4109
19	Chhidgaon	1,729.00	1976-1999	24	0.5824	0.5548
20	Ginnore	4,815.70	1979-1999	29	0.4380	0.4246
21	Kogaon	3,955.00	1978-1999	22	0.2756	0.2652
22	Ajandiman	997.00	1985-1993 1996-2000	14	0.2559	0.2455
23	Tikola	1,339.00	1985-1993 1996-1999	13	0.3974	0.3355
24	Chandwada	4,782.00	1979-1999	21	0.3044	0.3017
25	Sandalpur	552.00	1987-1993 1996-2000	12	0.4096	0.3692
26	Barmanghat	26,453.00	1988-1999	12	0.4779	0.3731
27	Balkheri	1,508.00	1977-1999	23	0.4789	0.4601
28	Barman	26,563.00	1970-1988 1991-1995	24	0.4364	0.3991
29	Bagratawa	6,018.00	1976-1991	16	0.2994	0.2910
30	Garudeshwar A.M	Not available	1970-1976	7	NA	NA

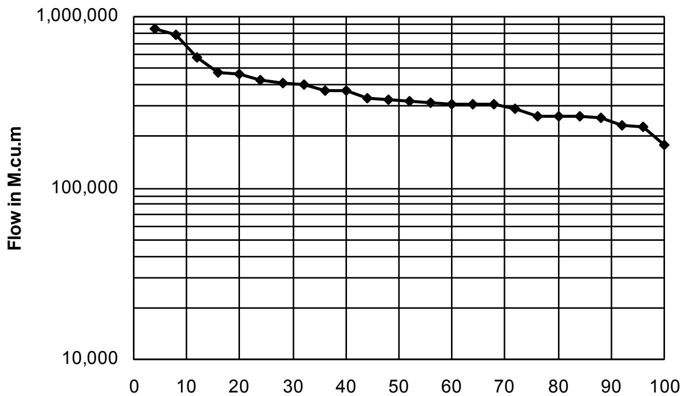
Flow duration curves



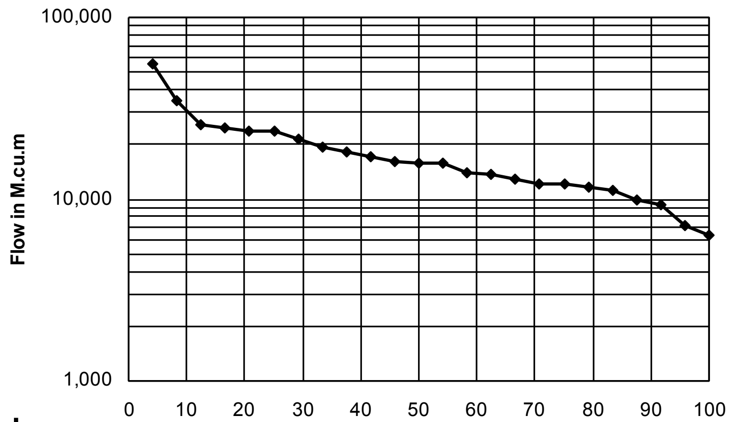
Hoshangabad



Mortakka



Garudeshwar



Hridayanagar

Data Consistency Checks

Consistency Checks for the basin:

(a) The sums of daily discharges (to get monthly values) as given with respect to the sums computed from the data.

(b) Consistency of flow data at a gauge site with the sum of flows from immediate upstream gauges,

(c) Consistency of flow data with respect to specific flows,

(d) Consistency of flow data with the flow data at an immediate neighboring (upstream) station, and

(e) Consistency of the reservoir inflow data where available, with the data from the surrounding gauge sites

Data Consistency Checks

(a) Mismatch of algebraic sums of daily flows:

- Daily discharge data as supplied are keyed in and the monthly flows are computed by summing the daily discharges
- The computed monthly flows are compared with the corresponding totals in the hard copies of the records
- high mismatches (of more than about 15%) occur mostly in the non-monsoon periods
- high mismatches occur during the monsoon periods for Hoshangabad, Barman, Handia, Mandleshwar and Ajandiman

Data Consistency Checks

(b) Homogeneity and consistency with respect to immediate upstream gauges :

- For checking the homogeneity of data at a site with respect to the data from upstream gauges, the double mass curve approach is used
- A double mass curve is plotted between accumulated monthly flows at the site being examined and the corresponding accumulated sum of monthly flows at immediate upstream gauge sites

Data Consistency Checks

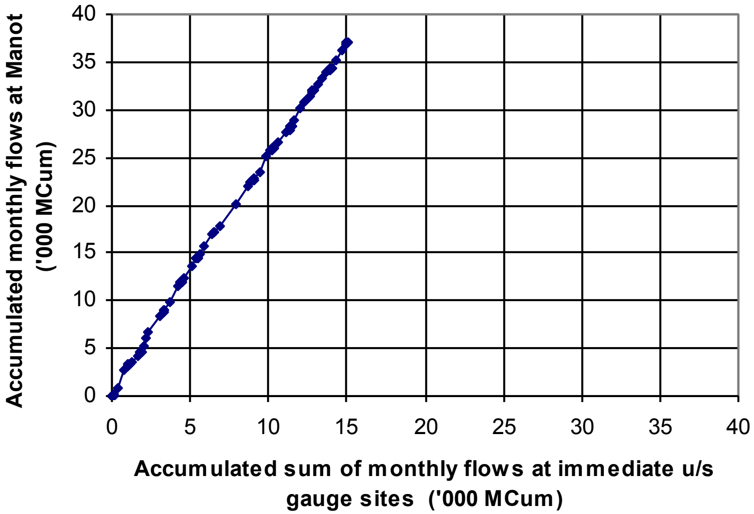
- If the net runoff in the intervening catchment (accounting for utilization) is added to the outflow at the upstream station/s, it should equal the flow at the downstream station.
- The double mass curve analysis is mainly useful in assessing whether the data at a given station is inhomogeneous

  i.e., whether data at the station has been affected due to circumstances like change in the method of measurement, shift in location etc., provided that the other stations are not so affected

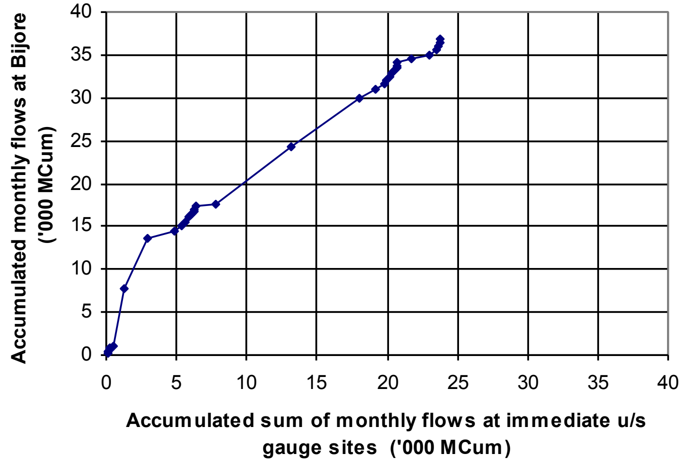
Data Consistency Checks

- The double mass curve does not directly indicate whether data at one station is hydrologically consistent with that at one or more upstream stations
- A nonlinear double mass curve can occur even when data at the two stations are consistent, if rainfall pattern in the intervening catchment differs from that in the catchment of the upstream station
- the double mass curves are used in this study primarily to examine the algebraic sums of flows at a downstream site with respect to the sums of flows at the upstream gauge sites

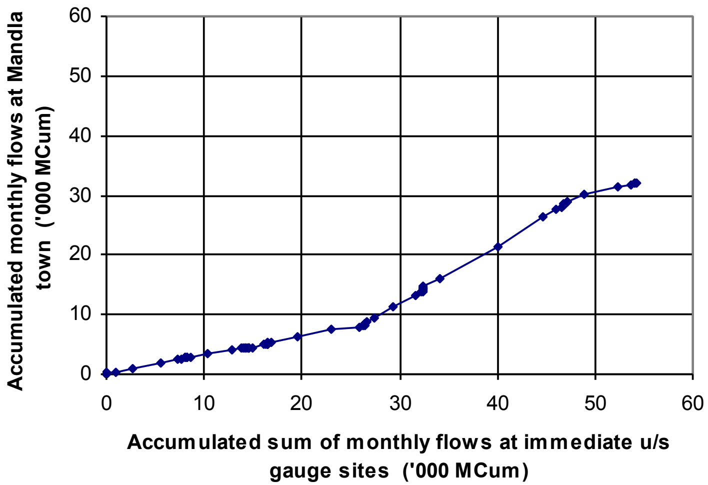
Double Mass Curve



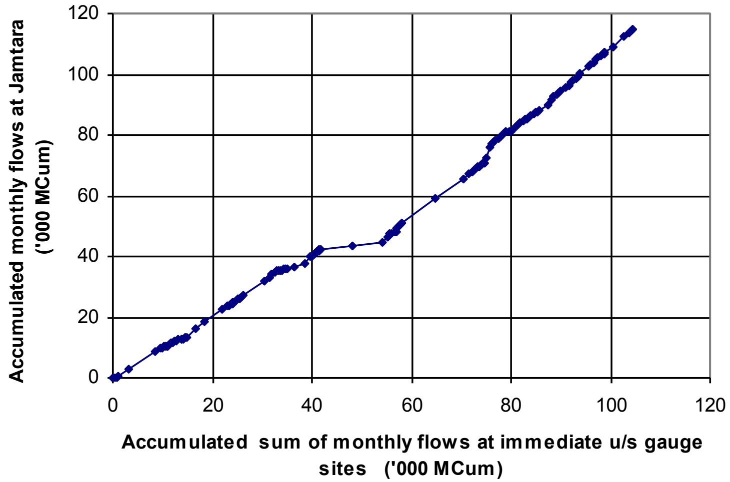
Manot – u/s Gauges



Bijore – u/s Gauges



Mandla Town – u/s Gauges



Jamtara – u/s Gauges

Data Consistency Checks

Few observations from double mass curve :

- The double mass curve for Manot gauge site indicates that the Manot flows are significantly higher than the flows at upstream gauge site
- The double mass curve for Mandla town gauge site indicates a change around June 1993
- The double mass curve for Bijore gauge site indicates that the flows are significantly higher than the aggregate flows at the upstream sites, implying high intermediate catchment flow
- The double mass curve for Jamtara indicates a break in slope in the year 1993

Data Consistency Checks

(c) Comparison of specific flows:

- For comparison, gauge sites are put in four different groups based on the range of annual specific flows
- annual specific flows of a downstream gauge site are compared with the those obtained for the surrounding upstream gauge sites

Annual average specific flow range (MCum/ sq.km)	Gauge site	Annual average specific flow (MCum/ sq.km)
0.2 to 0.3	Kogaon	0.2756
	Ajandiman	0.2559
	Bagratawa	0.2994
0.3 to 0.4	Garudeshwar	0.3670
	Patan	0.3895
	Tikola	0.3974
	Chandwada	0.3044
0.4 to 0.5	Sandia	0.4247
	Handia	0.4684
	Mortakka	0.4659
	Mandleshwar	0.4565
	Rajghat	0.4349
	Mohegaon	0.4758
	Hridayanagar	0.4598
	Maheshwar	0.4984
	Bareli	0.4760
	Ginnore	0.4380
	Sandalpur	0.4096
	Barmanghat	0.4779
	Balkheri	0.4789
	Barman	0.4364
0.5 to 0.7	Dindori	0.5460
	Bijore	0.5984
	Jamtara	0.5350
	Hoshangabad	0.5148
	Gadarwara	0.5749
	Chhidgaon	0.5824
	Manot	0.6519

Data Consistency Checks

Consistency of Specific Flows in Intervening Catchments:

Let specific flows at stations A and B be S_A and S_B, catchment areas C_A and C_B resp.

Flow at A = $C_A S_A$

Flow at B = $C_B S_B$

flow from intervening catchment bet. A and B = $C_B S_B - C_A S_A$

specific flow in the intervening catchment = $\dfrac{(C_B S_B - C_A S_A)}{(C_B - C_A)}$

Intervening Catchment Specific Flow Comparisons

S.No.	Description	Gauge site	Annual average sp. flow (MCum/ sq.km)	Catchment area (sq.km)	Remarks
1	**Dindori-Manot**				
	Upstream site	Dindori	0.5460	2,292.00	Either contributions from controlled flows, or a higher rainfall in the intervening catchment; Otherwise inconsistency is indicated.
	Downstream site	Manot	0.6519	4,667.00	
	Intervening Catchment = (Manot-Dindori)		0.7541	2,375.00	
2	**Manot-Bijore**				
	Upstream site	Manot	0.6519	4,667.00	
	Downstream site	Bijore	0.5984	14,561.00	
	Intervening Catchment = (Bijore-Manot)		0.5732	9,894.00	
3	**Bijore-Jamtara**				
	Upstream site	Bijore	0.5984	14,561.00	Either significant utilisation or lower rainfall in the catchment above Jamtara, or both. Otherwise, inconsistency is indicated
	Downstream site	Jamtara	0.5350	17,157.00	
	Intervening Catchment = (Jamtara-Bijore)		0.1794	2,596.00	

Data Consistency Checks

Few observations :

- The specific flow in the intermediate catchment between Dindori and Manot is 0.7541, compared to 0.546 at Dindori.
- This can happen if rainfall between Dindori and Manot is much larger than that above Dindori, or there is a contribution from controlled flows in the intervening catchment (or a combination of both).
- Otherwise, inconsistency is indicated

Data Consistency Checks

Reservoir inflow:

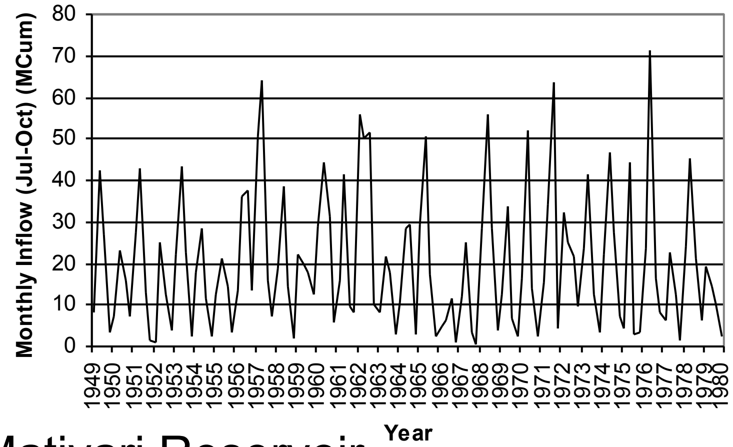
- Reservoirs considered in simulation studies

S.No.	Reservoir	Data used (period)	Type of data
1	Banjar	1981-2002	Daily
2	Matiyari	1949-1979	Monthly
3	Bargi	1948-1978	Monthly
4	Dukrikheda	1990-2004	Daily
5	Barna	1977-2002	Monthly
6	Tawa	1948-1993	Monthly
7	Kolar	1991-2000	Monthly
8	Sukta	1989-2003	Daily
9	Indira sagar	1988-2002	Monthly
10	Omkareshwar	-	-
11	Maheshwar	1950-1977	Monthly
12	Satak	-	-
13	Jobat	1961-1980	Monthly
14	Sardar sarovar	Flows at Garudeshwar will be used	Daily

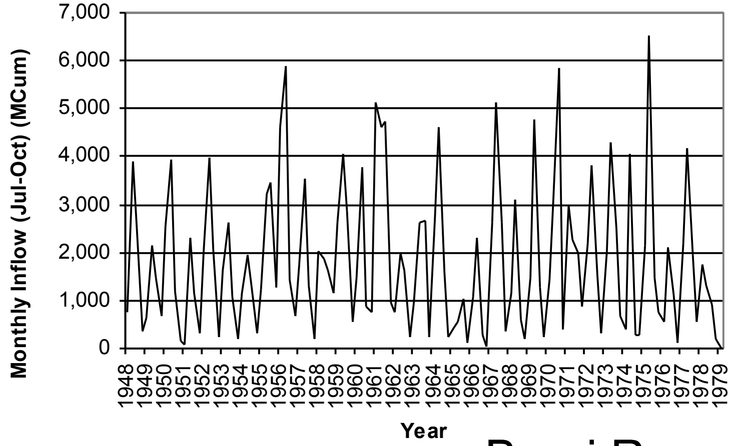
Statistics of Annual Inflows

S.No	Gauge site	Data used (Period)	Duration (years)	Annual Average (MCum)	Maximum (MCum)	Minimum (MCum)	Standard deviation (MCum)	Coeff. of variation (%)
1	Matiyari*	1949-1979	31	80.43	168.17	23.43	32.25	40.10
2	Bargi*	1948-1978	31	7,392.65	15,430.00	2,152.00	2,957.96	40.01
3	Barna#	1977-2002	26	500.12	1,208.03	67.14	269.11	53.81
4	Tawa	1948-1993	46	3,768.41	9,444.75	1,787.68	1,721.83	45.69
5	Kolar	1991-2000	10	219.09	470.17	78.34	119.71	54.64
6	Sukta	1989-2003	15	71.03	98.81	32.95	22.82	32.13
7	Indira Sagar+	1988-2002	15	10,594.85	23,737.80	4,036.20	5,854.42	55.26
8	Maheshwar	1950-1977	28	27,822.55	56,125.10	11,298.90	9,454.72	33.98
9	Jobat#	1961-1980	20	299.49	807.10	39.20	203.16	67.84
10	Sardar sarovar	Flows at Garudeshwar will be used	-	-	-	-	-	-

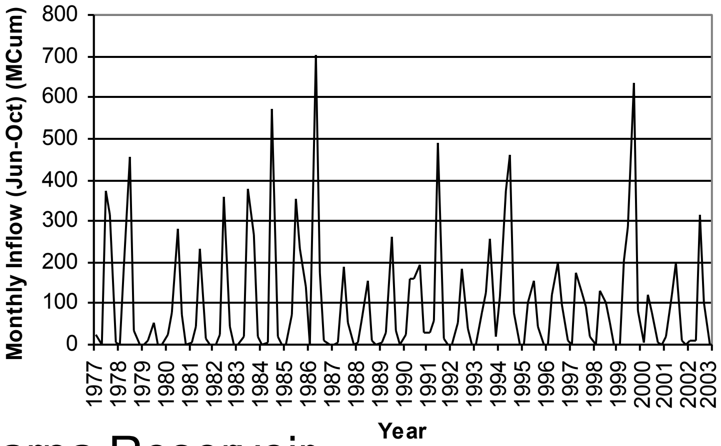
Monthly Inflows



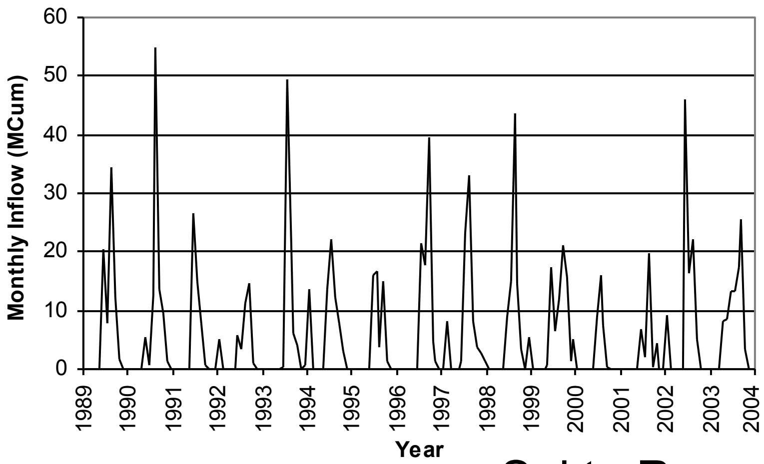
Matiyari Reservoir



Bargi Reservoir



Barna Reservoir



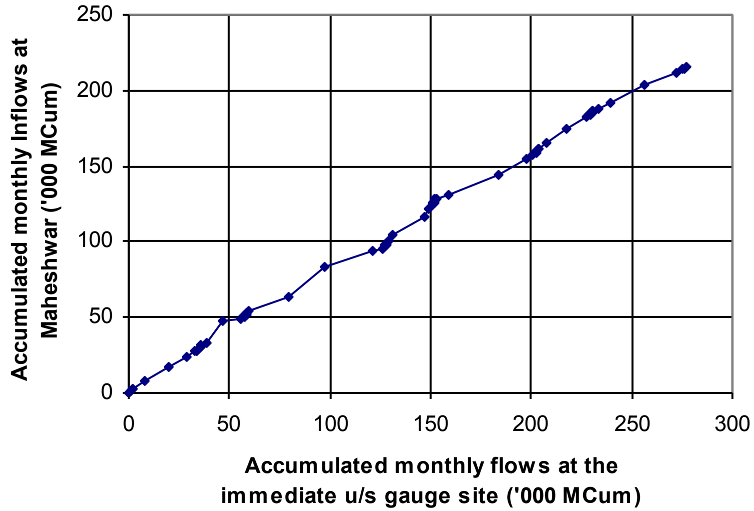
Sukta Reservoir

Data Consistency Checks

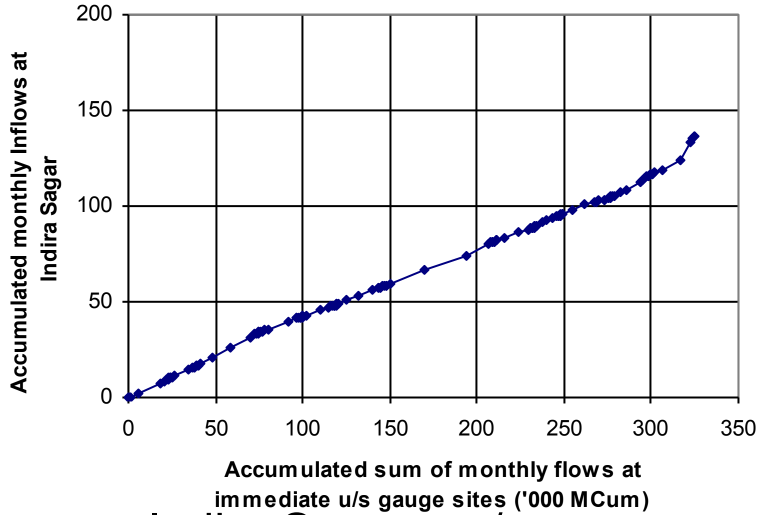
Consistency of Reservoir Inflow Data :

- Similar to the gauge discharge data, consistency checks are performed for the reservoir inflow data.
- Double mass curves for inflows are prepared
- The double mass curves do not indicate any obvious inconsistency in the data

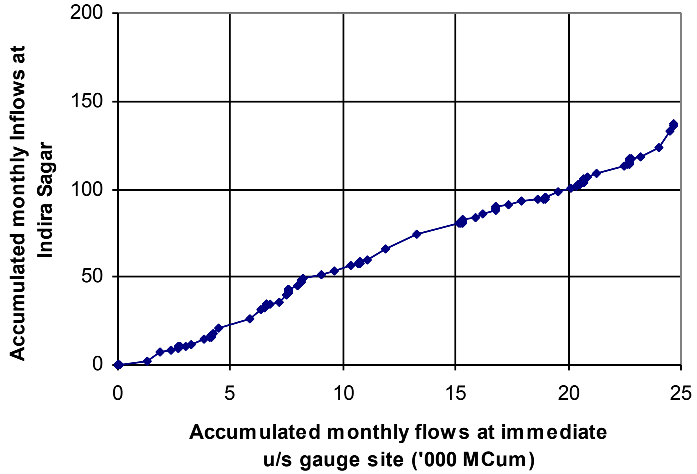
Double mass curves for inflows



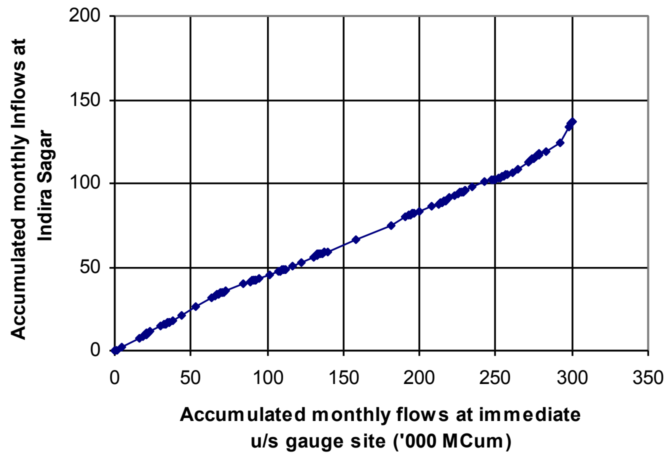
Maheshwar – u/s gauges



Indira Sagar – u/s gauges



Indira Sagar – Ginnore



Indira Sagar – Handia