INDIAN INSTITUTE OF SCIENCE

STOCHASTIC HYDROLOGY

Lecture -28

Course Instructor : Prof. P. P. MUJUMDAR

Department of Civil Engg., IISc.

Summary of the previous lecture

- Probability paper construction
  - Graphical construction
    - Normal distribution

- Plotting position
  - Weibull's formula

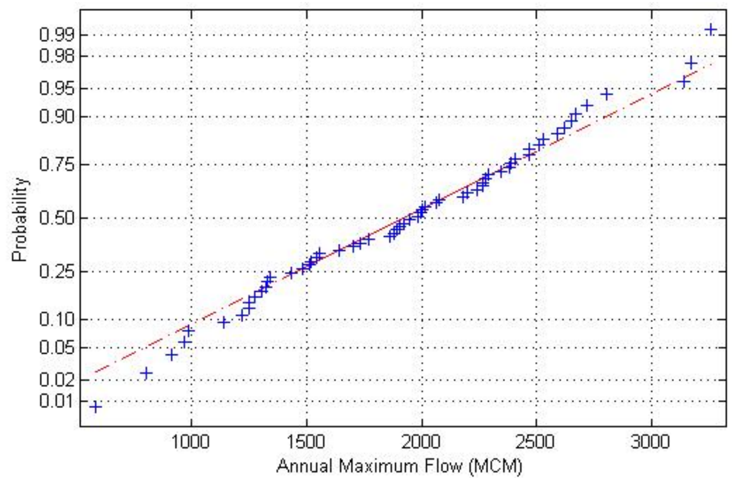
$$p\left(X \geq x_m\right) = \frac{m}{n+1}$$

GOODNESS OF FIT -
PROBABILITY DISTRIBUTIONS

Tests for Goodness of Fit

- Two ways of testing whether or not a particular distribution adequately fits a set of observations.
  - using probability paper.
  - compare the observed relative frequency with theoretical relative frequency.

$p_i = \frac{n_i}{n}$



Tests for Goodness of Fit

- Two tests that compare observed and expected relative frequencies.

  - Chi-Square test
  - Kolmogorov – Smirnov test

Tests for Goodness of Fit

Chi-Square Goodness of fit test:

- One of the most commonly used tests.
- The test makes a comparison between the actual number of observations and the expected number of observations.

Steps to be followed:

- Select the distribution whose adequacy is to be tested, and estimate its parameters from the sample data.

Tests for Goodness of Fit

- Divide the observed data into k class intervals.
- Count N_i , the number of observations falling in each class interval, i.
- Determine the probability with which the RV lies in each of the class interval using the selected distribution.
- Calculate E_i, the expected number of observations in the class interval i, by multiplying the probability with the number of sample values (n).

Tests for Goodness of Fit

- The Chi-square test statistic is calculated by

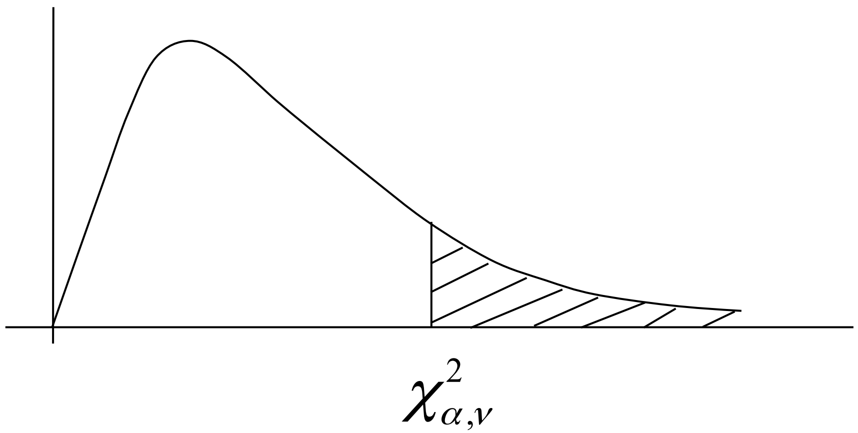
$$\chi^2_{data} = \sum_{i=1}^{k} \frac{\left(N_i - E_i\right)^2}{E_i}$$

- This statistic follows Chi-square distribution with number of degrees of freedom equal to k-p-1, where p is no. of parameters of the distribution.
- The hypothesis that the data follows a specified distribution is accepted if

$$\chi^2_{data} < \chi^2_{1-\alpha,k-p-1}$$

Tests for Goodness of Fit

- α is the significance level.
- Usually the tests are carried out at 10% or 5% significance level.
- The critical value is obtained from the Chi-Square distribution table.



$$\nu = k - p - 1$$

Tests for Goodness of Fit

ν \ α	0.995	0.95	0.9	0.1	0.05	0.025	0.01	0.005
1	3.9E-05	0.004	0.016	2.71	3.84	5.02	6.63	7.88
2	0.010	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.21	0.71	1.06	7.78	9.49	11.14	13.28	14.86
5	0.41	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	0.68	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.99	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	3.94	4.87	15.99	18.31	20.48	23.21	25.19
20	7.43	10.85	12.44	28.41	31.41	34.17	37.57	40.00
30	13.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67

Tests for Goodness of Fit

- The number of class intervals should be at least 5.
- If the sample size n is large, the number of class intervals may be approximately fixed by

$$k = 10 + 1.33\ln(n)$$

- Preferred to choose non-uniform class intervals with at least 5 observations in each class interval.
- If more than one distribution passes the test, then the distribution which gives the least value of χ^2 is selected.

Ref: Stochastic Hydrology by P.Jayarami Reddy (1997), Laxmi Publications, New Delhi.

Example – 1

Consider the annual maximum discharge x (in cumec) of a river for 40 years, Check whether the data follows a normal distribution using Chi-Square goodness of fit test at 10% significance level.

S.No.	x (x10^6)	S.No.	x (x10^6)	S.No.	x (x10^6)	S.No.	x (x10^6)
1	590	11	501	21	863	31	658
2	618	12	360	22	672	32	646
3	739	13	535	23	1054	33	1000
4	763	14	644	24	858	34	653
5	733	15	700	25	285	35	626
6	318	16	607	26	643	36	543
7	791	17	686	27	479	37	650
8	582	18	411	28	613	38	900
9	529	19	556	29	584	39	765
10	895	20	512	30	900	40	831

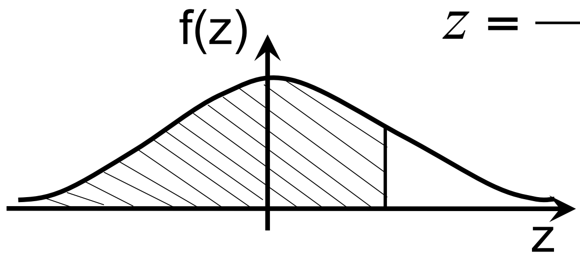
Example – 1 (Contd.)

Mean, $\bar{x}$ = 657.42

Standard deviation, s = 173.59

Data is divided into 8 class intervals

<400
400-500
500-620
620-740
740-850
850-960
960-1000
>1000



$$z = \frac{x - \bar{x}}{s}$$

Example – 1 (Contd.)

observed data — Expected

Class Interval	N_i	(z_i) of upper limit	$F(z_i)$	$p_i=F(z_i) - F(z_i-1)$	$E_i=np_i$	$\frac{(N_i - E_i)^2}{E_i}$
<400	3	$\frac{400-657.42}{173.59} = -1.483$	0.069	0.069	2.76	0.021
400-500	2	-0.907	0.182	0.113	4.52	1.405
500-620	12	-0.216	0.415	0.233	9.32	0.771
620-740	12	0.476	0.683	0.268	10.72	0.153
740-850	4	1.109	0.866	0.183	7.32	1.506
850-960	5	1.743	0.959	0.093	3.72	0.440
960-1000	1	1.974	0.976	0.017	0.68	0.151
>1000	1		1	0.024	0.96	0.002
Total	n =40			1	40	4.448

Example – 1 (Contd.)

$$\chi^2_{data} = \sum_{i=1}^{k} \frac{\left(N_i - E_i\right)^2}{E_i} = 4.448$$

No. of class intervals, k = 8

No. of parameters, p = 2

Therefore ν = k – p – 1

= 8 – 2 – 1

= 5

Significance level α = 10% = 0.1

Example – 1 (Contd.)

From the Chi-square distribution table,

$$\chi^2_{0.1,5} = 9.24$$

ν \ α	0.9	0.1	0.05
3	0.584	6.25	7.81
4	1.06	7.78	9.49
5	1.61	**9.24**	11.07
6	2.20	10.64	12.59

$$\chi^2_{data} < \chi^2_{0.1,5}$$

The hypothesis that the normal distribution fits the data can be accepted at 10% significance level.

Tests for Goodness of Fit

Kolmogorov – Smirnov Goodness of fit test:

- Alternative to the Chi-square test.
- The test is conducted as follows
  - The data is arranged in descending order of magnitude.
  - The cumulative probability $P(x_i)$ for each of the observations is calculated using the Weibull's formula.
  - The theoretical cumulative probability $F(x_i)$ for each of the observation is obtained using the assumed distribution.

Tests for Goodness of Fit

- The absolute difference of $P(x_i)$ and $F(x_i)$ is calculated.
- The Kolmogorov-Smirnov test statistic Δ is the maximum of this absolute difference.

$$\Delta = maximum \left| P\left(x_i\right) - F\left(x_i\right) \right|$$

- The critical value of Kolmogorov-Smirnov statistic Δ_o is obtained from the table for a given significance level α.
- If $\Delta < \Delta_o$, accept the hypothesis that the assumed distribution is a good fit at significance level α.

Tests for Goodness of Fit

Table for Kolmogorov-Smirnov statistic Δ_o :

Size of sample	Significance Level α				
	0.2	0.15	0.1	0.05	0.01
5	0.45	0.47	0.51	0.56	0.67
10	0.32	0.34	0.37	0.41	0.49
20	0.23	0.25	0.26	0.29	0.36
30	0.19	0.20	0.22	0.24	0.29
40	0.17	0.18	0.19	0.21	0.25
50	0.15	0.16	0.17	0.19	0.23
Asymptotic formula (n > 50)	$\frac{1.07}{\sqrt{n}}$	$\frac{1.14}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Tests for Goodness of Fit

- The advantage of Kolmogorov-Smirnov test over the Chi-square test is that it does not lump the data and compare only the discrete categories.
- It is easier to compute Δ than χ^2.
- This test is more convenient to adopt when the sample size is small.

Example – 2

Consider the annual maximum discharge x (in cumec) of a river for 20 years, Check whether the data follows a normal distribution using Kolmogorov-Smirnov goodness of fit test at 10% significance level.

S.No.	x (x10^6)	S.No.	x (x10^6)
1	590	11	501
2	618	12	360
3	739	13	535
4	763	14	644
5	733	15	700
6	318	16	607
7	791	17	686
8	582	18	411
9	529	19	556
10	895	20	831

Example – 2 (Contd.)

Mean, $\bar{x}$ = 619.62

Standard deviation, s = 153.32

Data is arranged in the descending order and a rank (m) is assigned to each data point. The probability is obtained using Weibull's formula

$$p = P\left(X \geq x_m\right) = \frac{m}{n+1}$$

$$z = \frac{x - \bar{x}}{s}$$

S.No. (i)	x ($x10^6$)	Descending order	Rank (m)	$p = \frac{m}{n+1}$	$P(x_i)=1-p$	(z_i)	$F(z_i)$	$\lvert P(x_i) - F(x_i) \rvert$
1	590	895	1	0.048	0.952	1.795	0.964	0.012
2	618	831	2	0.095	0.905	1.381	0.916	0.011
3	739	791	3	0.143	0.857	1.121	0.869	0.012
4	763	763	4	0.190	0.810	0.936	0.825	0.015
5	733	739	5	0.238	0.762	0.781	0.783	0.021
6	318	733	6	0.286	0.714	0.737	0.769	0.055
7	791	700	7	0.333	0.667	0.524	0.700	0.033
8	582	686	8	0.381	0.619	0.432	0.667	0.048
9	529	644	9	0.429	0.571	0.162	0.564	0.007
10	895	618	10	0.476	0.524	-0.011	0.495	0.029
11	501	607	11	0.524	0.476	-0.082	0.468	0.008
12	360	590	12	0.571	0.429	-0.192	0.424	0.005
13	535	582	13	0.619	0.381	-0.242	0.404	0.023
14	644	556	14	0.667	0.333	-0.412	0.340	0.007
15	700	535	15	0.714	0.286	-0.549	0.292	0.006
16	607	529	16	0.762	0.238	-0.589	0.278	0.040
17	686	501	17	0.810	0.190	-0.772	0.220	0.030
18	411	411	18	0.857	0.143	-1.361	0.087	**0.056**
19	556	360	19	0.905	0.095	-1.692	0.045	0.050
20	831	318	20	0.952	0.048	-1.965	0.025	0.023

Example – 2 (Contd.)

Maximum value $\Delta = 0.056$

From the Kolmogorov-Smirnov table,

$$\Delta_0 = 0.26$$

N \ α	0.9	0.1	0.05
10	0.34	0.37	0.41
20	0.25	**0.26**	0.29
30	0.20	0.22	0.24

Since $\Delta < \Delta_o$,

The hypothesis can be accepted that the normal distribution fits the data at 10% significance level.

Goodness of Fit of Data

General comments on Goodness of fit tests:

- Many hydrologists discourage the use of these tests when testing hydrologic frequency distributions for extreme values because of insensitivity of these tests in the tail regions of the distributions.
- Probability of accepting the hypothesis when it is in fact false is very high especially when small sample are used.

Ref: Statistical methods in Hydrology by C.T.Haan, Iowa state university press

INTENSITY-DURATION-FREQUENCY (IDF) CURVES

IDF Curves

- In many of the hydrologic projects, the first step is the determination of rainfall event
  - E.g., urban drainage system
- Most common way is using the IDF relationship
- An IDF curve gives the expected rainfall intensity of a given duration of storm having desired frequency of occurrence.
- IDF curve is a graph with duration plotted as abscissa, intensity as ordinate and a series of curves, one for each return period
- Standard IDF curves are available for a site.

IDF Curves

- The intensity of rainfall is the rate of precipitation, i.e., depth of precipitation per unit time
- This can be either instantaneous intensity or average intensity over the duration of rainfall
- The average intensity is commonly used.

$$i = \frac{P}{t}$$

where P is the rainfall depth

t is the duration of rainfall

IDF Curves

- The frequency is expressed in terms of return period (T) which is the average length of time between rainfall events that equal or exceed the design magnitude.
- If local rainfall data is available, IDF curves can be developed using frequency analysis.
- For the development of the curves, a minimum of 20 years data is required.

IDF Curves

Procedure for developing IDF curves:

Step 1: Preparation of annual maximum data series

- From the available rainfall data, rainfall series for different durations (e.g.,1H, 2H, 6H, 12H and 24H) are developed.
- For each selected duration, the annual maximum rainfall depths are calculated.

IDF Curves

Step 2: Fitting the probability distribution

- A suitable probability distribution is fitted to the each selected duration data series .
- Generally used probability distributions are
  - Gumbel's Extreme Value distribution
  - Gamma distribution (two parameter)
  - Log Pearson Type III distribution
  - Normal distribution
  - Log-normal distribution (two parameter)

IDF Curves

Statistical distributions and their functions:

Distribution	PDF
Gumbel's EVT-I	$f(x) = \exp\left\{-(x-\beta)/\alpha - \exp\left[-(x-\beta)/\alpha\right]\right\}/\alpha$ $\quad -\infty \le x \le \infty$
Gamma	$f(x) = \dfrac{\lambda^n x^{\eta-1} e^{-\lambda x}}{\Gamma(\eta)}$ $\quad x, \lambda, \eta > 0$
Log Pearson Type-III	$f(x) = \dfrac{\lambda^\beta (y-\varepsilon)^{\beta-1} e^{-\lambda(y-\varepsilon)}}{x\Gamma(\beta)}$ $\quad \log x \ge \varepsilon$
Normal	$f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^2\right\}$ $\quad -\infty < x < \infty$
Log-Normal	$f(x) = \dfrac{1}{\sqrt{2\pi} x \sigma_x} \mathrm{e}^{-(\ln x - \mu_x)^2 / 2\sigma_x^2}$ $\quad 0 < x < \infty$

IDF Curves

- Commonly used distribution is Gumbel's Extreme Value Type-I distribution
- Parameters of the distribution is calculated for the selected distribution using method of maximum likelihood method.
- The Kolmogorov-Smirnov or the Chi-Square goodness of fit test is used to evaluate the accuracy of the fitting of a distribution

IDF Curves

Step 3: Determining the rainfall depths

- Two ways of determining the rainfall depths
  - Using frequency factors or
  - Using the CDF of the distribution (by inversing the CDF).

IDF Curves

Using frequency factors:

- The precipitation depths is calculated for a given return period as

$$x_T = \bar{x} + K_T s$$

where

$\bar{x}$ is mean of the data,

s is the standard deviation and

K_T is the frequency factor

IDF Curves

Using CDF of a distribution:

- Using the parameters of the distribution which are caluclated in the previous step, a probability model is formulated.
- The formulated probability model is inversed and x_T value for a given return period is calculated by

$$P\left(X \geq x_T\right) = \frac{1}{T}; \qquad 1 - P\left(X < x_T\right) = \frac{1}{T}$$

$$1 - F\left(x_T\right) = \frac{1}{T}; \qquad F\left(x_T\right) = 1 - \frac{1}{T} = \frac{T-1}{T}$$

IDF Curves

- The precipitation depths calculated from the annual exceedence series is adjusted to match the depths derived from annual maximum series by multiplying a factor

Return Period	Factor
2	0.88
5	0.96
10	0.99

- No adjustment of the estimates is required for longer return periods (> 10 year return period)

Example – 3

Bangalore rainfall data is considered to demonstrate the construction of IDF curves.

The duration of the rainfall considered is : 1H, 2H, 6H, 12H and 24 H.

The return period considered is: 2Y, 5Y, 10Y, 50Y and 100Y.

The hourly rainfall data for 33years is collected at a rain gauge station. The data is as shown

Example – 3 (Contd.)

The part data is shown below.

Year	Month	Day	Hr1	Hr2	Hr3	Hr4	Hr5
1969	8	22	3.7	0	0.1	0	0
1969	8	23	0	1.2	2.8	5.3	4.5
1969	8	24	0.6	0.2	0	0	0
1969	8	25	5.2	3.3	4.5	1.6	0
1969	8	26	0	0	0	0	0
1969	8	27	0	0	0	0	0
1969	8	28	1.2	12.8	3.2	4	6.1
1969	8	29	0	0	0	0	0
1969	8	30	0.1	0.1	0	0	0

Example – 3 (Contd.)

- From the hourly data, the rainfall for different durations are obtained.
- The rainfall data is converted to intensity by dividing the rainfall with duration.
- The mean and standard deviation of the data for all the selected durations is calculated.
- The frequency factor is calculated for all the selected return periods, based on the distribution.
- The design rainfall intensity is obtained by

$$x_T = \bar{x} + K_T s$$

Example – 3 (Contd.)

Rainfall in mm for different durations.

Year	1H	2H	6H	12H	24H	Year	1H	2H	6H	12H	24H
1969	44.5	61.6	104.1	112.3	115.9	1986	65.2	73.7	97.9	103.9	104.2
1970	37	48.2	62.5	69.6	92.7	1987	47	55.9	64.8	65.6	67.5
1971	41	52.9	81.4	86.9	98.7	1988	148.8	210.8	377.6	432.8	448.7
1972	30	40	53.9	57.8	65.2	1989	41.7	47	51.7	53.7	78.1
1973	40.5	53.9	55.5	72.4	89.8	1990	40.9	71.9	79.7	81.5	81.6
1974	52.4	62.4	83.2	93.4	152.5	1991	41.1	49.3	63.6	93.2	147
1975	59.6	94	95.1	95.1	95.3	1992	31.4	56.4	76	81.6	83.1
1976	22.1	42.9	61.6	64.5	71.7	1993	34.3	36.7	52.8	68.8	70.5
1977	42.2	44.5	47.5	60	61.9	1994	23.2	38.7	41.9	43.4	50.8
1978	35.5	36.8	52.1	54.2	57.5	1995	44.2	62.2	72	72.2	72.4
1979	59.5	117	132.5	135.6	135.6	1996	57	74.8	85.8	86.5	90.4
1980	48.2	57	82	86.8	89.1	1997	50	71.1	145.9	182.3	191.3
1981	41.7	58.6	64.5	65.1	68.5	1998	72.1	94.6	111.9	120.5	120.5
1982	37.3	43.8	50.5	76.2	77.2	1999	59.3	62.9	82.3	90.7	90.9
1983	37	60.4	70.5	72	75.2	2000	62.3	78.3	84.3	84.3	97.2
1984	60.2	74.1	76.6	121.9	122.4	2001	46.8	70	95.9	95.9	100.8
						2003	53.2	86.5	106.1	106.2	106.8

Example – 3 (Contd.)

The rainfall intensity (mm/hr) for different durations.

Year	1H	2H	6H	12H	24H	Year	1H	2H	6H	12H	24H
1969	44.50	30.80	17.35	9.36	4.83	1986	65.20	36.85	16.32	8.66	4.34
1970	37.00	24.10	10.42	5.80	3.86	1987	47.00	27.95	10.80	5.47	2.81
1971	41.00	26.45	13.57	7.24	4.11	1988	148.80	105.40	62.93	36.07	18.70
1972	30.00	20.00	8.98	4.82	2.72	1989	41.70	23.50	8.62	4.48	3.25
1973	40.50	26.95	9.25	6.03	3.74	1990	40.90	35.95	13.28	6.79	3.40
1974	52.40	31.20	13.87	7.78	6.35	1991	41.10	24.65	10.60	7.77	6.13
1975	59.60	47.00	15.85	7.93	3.97	1992	31.40	28.20	12.67	6.80	3.46
1976	22.10	21.45	10.27	5.38	2.99	1993	34.30	18.35	8.80	5.73	2.94
1977	42.20	22.25	7.92	5.00	2.58	1994	23.20	19.35	6.98	3.62	2.12
1978	35.50	18.40	8.68	4.52	2.40	1995	44.20	31.10	12.00	6.02	3.02
1979	59.50	58.50	22.08	11.30	5.65	1996	57.00	37.40	14.30	7.21	3.77
1980	48.20	28.50	13.67	7.23	3.71	1997	50.00	35.55	24.32	15.19	7.97
1981	41.70	29.30	10.75	5.43	2.85	1998	72.10	47.30	18.65	10.04	5.02
1982	37.30	21.90	8.42	6.35	3.22	1999	59.30	31.45	13.72	7.56	3.79
1983	37.00	30.20	11.75	6.00	3.13	2000	62.30	39.15	14.05	7.03	4.05
1984	60.20	37.05	12.77	10.16	5.10	2001	46.80	35.00	15.98	7.99	4.20
						2003	53.20	43.25	17.68	8.85	4.45

Example – 3 (Contd.)

The mean and standard deviation for the data for different durations is calculated.

Duration	1H	2H	6H	12H	24H
Mean	48.70	33.17	14.46	8.05	4.38
Std. Dev.	21.53	15.9	9.59	5.52	2.86

Gumbell's distribution is considered

Example – 3 (Contd.)

K_T values are calculated for different return periods using

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

T (years)	2	5	10	50	100
K_T	-0.164	0.719	1.305	2.592	3.137

Example – 3 (Contd.)

The rainfall intensities are calculated using

$$x_T = \bar{x} + K_T s$$

For example,

For duration of 2 hour, and 10 year return period,

Mean $\bar{x}$ = 33.17 mm/hr,

Standard deviation s = 15.9 mm/hr

Frequency factor K_T = 1.035

Example – 3 (Contd.)

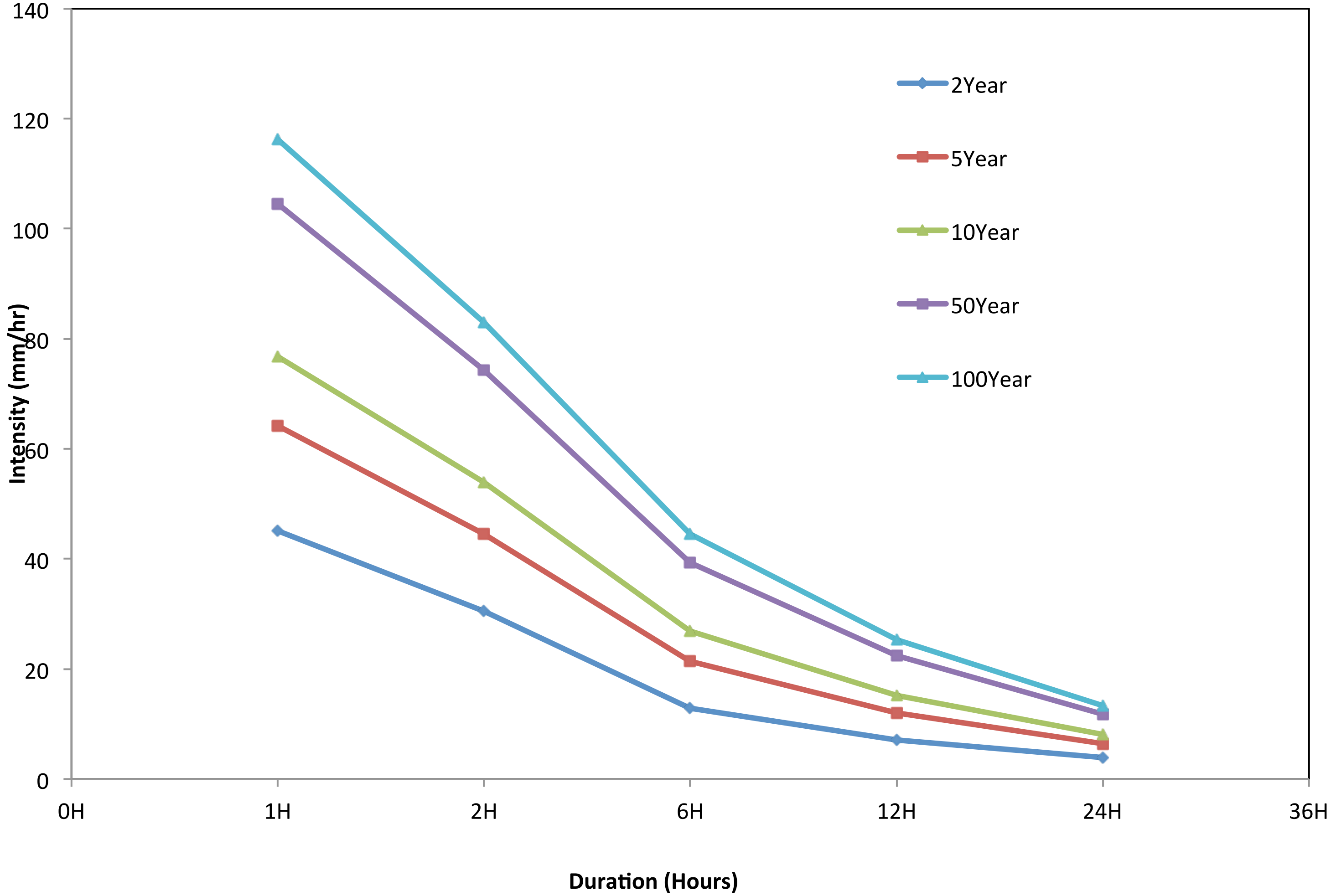
x_T = 33.17 + 1.035 * 15.9

= 53.9 mm/hr.

The values or other durations are tabulated.

Duration (hours)	Return Period T (Years)				
	2	5	10	50	100
1H	45.17	64.19	76.79	104.51	116.23
2H	30.55	44.60	53.90	74.36	83.02
6H	12.89	21.36	26.97	39.31	44.53
12H	7.14	12.02	15.25	22.36	25.37
24H	3.91	6.44	8.11	11.79	13.35

Example – 3 (Contd.)



IDF Curves

Equations for IDF curves:

- IDF curves can also be expressed as equations to avoid reading the design rainfall intensity from a graph

$$i = \frac{c}{t^e + f}$$

where

i is the design rainfall intensity,

t is the duration and

c, e and f are coefficients varying with location and return period.

Ref: Applied Hydrology by V.T.Chow, D.R.Maidment, L.W.Mays, McGraw-Hill 1998

IDF Curves

Equations for IDF curves:

- IDF curves can also be expressed as equations to avoid reading the design rainfall intensity from a graph

$$i = \frac{c}{t^e + f}$$

where

i is the design rainfall intensity,

t is the duration and

c, e and f are coefficients varying with location and return period.

Ref: Applied Hydrology by V.T.Chow, D.R.Maidment, L.W.Mays, McGraw-Hill 1998

IDF Curves

IDF Equation for Indian region:

- Rambabu et. al. (1979) developed equation after analyzing rainfall characteristics for 42 stations.

$$i = \frac{KT^a}{(t+b)^n}$$

where

i is the design rainfall intensity in cm/hr,

T is return period in years

t is the storm duration in hours and

K, a, b and n are coefficients varying with location.

Ref: Ram Babu, Tejwani, K. K., Agrawal, M. C. & Bhusan, L. S. (1979) - Rainfall intensityduration-return period equations & nomographs of India, CSWCRTI, ICAR, Dehradun, India

IDF Curves

Coefficients for few locations is given below

Location	K	a	b	n
Agra	4.911	0.167	0.25	0.629
New Delhi	5.208	0.157	0.5	1.107
Nagpur	11.45	0.156	1.25	1.032
Bhuj	3.823	0.192	0.25	0.990
Gauhati	7.206	0.156	0.75	0.940
Bangalore	6.275	0.126	0.5	1.128
Hyderabad	5.25	0.135	0.5	1.029
Chennai	6.126	0.166	0.5	0.803

Ref: Ram Babu, Tejwani, K. K., Agrawal, M. C. & Bhusan, L. S. (1979) - Rainfall intensityduration-return period equations & nomographs of India, CSWCRTI, ICAR, Dehradun, India

IDF Curves

IDF Equation for Indian region:

- Kothyari and Garde (1992) developed a general relationship on IDF after analyzing 80 rain gauge stations.
- Assumption that general properties of convective cells that are associated with short - period rainfalls are similar in different hydrologic regions is made while obtaining the formula.
- The developed equation results with less than ±30% error.

Ref: Kothyari, U.C., and Garde, R. J. (1992), - Rainfall intensity - duration-frequency formula for India, Journal of Hydraulics Engineering, ASCE, 118(2)

IDF Curves

$$i_t^T = C\frac{T^{0.20}}{t^{0.71}}\left(R_{24}^2\right)^{0.33}$$

where

i_t^T is the rainfall intensity in mm/hr for T year return period and t hour duration,

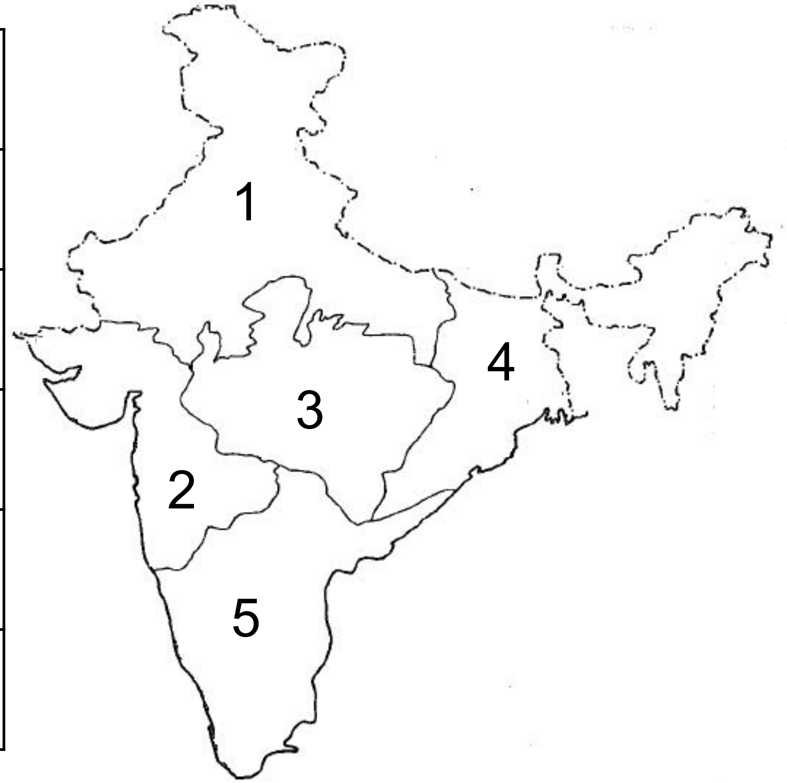
C is constant and

R_{24}^2 is rainfall for 2-year return period and 24 hour duration in mm.

Ref: Kothyari, U.C., and Garde, R. J. (1992), - Rainfall intensity - duration-frequency formula for India, Journal of Hydraulics Engineering, ASCE, 118(2)

IDF Curves

Zone	Location	C
1	Northern India	8.0
2	Western India	8.3
3	Central India	7.7
4	Eastern India	9.1
5	Southern India	7.1



Ref: Kothyari, U.C., and Garde, R. J. (1992), - Rainfall intensity - duration-frequency formula for India, Journal of Hydraulics Engineering, ASCE, 118(2)

Example – 2

Obtain the design rainfall intensity for 10 year return period with 24 hour duration for Bangalore.

Also obtain the design rainfall intensity for 100 year return period and compare with 10 year return period

Solution:

Formula for design rainfall intensity is

$$i = \frac{KT^a}{(t+b)^n}$$

Example – 2 (Contd.)

For Bangalore, the constants are as follows

K = 6.275

a = 0.126

b = 0.5

n = 1.128

For T = 10 Year and t = 24 hour,

$$i = \frac{6.275 \times 10^{0.126}}{(24 + 0.5)^{1.128}} = 0.227$$

Example – 2 (Contd.)

For T = 100 Year and t = 24 hour,

$$i = \frac{6.275 \times 100^{0.126}}{\left(24 + 0.5\right)^{1.128}} = 0.304$$

IDF Curves

Design precipitation Hyetographs from IDF relationships:

- One method is discussed
  - Alternating block method.

IDF Curves

Alternating block method :

- Simple way of developing a design hydrograph from an IDF curve.
- Design hyetograph developed by this method specifies the precipitation depth occurring in n successive time intervals of duration Δt over a total duration T_d.
- The design return period is selected and the intensity is read from the IDF curve for each of the durations

IDF Curves

- The corresponding precipitation depth found as the product of intensity and duration.
- By taking differences between successive precipitation depth values, the amount of precipitation to be added for each additional unit of time Δt is found.
- The increments are rearranged into a time sequence with maximum intensity occurring at the center of the duration and the remaining blocks arranged in descending order alternatively to the right and left of the central block to form the design hyetograph.

Example – 3

Obtain the design precipitation hyetograph for a 2-hour storm in 10 minute increments in Bangalore with a 10 year return period.

Solution:

The 10 year return period design rainfall intensity for a given duration is calculated using IDF formula by Rambabu et. al. (1979)

$$i = \frac{KT^a}{\left(t + b\right)^n}$$

Example – 3 (Contd.)

For Bangalore, the constants are

K = 6.275

a = 0.126

b = 0.5

n = 1.128

For T = 10 Year and duration, t = 10 min = 0.167 hr,

$$i = \frac{6.275 \times 10^{0.126}}{(0.167 + 0.5)^{1.128}} = 13.251$$

Example – 3 (Contd.)

- Similarly the values for other durations at interval of 10 minutes are calculated.
- The precipitation depth is obtained by multiplying the intensity with duration.

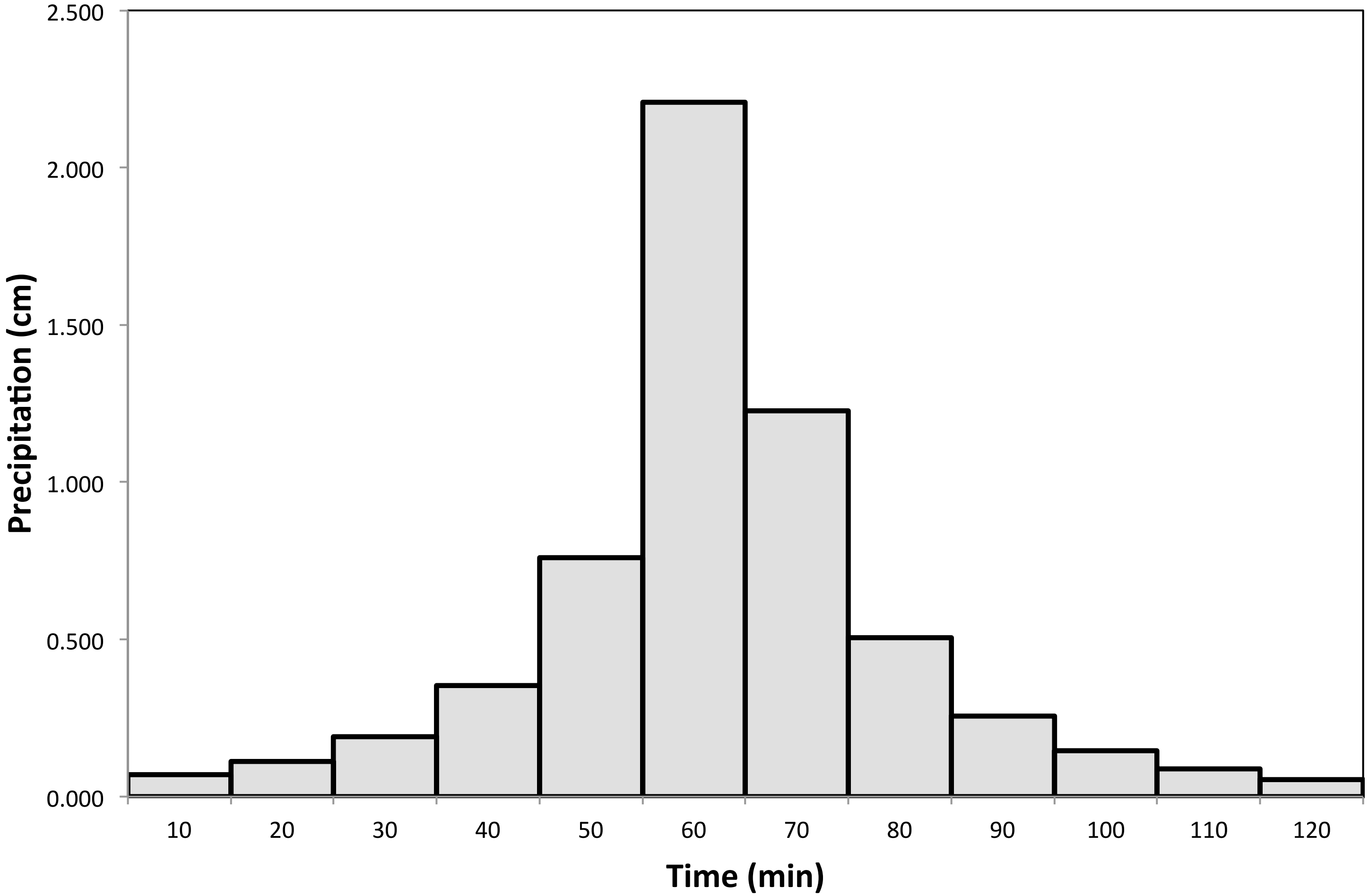
Precipitation = 13.251 * 0.167 = 2.208 cm

- The 10 minute precipitation depth is 2.208 cm compared with 3.434 cm for 20 minute duration, hence the most intense of 20 minutes of the design storm, 2.208 cm will fall in 10 minutes, the remaining 1.226 (= 3.434 – 2.208) cm will fall in the remaining 10 minutes.
- Similarly the other values are calculated and tabulated

Example – 3 (Contd.)

Duration (min)	Intensity (cm/hr)	Cumulative depth (cm)	Incremental depth (cm)	Time (min)	Precipitation (cm)
10	13.251	2.208	2.208	0 - 10	0.069
20	10.302	3.434	1.226	10 - 20	0.112
30	8.387	4.194	0.760	20 - 30	0.191
40	7.049	4.699	0.505	30 - 40	0.353
50	6.063	5.052	0.353	40 - 50	0.760
60	5.309	5.309	0.256	50 - 60	2.208
70	4.714	5.499	0.191	60 - 70	1.226
80	4.233	5.644	0.145	70 - 80	0.505
90	3.838	5.756	0.112	80 - 90	0.256
100	3.506	5.844	0.087	90 - 100	0.145
110	3.225	5.913	0.069	100 - 110	0.087
120	2.984	5.967	0.055	110 - 120	0.055

Example – 3 (Contd.)



ESTIMATED LIMITING VALUES

Estimated Limiting Values

- Estimated Limiting Values (ELV's) are developed as criteria for various types of hydraulic design.

- Commonly employed ELV's for design are
  - Probable Maximum Precipitation (PMP)
  - Probable Maximum Storm (PMS)
  - Probable Maximum Flood (PMF)

Probable Maximum Precipitation (PMP)

- PMP is the estimated limiting value of precipitation.
- PMP is defined as the estimated greatest depth of the precipitation for a given duration that is possible physically and reasonably characteristic over a particular geographic region at a certain time of year.
- PMP cannot be exactly estimated as its probability of occurrence is not known.
- PMP is useful in operational applications (e.g., design of large dams).
- Any allowance should not be made in the estimation of PMP for long term climate change.

Probable Maximum Precipitation (PMP)

- Various methods for determining PMP
- Application of storm models:
- Maximization of actual storms:
- Generalized PMP charts:

Probable Maximum Storm (PMS)

- PMS values are given as maximum accumulated depths for any specified duration.
- PMS involves temporal distribution of rainfall.
-