Finite Element Analysis

Assignment 1

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1. Obtain an approximate solution of the following boundary value problem using the Least Squares weighted residual method, the Collocation method, and the Galerkin method (basic form)

$$2u''(x) + 3u = 0 \qquad 1 < x < 3$$
$$u(1) = 1 \qquad \text{Essential boundary condition}$$
$$u'(3) = 1 \qquad \text{Natural boundary condition}$$

   (a) Assume a quadratic polynomial satisfying the essential boundary condition as a trial solution.

   (b) Assume a cubic polynomial satisfying the essential boundary condition as a trial solution. Compare graphically the two solutions and their first derivatives. A good approximation to the exact solution is given by

$$u(x) = 0.29986\, sin(1.2247x) + 2.1166\, cos(1.2247x).$$

2. Obtain an approximate solution of the following boundary value problem using the modified Galerkin method and the Rayleigh-Ritz method

$$u''(x) + u(x) + x = 0 \quad 0 < x < 1$$
$$u(0) = 0 \qquad u(1) = 0$$

   (a) Assume a quadratic polynomial as a trial solution.

   (b) Assume a cubic polynomial as a trial solution. Compare graphically the two solutions and their first derivatives with the exact solution $u(x) = -x + \dfrac{sin(x)}{sin(1.0)}$.

3. Obtain an approximate solution of the following boundary value problem using the modified Galerkin method and the Rayleigh-Ritz method

$$2u''(x) + 3u(x) = 0 \qquad 1 < x < 3$$
$$u(1) = 1 \qquad \text{Essential boundary condition}$$
$$u'(3) = 1 \qquad \text{Natural boundary condition}$$

   (a) Assume a quadratic polynomial satisfying the essential boundary condition as a trial solution.

(b) Assume a cubic polynomial satisfying the essential boundary condition as a trial solution. Compare graphically the two solutions and their first derivatives. A good approximation to the exact solution is given by

$$u(x) = 0.29986\,sin(1.2247x) + 2.1166\,cos(1.2247x).$$

4. Obtain an approximate solution of the following boundary value problem using the Rayleigh-Ritz method

$$x\frac{d^2u}{dx^2} + \frac{du}{dx} + x^3 = 4 \quad 1 < x < 2$$
$$u(1) = 2 \qquad u'(2) = 0$$

(a) Use a linear polynomial trial solution. (b) Use a quadratic polynomial trial solution. Compare graphically the two solutions and their first derivatives with the exact solution $u(x) = -\frac{1}{16}x^4 + 4x - \frac{31}{16} - 4\ln(x)$.

5. Obtain an approximate solution of the following boundary value problem using the Rayleigh-Ritz method

$$x^2\frac{d^2u}{dx^2} + 2x\frac{du}{dx} - xu + 4 = 0 \qquad 1 < x < 3$$
$$u(1) = 1 \qquad \frac{du(3)}{dx} - 2u(3) = 2$$

(a) Use a linear polynomial trial solution. (b) Use a quadratic polynomial trial solution. Compare graphically the two solutions and their first derivatives.

6. Find the lowest eigenvalue λ for the following problem using the Rayleigh-Ritz method and the modified Galerkin method

$$x^2\frac{d^2u(x)}{dx^2} + 2x\frac{du}{dx} - \lambda u(x) = 0 \quad 1 < x < 3$$
$$u(1) = 0 \qquad \frac{du(3)}{dx} = 0$$

(a) Use a linear polynomial trial solution. (b) Use a quadratic polynomial trial solution.

7. Find the lowest eigenvalue λ for the following problem using the Rayleigh-Ritz method and the modified Galerkin method

$$\frac{d^2u(x)}{dx^2} + \lambda u(x) = 0 \qquad 0 < x < 1$$
$$u(0) = 0 \qquad 2\frac{du(1)}{dx} - u(1) = 0$$

(a) Use a linear polynomial trial solution. (b) Use a quadratic polynomial trial solution.