Finite Element Analysis

Assignment 4

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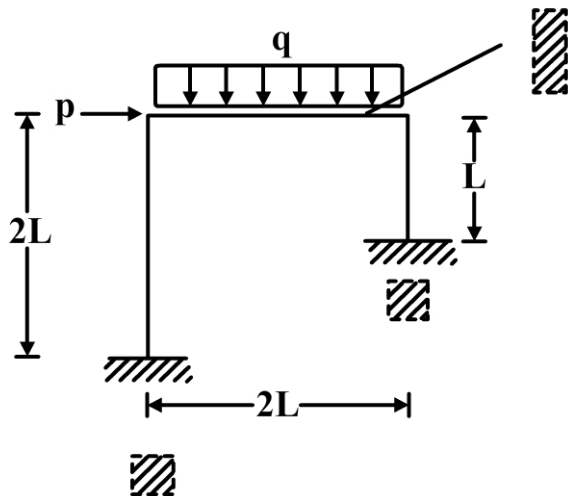
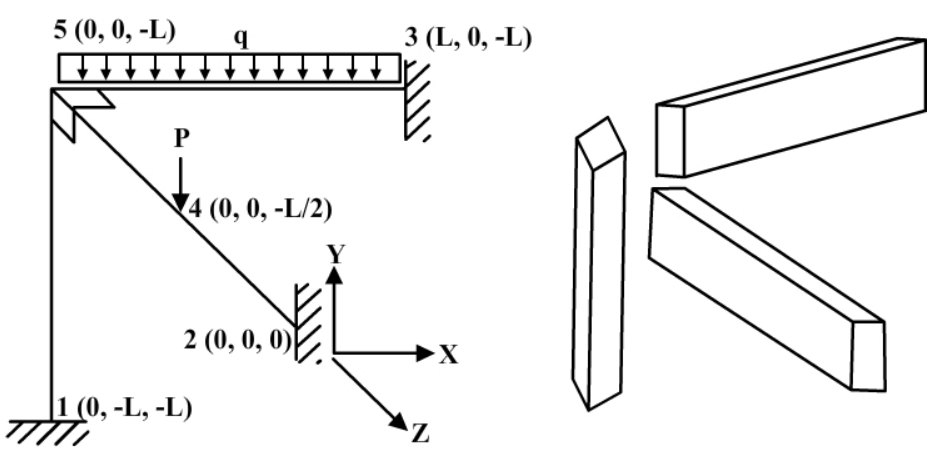
1. Determine nodal displacements and element forces in the plane frame as shown in the following figure. Draw free body diagram clearly showing all element forces and moments. Assume P = 10 kN, q = 10 kN/m, L = 2 m, E = 210 GPa. Columns are 25 cm × 25 cm square and the beam is 25 cm × 50 cm rectangular cross-section. Orientation of members is shown in the following figure.



2. Determine nodal displacements and element forces in a three dimensional frame shown in the following figure. The beams are of rectangular cross section 0.254 m × 0.381 m. The column is of square cross section 0.254 m × 0.254 m. Assume L = 4.572 m, q = 29.1878 N/m, P = 44.482 kN, E = 20.6842GPa, G = 6.895 GPa.



3. Consider transverse deflection of a flexible cable shown in the following figure. If the cable is stretched with a tension *T*, and is hanging under its own weight alone, the governing differential equation can be written as follows.

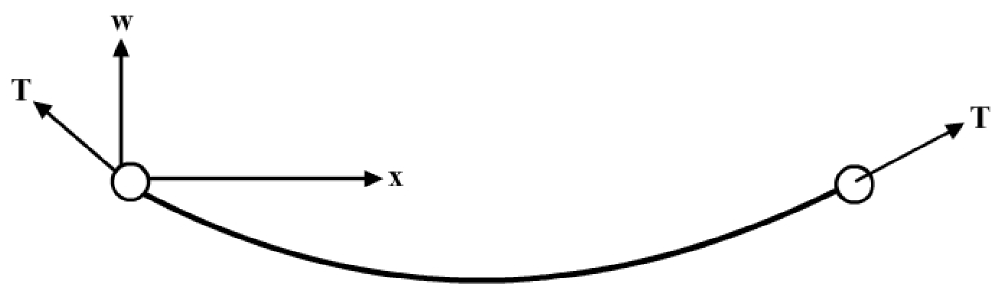
$$-\frac{d}{dx}\left(T\frac{dw}{dx}\right)+\rho g=0 \qquad 0<x<L$$

$$w(0)=w(L)=0$$

where $w(x)$ is the vertical deflection of the cable (positive upwards), $\rho(x)$ is mass per unit length, and g is acceleration due to gravity.

(a) Using the general form of element equations write down element equations for a linear element for this problem assuming T is constant over each element.

(b) Use at-least three elements to determine cable profile with the following numerical data. $L = 10$ m, $\rho = 1$ kg/m, $g = 9.8$ m/sec^2, and $T = 98$ N.



4. Axial deformation of a non-uniform elastic bar subjected to a concentrated load P at one end and a uniform temperature change ΔT over the entire length, is governed by the following boundary value problem

$$\frac{d}{dx}\left[EA\left(\frac{du}{dx}-\alpha\Delta T\right)\right]=0 \qquad 0<x<L$$

$$u=2 \text{ at } x=0 \qquad EA\left(\frac{du}{dx}-\alpha\Delta T\right)=P \text{ at } x=L$$

(a) Derive element equations for a linear element for this problem assuming EA is constant over each element.

(b) Using at-least three linear elements determine displacement and axial stress in the bar. Use the following data.

$$E = 6.895 MPa, \quad \alpha = 0.0296 \times 10^{-6} / {}^{0}C, \quad L = 25.4m, \quad \Delta T = 38\,{}^{0}C$$

$$P = 0.444 kN, \quad A(x) = 100 - 0.1x$$

5. Using Lagrange interpolation formula, write down trial solution $u(x)$ in terms of nodal variables for a four node cubic element with nodes at $x_1 = 0$, $x_2 = 2$, $x_3 = 3$ and $x_4 = 6$. Show a plot $u(x)$ if the nodal values are $u_1 = 1$, $u_2 = 3$, $u_3 = -1$ and $u_4 = 1$.
6. Using Lagrange interpolation formula, write down trial solution $u(x)$ in terms of nodal variables for a five node quartic element with nodes at $x = \{0, 2, 3, 5, 6\}$. Show a plot $u(x)$ if the nodal values are $\{1, 1.2, 1.4, 1.3, 1.1\}$.
