INDIAN INSTITUTE OF SCIENCE

STOCHASTIC HYDROLOGY

Lecture -27

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Summary of the previous lecture

- Frequency factors
  - Normal distribution
  - Gumbel's Extreme Value Type-I distribution
  - Log Pearson Type-III distribution
- Probability plotting
- Probability paper construction
  - Mathematical construction

$x_T = \bar{x} + K_T S$

$T = \frac{1}{p}$

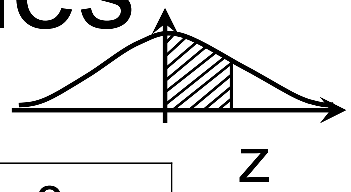
$p = p[X \geq x_T]$

Probability Plotting

Graphical construction:

- Graphical construction is done by transforming the arithmetic scale to probability scale so that a straight line is obtained when cumulative distribution function is plotted.
- The transformation technique is explained with the normal distribution.
- Consider the coordinates from the standardized normal distribution table.

Normal Distribution Tables



(Partial tables shown here)

z	0	2	4	6	8
0	0	0.008	0.016	0.0239	0.0319
0.1	0.0398	0.0478	0.0557	0.0636	0.0714
0.2	0.0793	0.0871	0.0948	0.1026	0.1103
0.3	0.1179	0.1255	0.1331	0.1406	0.148
0.4	0.1554	0.1628	0.17	0.1772	0.1844
0.5	0.1915	0.1985	0.2054	0.2123	0.219
0.6	0.2257	0.2324	0.2389	0.2454	0.2517
0.7	0.258	0.2642	0.2704	0.2764	0.2823
0.8	0.2881	0.2939	0.2995	0.3051	0.3106
0.9	0.3159	0.3212	0.3264	0.3315	0.3365
1	0.3413	0.3461	0.3508	0.3554	0.3599

Probability Plotting

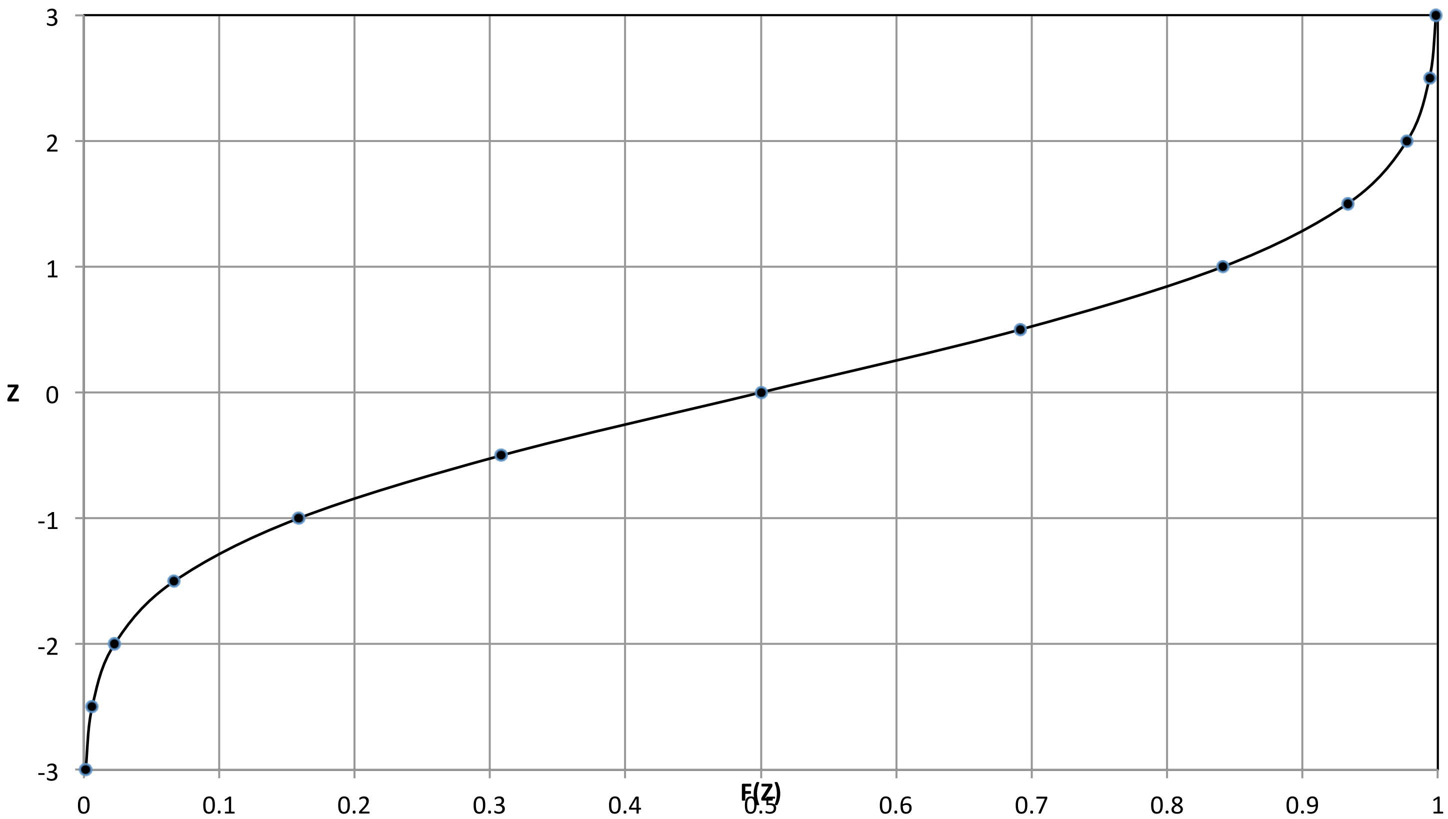
Normal distribution table:

Z	F(z)
-3	0.0013
-2.5	0.0062
-2	0.0227
-1.5	0.0668
-1	0.1587
-0.5	0.3085

Z	F(z)
0	0.5
0.5	0.6915
1	0.8413
1.5	0.9332
2	0.9772
2.5	0.9938
3	0.9987

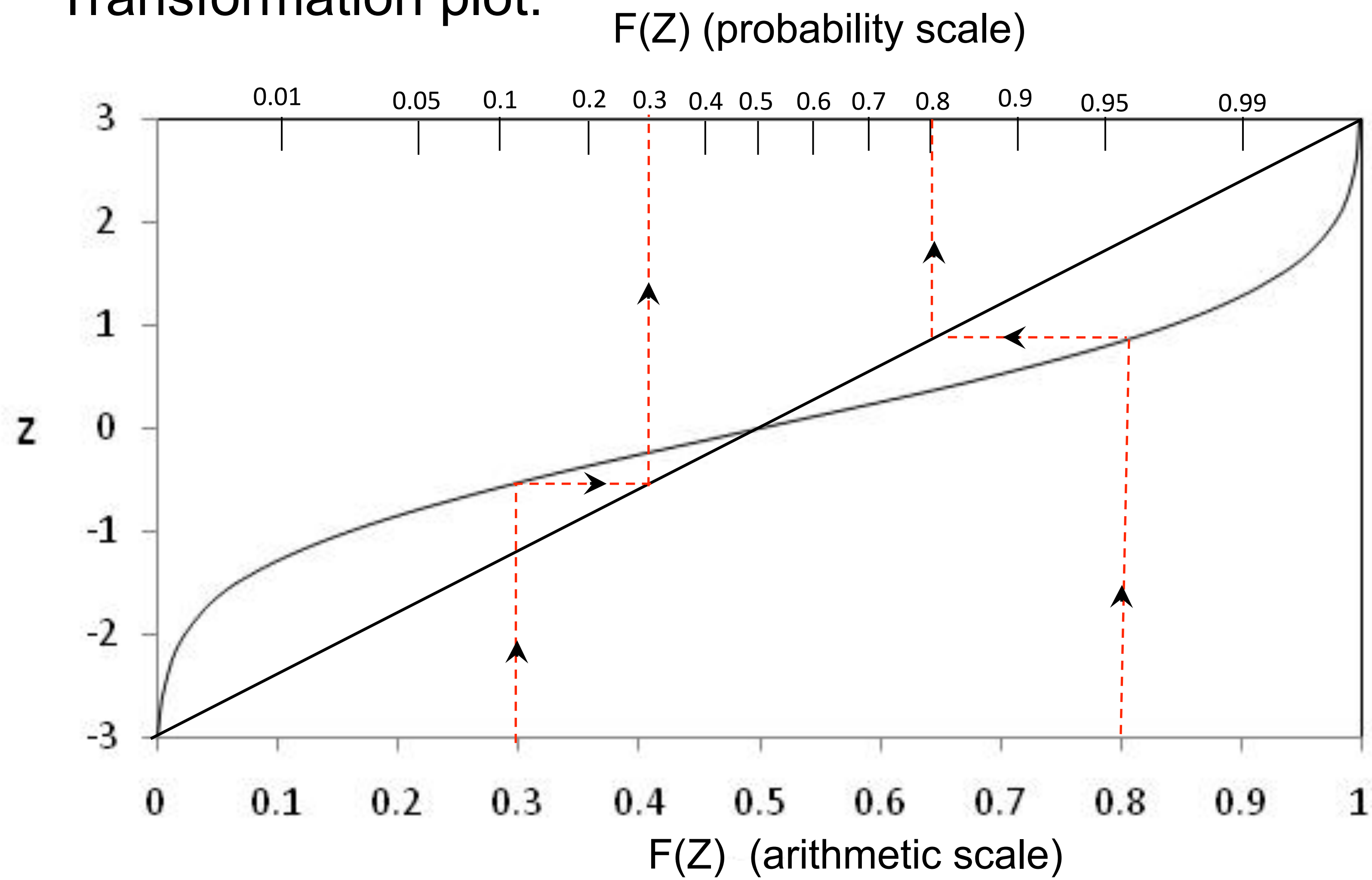
Probability Plotting

Arithmetic scale plot:



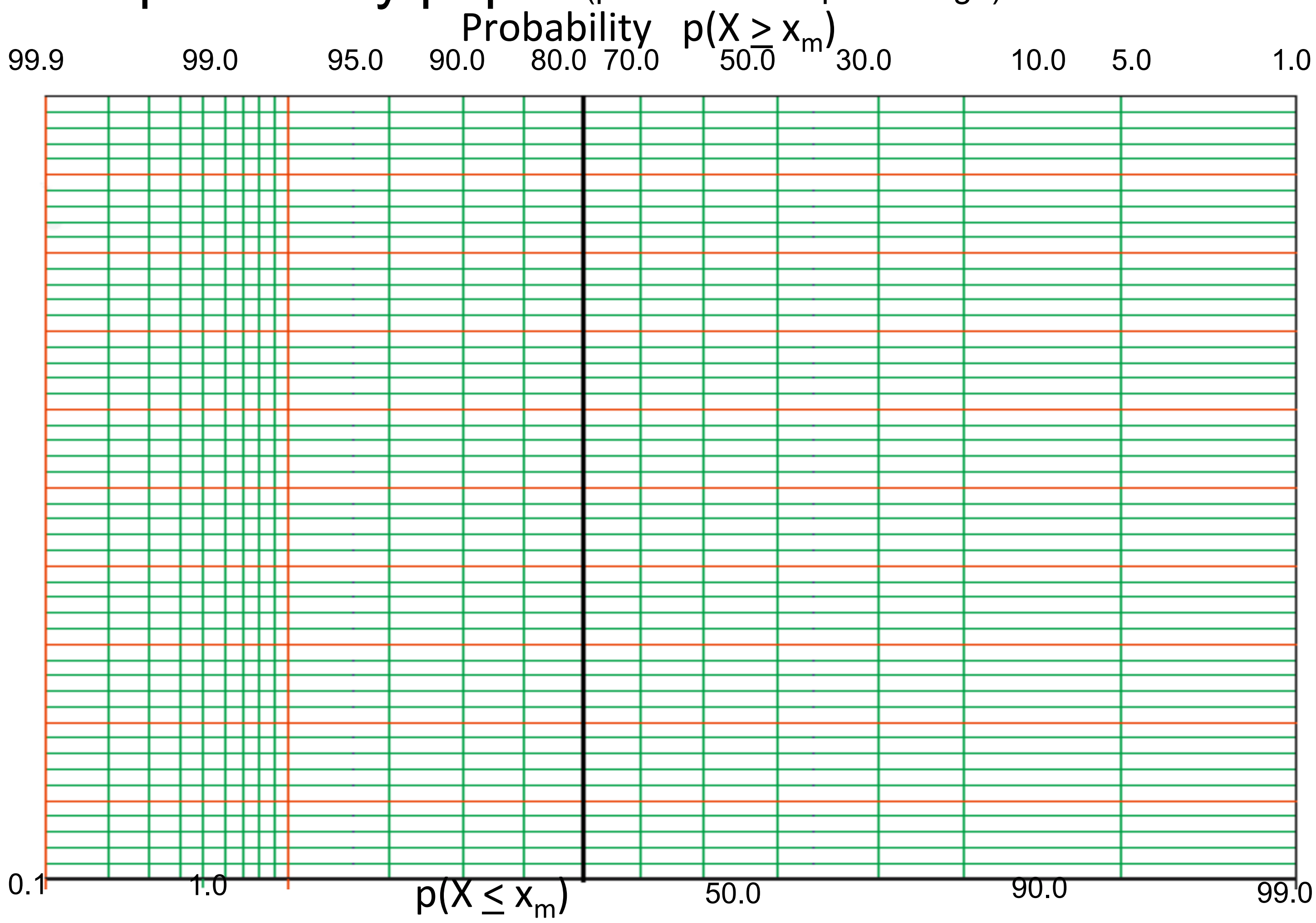
Probability Plotting

Transformation plot:



Probability Plotting

Normal probability paper (probabilities in percentage) :



Redrawn from source: http://www.weibull.com/GPaper/

Probability Plotting

- The purpose is to check if a data set fits the particular distribution.
- The plot can be used for interpolation, extrapolation and comparison purposes.
- The plot can be used for estimating magnitudes with specified return periods.
- Extrapolation must be attempted only when a reasonable fit is assured for the distribution.

PLOTTING POSITION

Plotting Position

- Plotting position is a simple empirical technique
- Relation between the magnitude of an event verses its probability of exceedence.
- Plotting position refers to the probability value assigned to each of the data to be plotted.
- Several empirical methods to determine the plotting positions.

Plotting Position

- Arrange the given series of data in descending order.
- Assign a order number to each of the data (termed as rank of the data).
- Let ‘n’ is the total no. of values to be plotted and ‘m’ is the rank of a value, the exceedence probability (p) of the m^{th} largest value is obtained by various plotting position formulae.
- The return period (T) of the event is calculated by T = 1/p
- Plot magnitude of event verses the exceedence probability (p) or 1 – p or the return period T.

Plotting Position

Formulae for exceedence probability:

California Method:

$$p\left(X \geq x_m\right) = \frac{m}{n}$$

Limitations

– Produces a probability of 100% for m = n

Plotting Position

Modification to California Method:

$$p\left(X \geq x_m\right) = \frac{m-1}{n}$$

Limitations

2 – Formula does not produce 100% probability

1 – If m = 1, probability is zero

Not a limitation

Plotting Position

Hazen's formula:

$$p\left(X \geq x_m\right) = \frac{m - 0.5}{n}$$

Chegodayev's formula:

$$p\left(X \geq x_m\right) = \frac{m - 0.3}{n + 0.4}$$

Plotting Position

Weibull's formula:

– Most commonly used method.

– If 'n' values are distributed uniformly between 0 and 100 percent probability, then there must be n+1 intervals, n–1 between the data points and 2 at the ends.

$$p\left(X \geq x_m\right) = \frac{m}{n+1}$$

– Indicates a return period T one year longer than the period of record for the largest value.

Plotting Position

Most plotting position formulae are represented by:

$$p\left(X \geq x_m\right) = \frac{m-b}{n+1-2b}$$

where b is a parameter

- e.g., b = 0.5 for Hazen's formula, b = 0.5 for Chegodayev's formula, b = 0 for Weibull's formula
- b = 3/8 ~~0.5~~ for Blom's formula
- b = 1/3 ~~0.5~~ for Tukey's formula
- b = 0.44 ~~0.5~~ for Gringorten's formula

Plotting Position

- Cunnane (1978) studied the various available plotting position methods based on unbiasedness and minimum variance criteria.
- If large number of equally sized samples are plotted, the average of the plotted points for each value of m lie on the theoretical distribution line.
- Minimum variance plotting minimizes the variance of the plotted points about the theoretical line.

Ref: Cunnane, C., Unbiased plotting positions – a review, J,Hydrol., Vol. 37, pp.205-222, 1978

Plotting Position

- For normally distributed data, Blom's plotting position formula (b = 3/8) is commonly used.
- For Extreme Value Type I distribution, the Gringorten formula (b = 0.44) is used.

- All the relationships give similar values near the center of the distribution but may vary near the tails considerably.
- Predicting extreme events depend on the tails of the distribution.

Plotting Position

Gumbel (1958) stated the following criteria for plotting position reationships.

1. The plotting position must be such that all the observations can be plotted.
2. The plotting position should lie between the observed frequencies of $(m - 1)/n$ and m/n
3. The return period of a value equal to or greater than the largest observation and the return period of a value equal to or smaller than the smallest observation should converge to n.

Ref: Gumbel, E. J., 1958. Statistics of Extreme Values, Columbia University Press, New York

Plotting Position

4. The observations should be equally spaced on the frequency scale
5. The plotting position should be analytically simple and easy to use.

Ref: Gumbel, E. J., 1958. Statistics of Extreme Values, Columbia University Press, New York

Plotting Position

The Weibull plotting position formula meets all the 5 of the above criteria.

$$p\left(X \geq x_m\right) = \frac{m}{n+1}$$

1.All the observations can be plotted since the plotting positions range from $1/(n+1)$ (which is greater than 0) to $n/(n+1)$ (which is less than 1).

2.The relationship lies between $(m-1)/n$ and m/n for all values of m and n.

Ref: Statistical methods in Hydrology by C.T.Haan, Iowa state university press

Plotting Position

3. The return period of the largest value is $(n+1)/1$, which approaches n as n tends to infinity and the return period of the smallest value is $(n+1)/n$, which approaches 1 as n tends to infinity.
4. The difference between the plotting position of the $(m+1)^{st}$ and m^{th} value is $1/(n+1)$ for all values of m and n
5. The formula is simple and easy to use.

$T = 1/p$

Ref: Statistical methods in Hydrology by C.T.Haan, Iowa state university press

Example – 1

Consider the annual maximum flow of a river (in MCM) for 60years.

1.Perform the probability plotting analysis using Hazen's formula.

2.Compare the plotted data with the normal distribution.

Example – 1 (Contd.)

Year	Q (MCM)	Year	Q (MCM)	Year	Q (MCM)	Year	Q (MCM)
1950	1982	1965	1246	1980	2291	1995	1252
1951	1705	1966	2469	1981	3143	1996	983
1952	2277	1967	3256	1982	2619	1997	1339
1953	1331	1968	1860	1983	2268	1998	2721
1954	915	1969	1945	1984	2064	1999	2653
1955	1557	1970	2078	1985	1877	2000	2407
1956	1430	1971	2243	1986	1303	2001	2591
1957	583	1972	3171	1987	1141	2002	2347
1958	1325	1973	2381	1988	1642	2003	2512
1959	2200	1974	2670	1989	2016	2004	2005
1960	1736	1975	1894	1990	2265	2005	1920
1961	804	1976	1518	1991	2806	2006	1773
1962	2180	1977	1218	1992	2532	2007	1274
1963	1515	1978	966	1993	1996	2008	2466
1964	1903	1979	1484	1994	1540	2009	2387

Example – 1 (Contd.)

- The data is arranged in descending order.
- Rank is assigned to the arranged data.
- The probability is obtained using

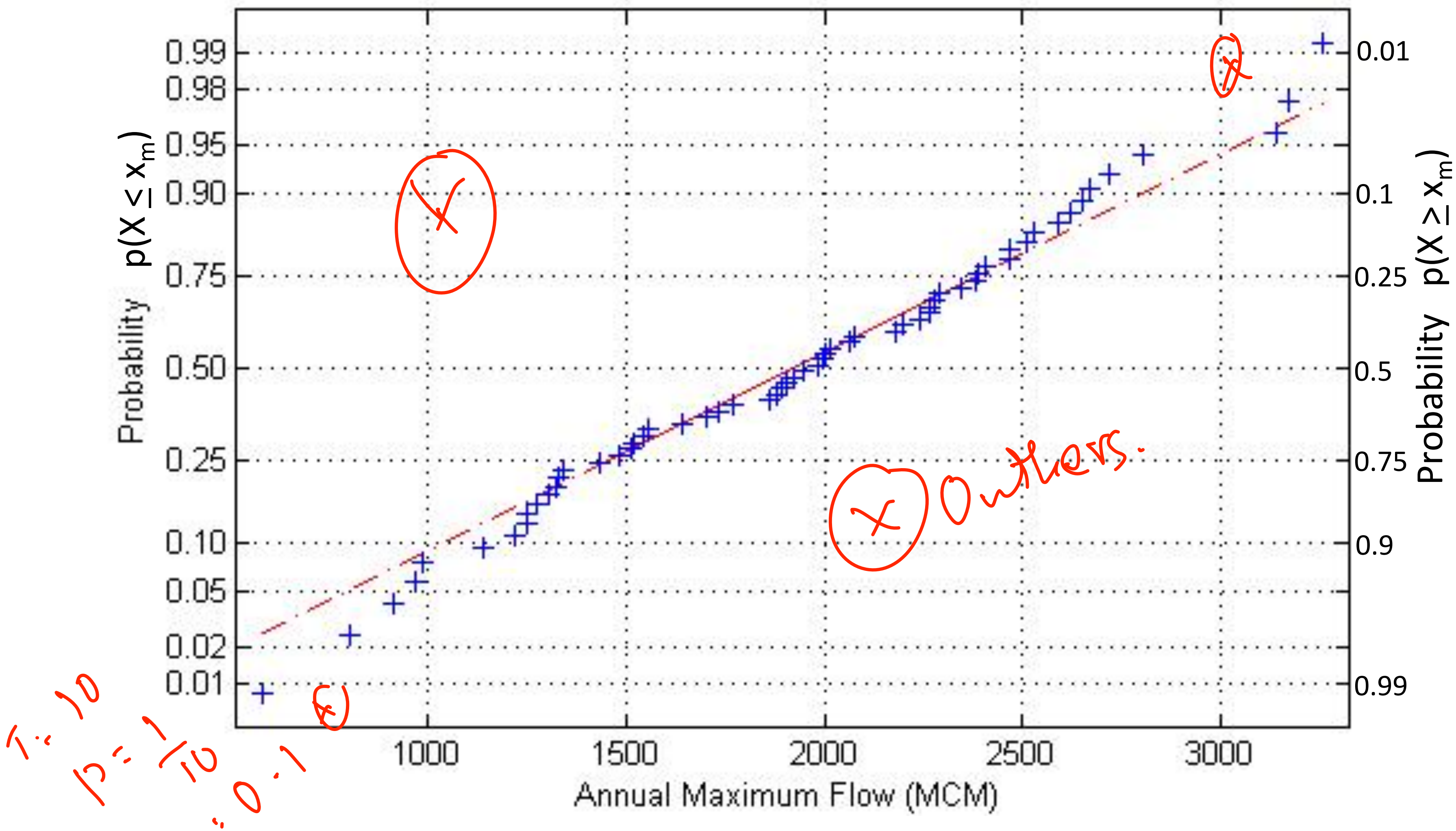
$$p\left(X \geq x_m\right) = \frac{m - 0.5}{n}$$

- The maximum annual discharge verses the return period is plotted on a normal probability paper.

Arranged data	Rank (m)	$p(X \geq x_m)$	Arranged data	Rank (m)	$p(X \geq x_m)$	Arranged data	Rank (m)	$p(X \geq x_m)$
3256	1	0.008	2265	21	0.342	1557	41	0.675
3171	2	0.025	2243	22	0.358	1540	42	0.692
3143	3	0.042	2200	23	0.375	1518	43	0.708
2806	4	0.058	2180	24	0.392	1515	44	0.725
2721	5	0.075	2078	25	0.408	1484	45	0.742
2670	6	0.092	2064	26	0.425	1430	46	0.758
2653	7	0.108	2016	27	0.442	1339	47	0.775
2619	8	0.125	2005	28	0.458	1331	48	0.792
2591	9	0.142	1996	29	0.475	1325	49	0.808
2532	10	0.158	1982	30	0.492	1303	50	0.825
2512	11	0.175	1945	31	0.508	1274	51	0.842
2469	12	0.192	1920	32	0.525	1252	52	0.858
2466	13	0.208	1903	33	0.542	1246	53	0.875
2407	14	0.225	1894	34	0.558	1218	54	0.892
2387	15	0.242	1877	35	0.575	1141	55	0.908
2381	16	0.258	1860	36	0.592	983	56	0.925
2347	17	0.275	1773	37	0.608	966	57	0.942
2291	18	0.292	1736	38	0.625	915	58	0.958
2277	19	0.308	1705	39	0.642	804	59	0.975
2268	20	0.325	1642	40	0.658	583	60	0.992

Example – 1 (Contd.)

MATLAB Syntax: normplot(data)



Plotting Position

- When probability plots of hydrologic data are made, one or more extreme events are present that appear to form a different population because they plot far off the line defined by the other points.
- The separate treatment is required for these outliers.
- Benson (1962c) has stated that these extremes can be treated if the historical information is available.

Ref: Benson, M.A., 1962c. Evolution of methods for evaluating the occurrence of floods, water supply paper 1580-A, U.S. Geological survey, Washington, D.C.

TESTING GOODNESS OF FIT

Tests for Goodness of Fit

- Two ways of testing whether or not a particular distribution adequately fits a set of observations.
- One method is to plot the data on appropriate probability paper and judge whether or not the resulting plot is a straight line.
- Second method is to compare the observed relative frequency curve with the theoretical relative frequency curve.

Tests for Goodness of Fit

- Two tests are discussed
  - Chi-Square test
  - Kolmogorov – Smirnov test

Tests for Goodness of Fit

Chi-Square Goodness of fit test:

- One of the most commonly used tests for goodness of fit of empirical data to specified theoretical frequency distributions.
- The test makes a comparison between the actual number of observations and the expected number of observations that fall in the class intervals.
- The expected numbers are calculated by multiplying the expected relative frequency by total number of observations.

Tests for Goodness of Fit

Steps to be followed:

- Divide the data into k class intervals.
- N_i is the number of observations falling in each class interval.
- Select the distribution whose adequacy is to be tested.
- Determine the probability with which the RV lies in each of the class interval using the selected distribution.
- Calculate the expected number of observations in any class interval E_i, by multiplying the probability with the total length of sample.

Tests for Goodness of Fit

- The Chi-square test statistic is calculated by

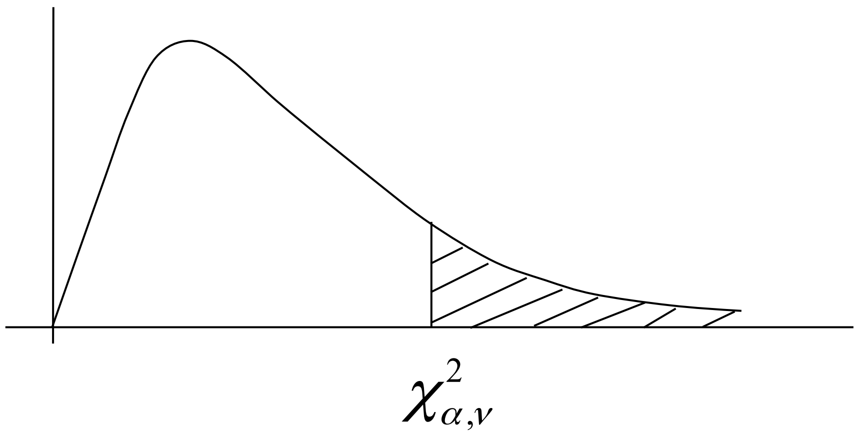
$$\chi^2 = \sum_{i=1}^{k} \frac{\left(N_i - E_i\right)^2}{E_i}$$

- This parameter follow Chi-square distribution with the number of degrees of freedom equal to k-p-1, where p is no. of parameters of the distribution.
- The hypothesis that the data follows a specified distribution is accepted if

$$\chi^2 < \chi^2_{1-\alpha,k-p-1}$$

Tests for Goodness of Fit

- α is the significance level.
- Usually the significance level is considered as 10% or 5%.
- The critical value is obtained from the Chi-Square distribution table.



$$\nu = k - p - 1$$

Tests for Goodness of Fit

ν \ α	0.995	0.95	0.9	0.1	0.05	0.025	0.01	0.005
1	3.9E-05	0.004	0.016	2.71	3.84	5.02	6.63	7.88
2	0.010	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.21	0.71	1.06	7.78	9.49	11.14	13.28	14.86
5	0.41	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	0.68	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.99	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	2.73	3.49	13.36	15.51	17.53	20.09	21.95
9	1.73	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	3.94	4.87	15.99	18.31	20.48	23.21	25.19
20	7.43	10.85	12.44	28.41	31.41	34.17	37.57	40.00
30	13.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67

Tests for Goodness of Fit

- The number of class intervals should be at least 5.
- If the sample size n is large, the number of class intervals may be approximately fixed by

$$k = 10 + 1.33\ln(n)$$

- It is better to choose non-uniform class intervals so that at least 5 observations in each class interval.
- If more than one distribution passes the test, then the distribution which gives the least value of χ^2 is considered.

Example – 2

Consider the annual maximum discharge Q (in cumec) of a river for 40years, Check whether the data follows a normal distribution using Chi-Square goodness of fit test at 10% significance level.

S.No.	Q ($x10^6$)	S.No.	Q ($x10^6$)	S.No.	Q ($x10^6$)	S.No.	Q ($x10^6$)
1	590	11	501	21	863	31	658
2	618	12	360	22	672	32	646
3	739	13	535	23	1054	33	1000
4	763	14	644	24	858	34	653
5	733	15	700	25	285	35	626
6	318	16	607	26	643	36	543
7	791	17	686	27	479	37	650
8	582	18	411	28	613	38	900
9	529	19	556	29	584	39	765
10	895	20	512	30	900	40	831

Example – 2 (Contd.)

Mean, $\bar{x}$ = 657.42

Standard deviation, s = 173.59

Data is divided into 8 class intervals

<400
400-500
500-620
620-740
740-850
850-960
960-1000
>1000

Example – 2 (Contd.)

Class Interval	n_i	(z_i) of upper limit	$F(z_i)$	$p_i=F(z_i) - F(z_i-1)$	$e_i=np_i$	$\frac{(n_i-e_i)^2}{e_i}$
<400	3	-1.483	0.069	0.069	2.76	0.021
400-500	2	-0.907	0.182	0.113	4.52	1.405
500-620	12	-0.216	0.415	0.233	9.32	0.771
620-740	12	0.476	0.683	0.268	10.72	0.153
740-850	4	1.109	0.866	0.183	7.32	1.506
850-960	5	1.743	0.959	0.093	3.72	0.440
960-1000	1	1.974	0.976	0.017	0.68	0.151
>1000	1		1	0.024	0.96	0.002
Total	40			1	40	4.448

Example – 2 (Contd.)

From the table, $\chi^2 = 4.448$

No. of class intervals, $k = 8$

No. of parameters estimated, $p = 2$

Therefore $\nu = k - p - 1$

$= 8 - 2 - 1$

$= 5$

Significance level $\alpha = 10\% = 0.1$

Example – 2 (Contd.)

From the Chi-square distribution table,

$$\chi^2_{0.1,5} = 9.24$$

ν \ α	0.9	0.1	0.05
3	0.584	6.25	7.81
4	1.06	7.78	9.49
5	1.61	9.24	11.07
6	2.20	10.64	12.59

$$\chi^2 < \chi^2_{0.1,5}$$

The hypothesis that the normal distribution fits the data can be accepted.

Testing – Goodness of Fit of Data

Kolmogorov – Smirnov Goodness of fit test:

- This is an alternative to the Chi-square test.
- The test is conducted as follows
  - The data is arranged in descending order of magnitude.
  - The cumulative probability $P(x_i)$ for each of the observations are calculated using the Weibull's formula.
  - The theoretical cumulative probability $F(x_i)$ for each of the observation is obtained using the assumed distribution.

Testing – Goodness of Fit of Data

- The absolute difference of $P(x_i)$ and $F(x_i)$ is calculated.
- The Kolmogorov-Smirnov test statistic Δ is the maximum of this absolute difference.

$$\Delta = maximum\ of \left| P(x_i) - F(x_i) \right|$$

- The critical value of Kolmogorov-Smirnov statistic Δ_o is obtained from the table for a given significance value α.
- If $\Delta < \Delta_o$, accept the hypothesis that the assumed distribution is a good fit.

Testing – Goodness of Fit of Data

Size of sample	Significance Level α				
	0.2	0.15	0.1	0.05	0.01
5	0.45	0.47	0.51	0.56	0.67
10	0.32	0.34	0.37	0.41	0.49
20	0.23	0.25	0.26	0.29	0.36
30	0.19	0.20	0.22	0.24	0.29
40	0.17	0.18	0.19	0.21	0.25
50	0.15	0.16	0.17	0.19	0.23
Asymptotic formula (n > 50)	$\frac{1.07}{\sqrt{n}}$	$\frac{1.14}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

Testing – Goodness of Fit of Data

- The advantage of Kolmogorov-Smirnov test over the Chi-square test is that it does not lump the data and compare only the discrete categories.
- It is easier to compute Δ than X^2.
- This test is more convenient to adopt when the sample size is small.

Example – 3

Consider the annual maximum discharge Q (in cumec) of a river for 20years, Check whether the data follows a normal distribution using Kolmogorov-Smirnov goodness of fit test at 10% significance level.

S.No.	Q ($x10^6$)	S.No.	Q ($x10^6$)
1	590	11	501
2	618	12	360
3	739	13	535
4	763	14	644
5	733	15	700
6	318	16	607
7	791	17	686
8	582	18	411
9	529	19	556
10	895	20	831

Example – 3 (Contd.)

Mean, $\bar{x}$ = 619.62

Standard deviation, s = 153.32

Data is arranged in the descending order and a rank is assigned. The probability is obtained using Weibull's formula

$$P\left(X \geq x_m\right) = \frac{m}{n+1}$$

S.No.	Q (x10^6)	Descending order	Rank (m)	$p = \frac{m}{n+1}$	$P(x_i)=1-p$	(z_i)	$F(z_i)$	$\left\|P(x_i)-F(x_i)\right\|$
1	590	895	1	0.048	0.952	1.795	0.964	0.011
2	618	831	2	0.095	0.905	1.381	0.916	0.012
3	739	791	3	0.143	0.857	1.121	0.869	0.012
4	763	763	4	0.190	0.810	0.936	0.825	0.016
5	733	739	5	0.238	0.762	0.781	0.783	0.021
6	318	733	6	0.286	0.714	0.737	0.769	0.055
7	791	700	7	0.333	0.667	0.524	0.700	0.033
8	582	686	8	0.381	0.619	0.432	0.667	0.048
9	529	644	9	0.429	0.571	0.162	0.564	0.007
10	895	618	10	0.476	0.524	-0.011	0.495	0.028
11	501	607	11	0.524	0.476	-0.082	0.468	0.009
12	360	590	12	0.571	0.429	-0.192	0.424	0.005
13	535	582	13	0.619	0.381	-0.242	0.404	0.023
14	644	556	14	0.667	0.333	-0.412	0.340	0.007
15	700	535	15	0.714	0.286	-0.549	0.292	0.006
16	607	529	16	0.762	0.238	-0.589	0.278	0.040
17	686	501	17	0.810	0.190	-0.772	0.220	0.029
18	411	411	18	0.857	0.143	-1.361	0.087	**0.056**
19	556	360	19	0.905	0.095	-1.692	0.045	0.050
20	831	318	20	0.952	0.048	-1.965	0.025	0.023

Example – 2 (Contd.)

Maximum value $\Delta = 0.056$

From the Kolmogorov-Smirnov table,

$$\Delta_0 = 0.26$$

N \ α	0.9	0.1	0.05
10	0.34	0.37	0.41
20	0.25	0.26	0.29
30	0.20	0.22	0.24

$\Delta < \Delta_o$,

The hypothesis can be accepted that the normal distribution fits the data.

Testing – Goodness of Fit of Data

General comments on Goodness of fit tests:

- Many hydrologists discourage the use of these tests when testing hydrologic frequency distributions
- The reason is the importance of the tails of hydrologic frequency distributions and insensitivity of these tests in the tails of the distributions.
- The -------- (TO BE CHECKED)

Ref: Statistical methods in Hydrology by C.T.Haan, Iowa state university press

INTENSITY-DURATION-FREQUENCY (IDF) CURVES

IDF Curves

- In many of the hydrologic projects, the first step is the determination of rainfall event
  - E.g., urban drainage system
- Most common way is using the IDF relationship
- An IDF curve gives the expected rainfall intensity of a given duration of storm having desired frequency of occurrence.
- IDF curve is a graph with duration plotted as abscissa, intensity as ordinate and a series of curves, one for each return period
- Standard IDF curves are available for a site.

IDF Curves

- The intensity of rainfall is the rate of precipitation, i.e., depth of precipitation per unit time
- This can be either instantaneous intensity or average intensity over the duration of rainfall
- The average intensity is commonly used.

$$i = \frac{P}{t}$$

where P is the rainfall depth

t is the duration of rainfall

IDF Curves

- The frequency is expressed in terms of return period (T) which is the average length of time between rainfall events that equal or exceed the design magnitude.
- If local rainfall data is available, IDF curves can be developed using frequency analysis.
- For the development of the curves, a minimum of 20 years data is required.

IDF Curves

Procedure for developing IDF curves:

Step 1: Preparation of annual maximum data series

- From the available rainfall data, rainfall series for different durations (e.g.,1H, 2H, 6H, 12H and 24H) are developed.
- For each selected duration, the annual maximum rainfall depths are calculated.

IDF Curves

Step 2: Fitting the probability distribution

- A suitable probability distribution is fitted to the each selected duration data series .
- Generally used probability distributions are
  - Gumbel’s Extreme Value distribution
  - Gamma distribution (two parameter)
  - Log Pearson Type III distribution
  - Normal distribution
  - Log-normal distribution (two parameter)

IDF Curves

Statistical distributions and their functions:

Distribution	PDF
Gumbel's EVT-I	$f(x) = \exp\left\{-\left(x-\beta\right)/\alpha - \exp\left[-\left(x-\beta\right)/\alpha\right]\right\}/\alpha$ $\quad -\infty \le x \le \infty$
Gamma	$f(x) = \dfrac{\lambda^{n} x^{\eta-1} e^{-\lambda x}}{\Gamma(\eta)}$ $\quad x, \lambda, \eta > 0$
Log Pearson Type-III	$f(x) = \dfrac{\lambda^{\beta} \left(y-\varepsilon\right)^{\beta-1} e^{-\lambda\left(y-\varepsilon\right)}}{x\Gamma(\beta)}$ $\quad \log x \ge \varepsilon$
Normal	$f(x) = \dfrac{1}{\sqrt{2\pi\sigma}} \exp\left\{-\dfrac{1}{2}\left(\dfrac{x-\mu}{\sigma}\right)^{2}\right\}$ $\quad -\infty < x < \infty$
Log-Normal	$f(x) = \dfrac{1}{\sqrt{2\pi x \sigma_x}} \mathrm{e}^{-\left(\ln x - \mu_x\right)^2 / 2\sigma_x^2}$ $\quad 0 < x < \infty$

IDF Curves

- Commonly used distribution is Gumbel’s Extreme Value Type-I distribution
- Parameters of the distribution is calculated for the selected distribution using method of maximum likelihood method.
- The Kolmogorov-Smirnov or the Chi-Square goodness of fit test is used to evaluate the accuracy of the fitting of a distribution

IDF Curves

Step 3: Determining the rainfall depths

- Two ways of determining the rainfall depths
  - Using frequency factors or
  - Using the CDF of the distribution (by inversing the CDF).

IDF Curves

Using frequency factors:

- The precipitation depths is calculated for a given return period as

$$x_T = \bar{x} + K_T s$$

where

$\bar{x}$ is mean of the data,

s is the standard deviation and

K_T is the frequency factor

IDF Curves

Using CDF of a distribution:

- Using the parameters of the distribution which are caluclated in the previous step, a probability model is formulated.
- The formulated probability model is inversed and x_T value for a given return period is calculated by

$$P\left(X \geq x_T\right) = \frac{1}{T}; \qquad 1 - P\left(X < x_T\right) = \frac{1}{T}$$

$$1 - F\left(x_T\right) = \frac{1}{T}; \qquad F\left(x_T\right) = 1 - \frac{1}{T} = \frac{T-1}{T}$$

IDF Curves

- The precipitation depths calculated from the annual exceedence series is adjusted to match the depths derived from annual maximum series by multiplying a factor

Return Period	Factor
2	0.88
5	0.96
10	0.99

- No adjustment of the estimates is required for longer return periods (> 10 year return period)

Example – 4

Bangalore rainfall data is considered to demonstrate the construction of IDF curves.

The duration of the rainfall considered is : 1H, 2H, 6H, 12H and 24 H.

The return period considered is: 2Y, 5Y, 10Y, 50Y and 100Y.

The hourly rainfall data for 33years is collected at a rain gauge station. The data is as shown

Example – 4 (Contd.)

The part data is shown below.

Year	Month	Day	Hr1	Hr2	Hr3	Hr4	Hr5
1969	8	22	3.7	0	0.1	0	0
1969	8	23	0	1.2	2.8	5.3	4.5
1969	8	24	0.6	0.2	0	0	0
1969	8	25	5.2	3.3	4.5	1.6	0
1969	8	26	0	0	0	0	0
1969	8	27	0	0	0	0	0
1969	8	28	1.2	12.8	3.2	4	6.1
1969	8	29	0	0	0	0	0
1969	8	30	0.1	0.1	0	0	0

Example – 4 (Contd.)

- From the hourly data, the rainfall for different durations are obtained.

Example – 4 (Contd.)

Rainfall in mm for different durations.

Year	1H	2H	6H	12H	24H	Year	1H	2H	6H	12H	24H
1969	44.5	61.6	104.1	112.3	115.9	1986	65.2	73.7	97.9	103.9	104.2
1970	37	48.2	62.5	69.6	92.7	1987	47	55.9	64.8	65.6	67.5
1971	41	52.9	81.4	86.9	98.7	1988	148.8	210.8	377.6	432.8	448.7
1972	30	40	53.9	57.8	65.2	1989	41.7	47	51.7	53.7	78.1
1973	40.5	53.9	55.5	72.4	89.8	1990	40.9	71.9	79.7	81.5	81.6
1974	52.4	62.4	83.2	93.4	152.5	1991	41.1	49.3	63.6	93.2	147
1975	59.6	94	95.1	95.1	95.3	1992	31.4	56.4	76	81.6	83.1
1976	22.1	42.9	61.6	64.5	71.7	1993	34.3	36.7	52.8	68.8	70.5
1977	42.2	44.5	47.5	60	61.9	1994	23.2	38.7	41.9	43.4	50.8
1978	35.5	36.8	52.1	54.2	57.5	1995	44.2	62.2	72	72.2	72.4
1979	59.5	117	132.5	135.6	135.6	1996	57	74.8	85.8	86.5	90.4
1980	48.2	57	82	86.8	89.1	1997	50	71.1	145.9	182.3	191.3
1981	41.7	58.6	64.5	65.1	68.5	1998	72.1	94.6	111.9	120.5	120.5
1982	37.3	43.8	50.5	76.2	77.2	1999	59.3	62.9	82.3	90.7	90.9
1983	37	60.4	70.5	72	75.2	2000	62.3	78.3	84.3	84.3	97.2
1984	60.2	74.1	76.6	121.9	122.4	2001	46.8	70	95.9	95.9	100.8
						2003	53.2	86.5	106.1	106.2	106.8

Example – 4 (Contd.)

The rainfall intensity (mm/hr) for different durations.

Year	1H	2H	6H	12H	24H	Year	1H	2H	6H	12H	24H
1969	44.50	30.80	17.35	9.36	4.83	1986	65.20	36.85	16.32	8.66	4.34
1970	37.00	24.10	10.42	5.80	3.86	1987	47.00	27.95	10.80	5.47	2.81
1971	41.00	26.45	13.57	7.24	4.11	1988	148.80	105.40	62.93	36.07	18.70
1972	30.00	20.00	8.98	4.82	2.72	1989	41.70	23.50	8.62	4.48	3.25
1973	40.50	26.95	9.25	6.03	3.74	1990	40.90	35.95	13.28	6.79	3.40
1974	52.40	31.20	13.87	7.78	6.35	1991	41.10	24.65	10.60	7.77	6.13
1975	59.60	47.00	15.85	7.93	3.97	1992	31.40	28.20	12.67	6.80	3.46
1976	22.10	21.45	10.27	5.38	2.99	1993	34.30	18.35	8.80	5.73	2.94
1977	42.20	22.25	7.92	5.00	2.58	1994	23.20	19.35	6.98	3.62	2.12
1978	35.50	18.40	8.68	4.52	2.40	1995	44.20	31.10	12.00	6.02	3.02
1979	59.50	58.50	22.08	11.30	5.65	1996	57.00	37.40	14.30	7.21	3.77
1980	48.20	28.50	13.67	7.23	3.71	1997	50.00	35.55	24.32	15.19	7.97
1981	41.70	29.30	10.75	5.43	2.85	1998	72.10	47.30	18.65	10.04	5.02
1982	37.30	21.90	8.42	6.35	3.22	1999	59.30	31.45	13.72	7.56	3.79
1983	37.00	30.20	11.75	6.00	3.13	2000	62.30	39.15	14.05	7.03	4.05
1984	60.20	37.05	12.77	10.16	5.10	2001	46.80	35.00	15.98	7.99	4.20
						2003	53.20	43.25	17.68	8.85	4.45

Example – 4 (Contd.)

The mean and standard deviation for the data for different durations is calculated.

Duration	1H	2H	6H	12H	24H
Mean	48.70	33.17	14.46	8.05	4.38
Std. Dev.	21.53	15.9	9.59	5.52	2.86

Gumbell's distribution is considered

Example – 4 (Contd.)

K_T values are calculated for different return periods using

$$K_T = -\frac{\sqrt{6}}{\pi}\left\{0.5772 + \ln\left[\ln\left(\frac{T}{T-1}\right)\right]\right\}$$

T (years)	2	5	10	50	100
K_T	-0.164	0.719	1.305	2.592	3.137

Example – 4 (Contd.)

The rainfall intensities are calculated using

$$x_T = \overline{x} + K_T s$$

For example,

For duration of 2 hour, and 10 year return period,

Mean $\overline{x}$ = 33.17 mm/hr,

Standard deviation s = 15.9 mm/hr

Frequency factor K_T = 1.035

Example – 4 (Contd.)

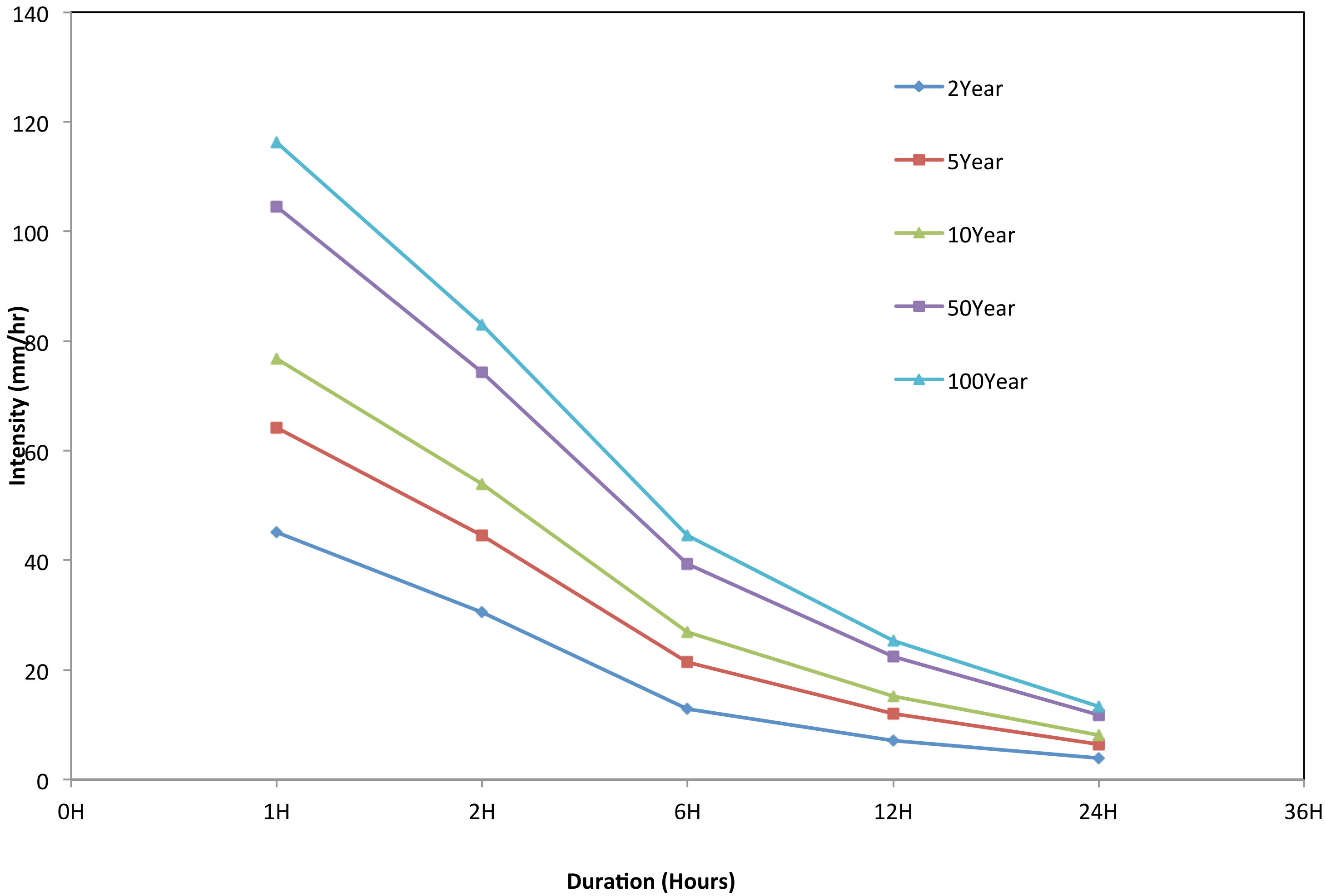
x_T = 33.17 + 1.035 * 15.9

= 53.9 mm/hr.

The values or other durations are tabulated.

Duration (hours)	Return Period T (Years)				
	2	5	10	50	100
1H	45.17	64.19	76.79	104.51	116.23
2H	30.55	44.60	53.90	74.36	83.02
6H	12.89	21.36	26.97	39.31	44.53
12H	7.14	12.02	15.25	22.36	25.37
24H	3.91	6.44	8.11	11.79	13.35

Example – 4 (Contd.)



IDF Curves

Equations for IDF curves:

- IDF curves can also be expressed as equations to avoid reading the design rainfall intensity from a graph

$$i = \frac{c}{t^e + f}$$

where

i is the design rainfall intensity,

t is the duration and

c, e and f are coefficients varying with location and return period.