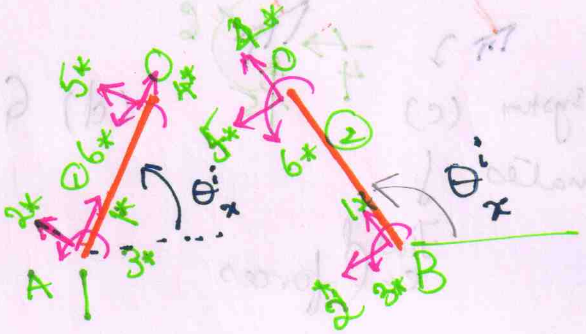
CE3310 Tutorial- 6 Plane and Space Frames M.S. SARVANI CE09B030

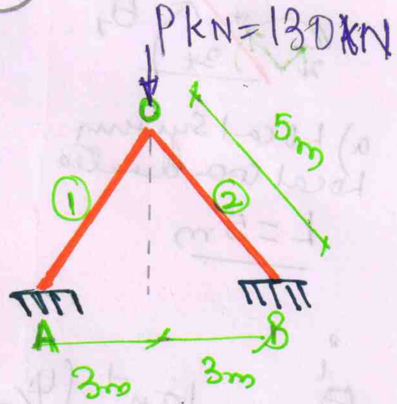
20

Q.1. $P = 130\,kN$

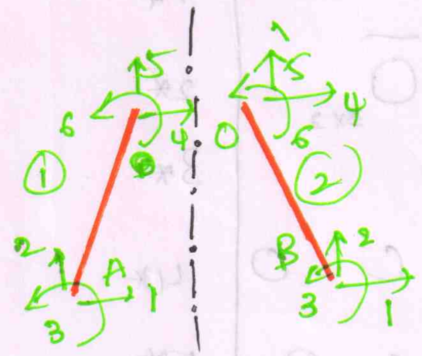
(a)



Local Axis System

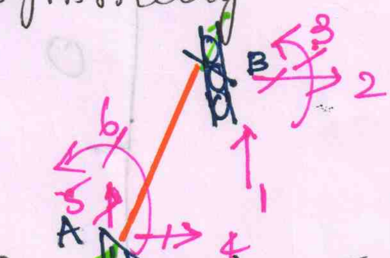


Structure



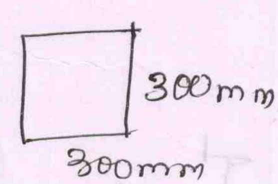
Global Axes System (Local Coordinates)

Taking advantage of Symmetry



Global Coordinates Global axes

$E = 2.5\times10^4\ N/mm^2$



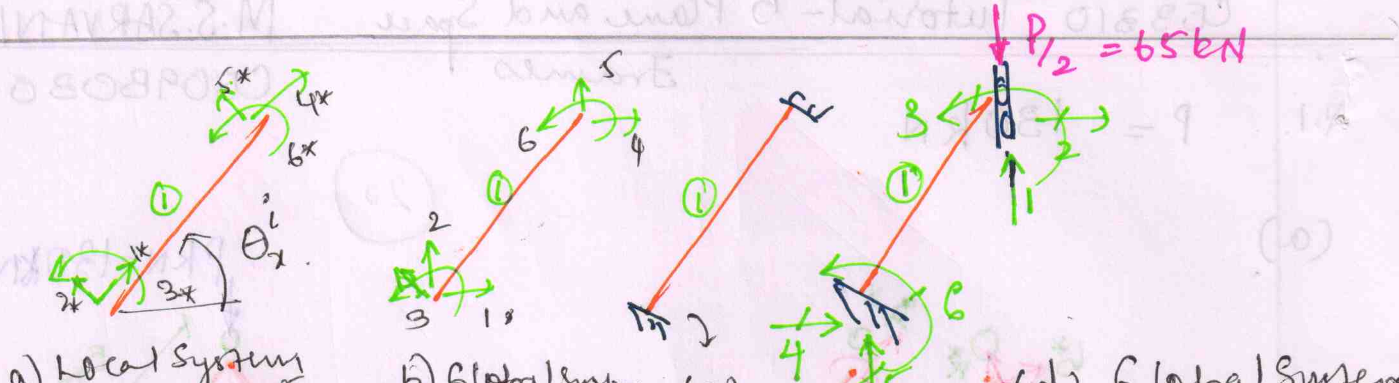
$$I = \frac{bh^3}{12} = \frac{0.3^4}{12} = 6.75\times10^{-4}\,m^4 = 6.75\times10^{-4}\times10^{12}\,mm^4 = 6.75\times10^{8}\,mm^4$$

$$EI = 2.5\times10^4\times6.75\times10^8\ N\,mm^2$$

$$= 16.875\times10^6\ Nm^2$$

$$EI = 16875\ kNm^2$$

$$EA = 2.5\times10^4\times 3^2 = 2250000\ kN/mm^2\cdot mm^2$$



a) Local System Local coordinates $L = 5\,m$

b) Global system Local coordinates

(c) Fixed end forces.

(d) Global System global coord.

$$\theta^i_x = \tan^{-1}(4/3)$$

$$T^1 = \begin{array}{c} \\ \left[\begin{array}{cccccc} (4) & (5) & (6) & (2) & (1) & (3) \\ s & c & 0 & & & \\ -c & s & 0 & & \bar{O}_{3\times3} & \\ 0 & 0 & 1 & & & \\ & & & s & c & 0 \\ & \bar{O}_{3\times3} & & -c & s & 0 \\ & & & 0 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} \\ 1* \\ 2* \\ 3* \\ 4* \\ 5* \\ 6* \end{array}$$

$$D'_* = T'D'$$

$$T^1 = \begin{bmatrix} 0.8 & 0.6 & 0 & & & \\ -0.6 & 0.8 & 0 & & \bar{O} & \\ 0 & 0 & 1 & & & \\ & & & 0.8 & 0.6 & 0 \\ & \bar{O} & & -0.6 & 0.8 & 0 \\ & & & 0 & 0 & 1 \end{bmatrix}$$

Fixed End forces: No fixed end forces.

$$k'_{*} = \begin{bmatrix} \alpha_1 & 0 & 0 & -\alpha_1 & 0 & 0 \\ 0 & \beta_1 & x_1 & 0 & -\beta_1 & x_1 \\ 0 & x_1 & 4\delta_1 & 0 & -x_1 & 2\delta_1 \\ -\alpha_1 & 0 & 0 & \alpha_1 & 0 & 0 \\ 0 & -\beta_1 & -x_1 & 0 & \beta_1 & -x_1 \\ 0 & x_1 & 2\delta_1 & 0 & -x_1 & 4\delta_1 \end{bmatrix}$$

$$\alpha_1 = \left(\frac{EA}{L}\right)_1 = \quad \beta_1 = \frac{12(EI)}{L^3} = 12\left(\frac{EI}{L^3}\right)_1$$

$$x_1 = 6\left(\frac{EI}{L^2}\right)_1, \quad \delta_1 = \left(\frac{EI}{L}\right)_1$$

Here $\alpha_1 = 450000$ kN ; $\beta_1 = 1620$ kN/m ; $x_1 = 4050$ kN

$\delta_1 = 3375$ kN m

$$k' = T'^{T} k' T' = 1000 \times \begin{array}{c} \\ (4) \\ (5) \\ (6) \\ (2) \\ (1) \\ (3) \end{array} \begin{array}{c} \begin{array}{cccccc} (4) & (5) & (6) & (2) & (1) & (3) \end{array} \\ \begin{bmatrix} 288.5832 & 215.2224 & -2.43 & -288.5832 & -215.2224 & -2.43 \\ 215.2224 & 163.0368 & 3.24 & -215.2224 & -163.0368 & 3.24 \\ -2.43 & 3.24 & 13.5 & 2.43 & -3.24 & 6.75 \\ -288.5832 & -215.2224 & 2.430 & 288.532 & 215.224 & +2.43 \\ -215.2224 & -163.0368 & -3.24 & 215.2224 & \boxed{163.0368} & -3.24 \\ -2.43 & 3.24 & 6.75 & 2.43 & -3.24 & 13.5 \end{bmatrix} \end{array}$$

a) Net Load Vector :
$$F_{net} = \begin{Bmatrix} -65 \text{ kN} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \end{matrix}$$

∴ Displacements @ Nodes are:

$$D = \begin{Bmatrix} D_A \\ \hline D_R = 0 \end{Bmatrix} = \begin{Bmatrix} D_1 = ? \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F_A \\ F_R \end{Bmatrix} = \begin{bmatrix} k_{AA} & k_{AR} \\ k_{RA} & k_{RR} \end{bmatrix} \begin{Bmatrix} D_A \\ D_R \end{Bmatrix}$$

$$F_A = k_{AA} D_A$$

$$-65\,kN = (163.0368 \times 10^{3}\ kN/m) \times D_A$$

$$D_A = -0.39868 \times 10^{-3}\ m$$

$$\boxed{D_A = D_1 = -0.399\ mm \downarrow}$$

$$F_R = k_{RA} D_A = \cancel{10^3} \times \begin{array}{c} \begin{bmatrix} -215.2224 \\ -163.0368 \\ -3.24 \\ 215.224 \\ -3.24 \end{bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \\ 2 \\ 3 \end{array} \times \left(-0.39868 \cancel{\times 10^{-3}}\right)$$

Support Reactions

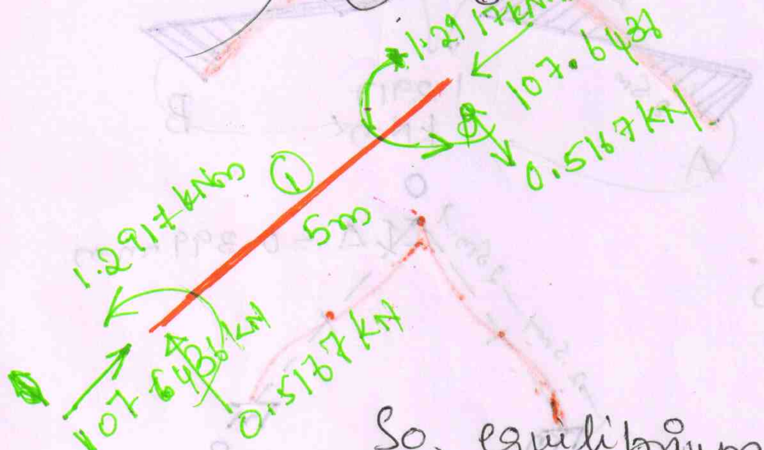
$$F_R = \begin{bmatrix} 85.805\ kN \\ 64.999\ kN \\ 1.292\ kNm \\ -85.805\ kN \\ 1.292\ kNm \end{bmatrix} \begin{array}{c} 4 \\ 5 \\ 6 \\ 2 \\ 3 \end{array}$$

$$F_{*}^{*\prime} = F'_{*f} + (k'_{*} T') D'$$

$$= \bar{O} + k'_{*} T' \begin{Bmatrix} -0.39868 \times 10^{-3}\ \text{m} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$= k'_{*} T' * \begin{array}{c} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -0 \\ -0.39868 \times 10^{-3}\ \text{m} \\ 0 \end{Bmatrix} \end{array} \begin{array}{c} 4 \\ 5 \\ 6 \\ 2 \\ 1 \\ 3 \end{array}$$

$$= \begin{Bmatrix} 107.6436\ \text{kN} \\ 0.5167\ \text{kN} \\ 1.2917\ \text{kNm} \\ -107.6436\ \text{kN} \\ -0.5167\ \text{kN} \\ 1.2917\ \text{kNm} \end{Bmatrix} \begin{array}{cc} \cancel{4} & 1* \\ \cancel{5} & 2* \\ \cancel{6} & 3* \\ \cancel{2} & 4* \\ \cancel{1} & 5* \\ \cancel{3} & 6* \end{array}$$

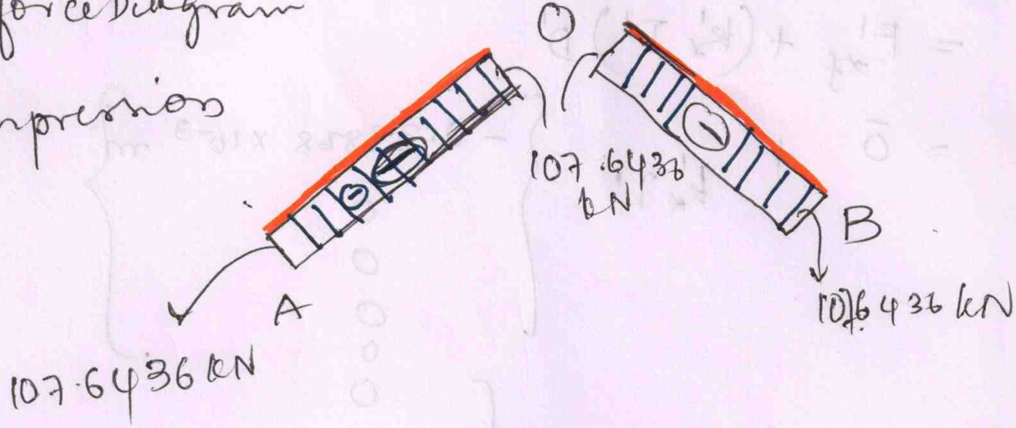


$$\frac{2 \times 1.2917\ \text{kNm}}{5\ \text{m}} = 0.51668 \approx 0.5167\ \text{kN}$$

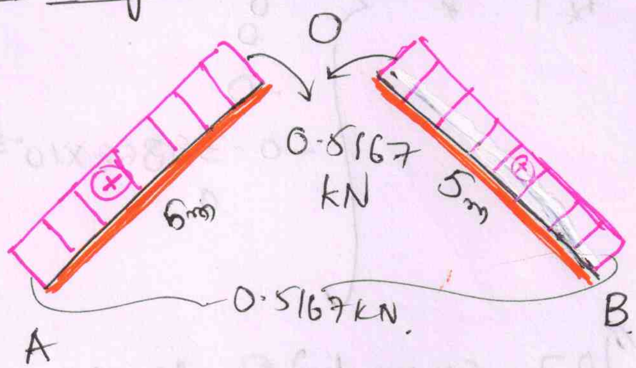
So, equilibrium is satisfied.

Axial force Diagram

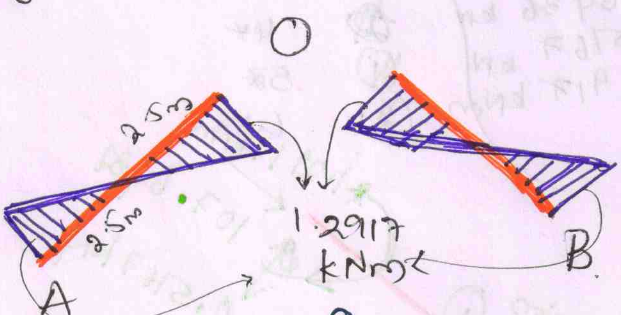
compression



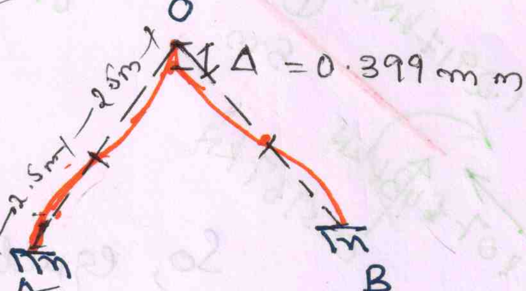
Shear force Diagram



Bending Moment Diagram



Deflection Diagram



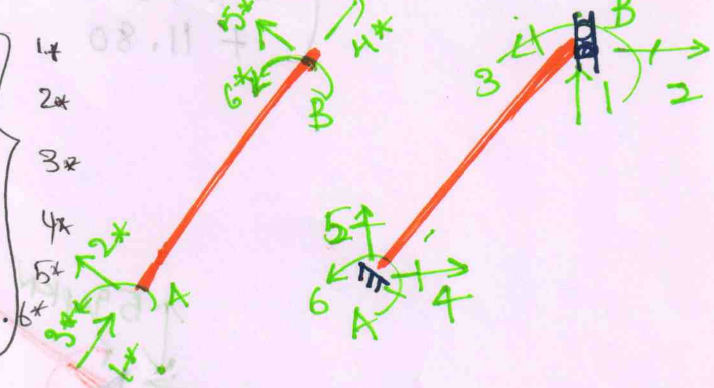
(b) Uniform Temperature Drop of 40°C

$\alpha = 11 \times 10^{-6}/°C$, No external load.

$$e_0 = \begin{Bmatrix} e_{10} \\ e_{20} \end{Bmatrix} = \begin{Bmatrix} L\alpha\Delta T \\ L\alpha\Delta T \end{Bmatrix} = \begin{Bmatrix} -2.2\ mm \\ -2.2\ mm \end{Bmatrix}$$

$$\Delta N'_{f*} = \left\{ \frac{EA}{L} \times (L\alpha\Delta T) \right\} = +990\ kN$$ (Not Compression) but tension

$$F'_{*f} = \begin{array}{l} \begin{Bmatrix} 990\ kN \\ 0 \\ 0 \\ 990\ kN \\ 0 \\ 0 \end{Bmatrix} \end{array} \begin{array}{l} 1* \\ 2* \\ 3* \\ 4* \\ 5* \\ 6* \end{array}$$



$$F_* = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Net load vector: $F_f = (T_i^{\prime\,0})^T F'_{*f} = \begin{Bmatrix} -792\ kN \\ -594\ kN \\ 0 \\ 792\ kN \\ 594\ kN \end{Bmatrix}$

$F_{net} = F - F_f$

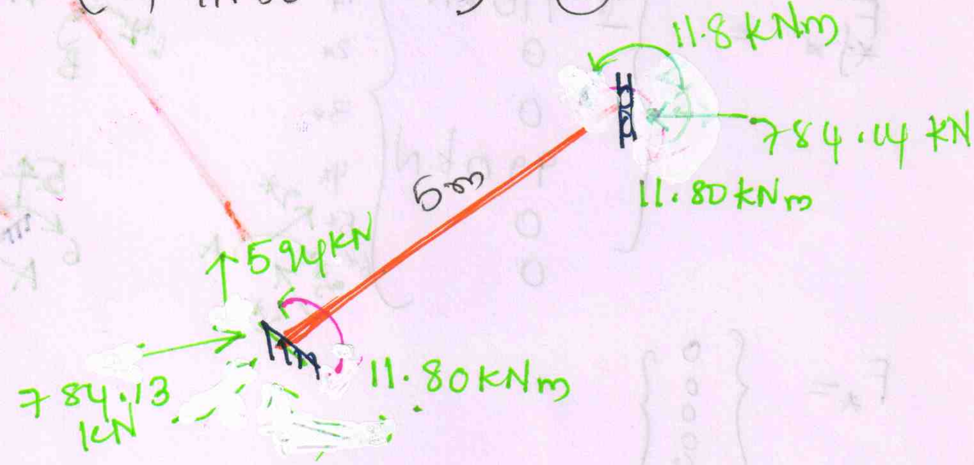
$$= \begin{Bmatrix} 792\ kN \\ 594\ kN \\ 0 \\ -792\ kN \\ -594\ kN \\ 0 \end{Bmatrix} \begin{array}{l} (4) \\ (5) \\ (6) \\ (2) \\ (1) \\ (3) \end{array}$$

$F_{net} = kD$.

$$D_1 = -\frac{594\ kN}{163.0368 \times 10^3\ kN/m} = -3.64335 \times 10^{-3}\ m$$

$$F_R = 10^3 \times \begin{bmatrix} -215.2224 \\ -163.6368 \\ -3.24 \\ 215.2224 \\ -3.24 \end{bmatrix} * (-3.64335 \times 10^{-2})$$

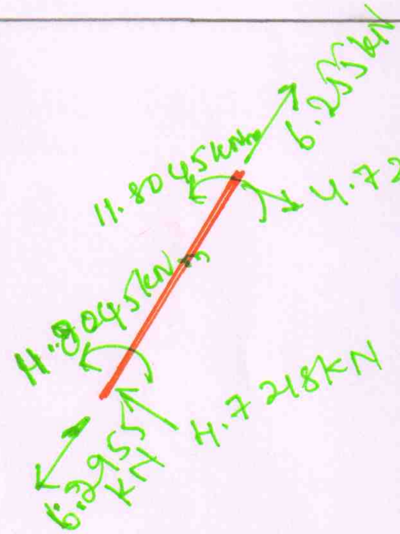
$$F_R = \begin{Bmatrix} +784.13 \text{ kN} \\ +594 \text{ kN} \\ +11.80 \text{ kNm} \\ -784.13 \text{ kN} \\ +11.80 \text{ kNm} \end{Bmatrix} \begin{array}{c} (4) \\ (5) \\ (6) \\ (2) \\ (3) \end{array}$$

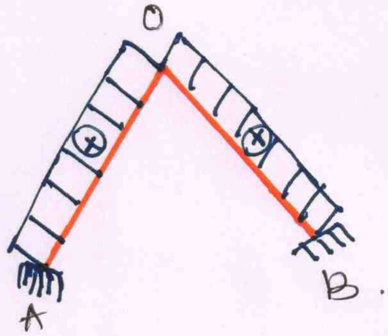


$$F'_{*} = F^{*}_{*f} + k'_{*} T' D'$$

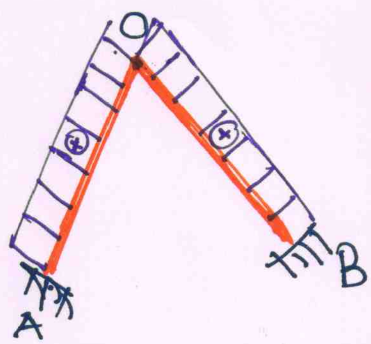
$$= \begin{Bmatrix} -990 \\ 0 \\ 0 \\ -990 \\ 0 \\ 0 \end{Bmatrix} + k'_{*} T' \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -3.64335 \times 10^{-3} \text{ m} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -990 \\ 0 \\ 0 \\ -990 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} +983.7045 \\ +4.7218 \\ +11.8045 \\ -983.7046 \\ -4.7218 \\ +11.8045 \end{Bmatrix} \begin{array}{c} 1* \\ 2* \\ 3* \\ 4* \\ 5* \\ 6* \end{array} = \begin{Bmatrix} -6.2955 \text{ kN} \\ 4.7218 \text{ kN} \\ 11.8045 \text{ kNm} \\ 6.2955 \text{ kN} \\ -4.7218 \text{ kN} \\ 11.8045 \text{ kNm} \end{Bmatrix}$$

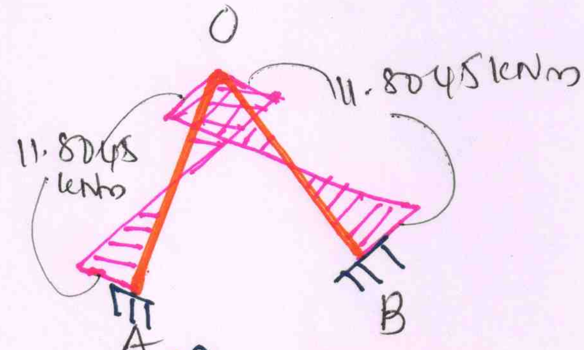




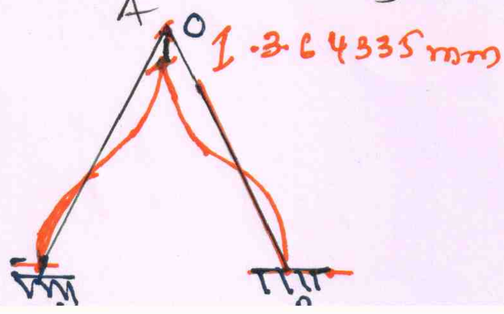
Axialforce diagram (Tension)



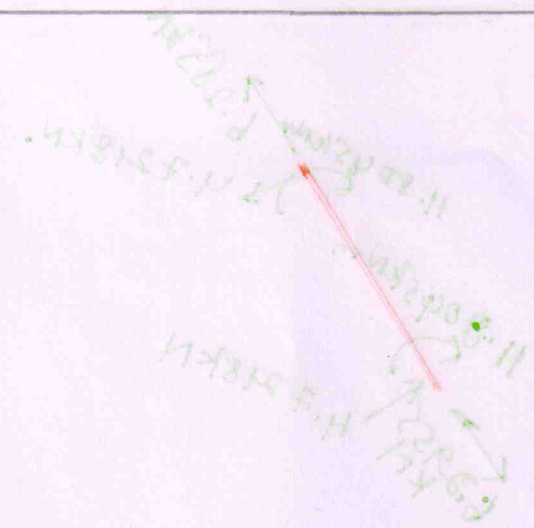
Shear force diagram



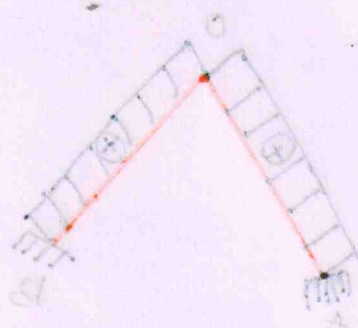
Bending Moment Diagram



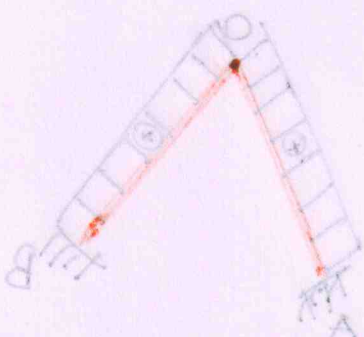
Deflection Diagram.



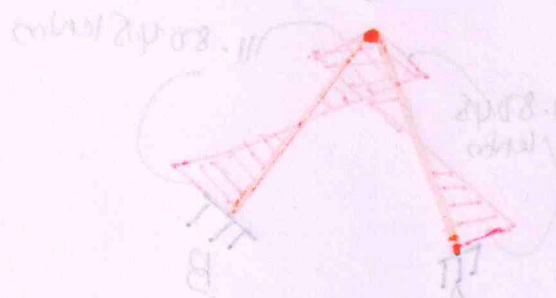
Axial force diagram
(Tension)



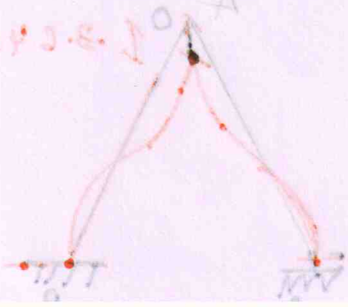
Shear force diagram



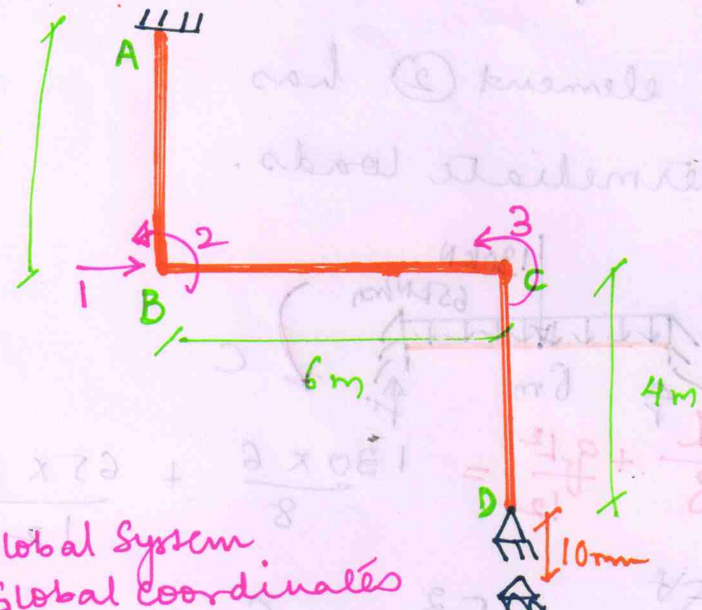
Bending Moment Diagram



Deflection Diagram



Q2) Given structure :



a) Global System
Global coordinates

In reduced stiffness method, if axial deformations are ignored, vertical degrees of freedom at B and C can be eliminated.

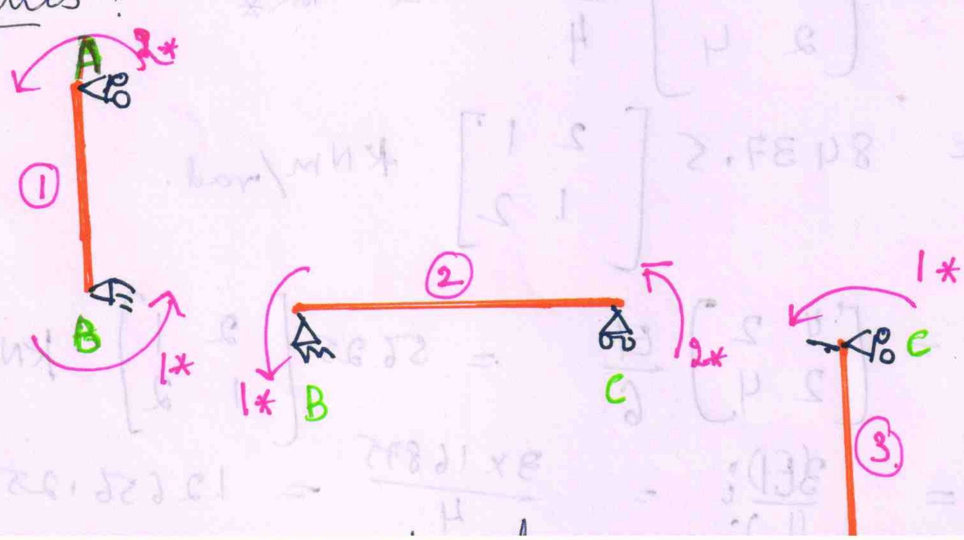
$E = 2.5 \times 10^4 \ N/mm^2$,

All elements have a size of 300×300 mm

$$I = \frac{0.3 \times 0.3^3}{12} = 6.75 \times 10^{-4} \ m^4$$

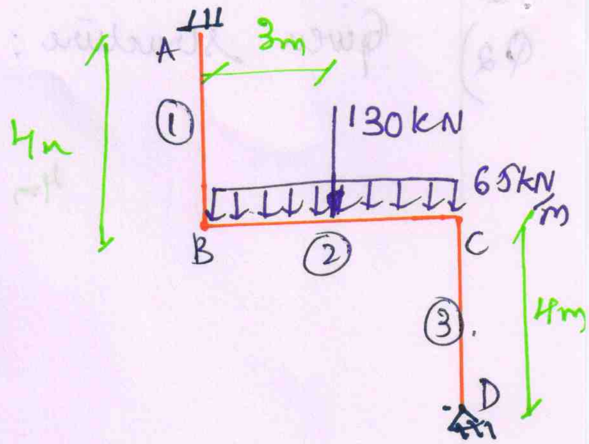
$$EI = 2.5 \times 10^4 \times 10^6 \times 6.75 \times 10^{-4} \ Nm^2$$
$$= 16.875 \times 10^6 \ Nm^2 = \underline{16875 \ kNm^2}$$

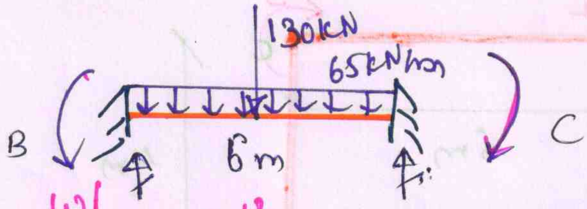
Elements :



Fixed end forces

Only element ② has intermediate loads.





$$\frac{WL}{8} + \frac{q_f L^2}{12} = \frac{130 \times 6}{8} + \frac{65 \times 6^2}{12} = 97.5 + 195 = 292.5 \text{ kNm}$$

$$\cancel{F^f_{*}} \quad F^2_{*f,\text{loads}} = \begin{Bmatrix} +292.5 \text{ kNm} \\ -292.5 \text{ kNm} \end{Bmatrix}$$

In global coordinates,

$$F_{f,\text{loads}} = \begin{Bmatrix} 0 \\ +292.5 \text{ kNm} \\ -292.5 \text{ kNm} \end{Bmatrix}$$

Element Stiffness Matrices

$$k^1_* = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \frac{EI}{4} \;\cancel{=}\; \cancel{k^3_*}$$

$$\Rightarrow k^1_* = 8437.5 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ kNm/rad.}$$

$$k^2_* = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \frac{EI}{6} = 5625 \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ kNm/rad.}$$

$$k^3_* = \frac{3EI_i}{(L)_i} = \frac{3 \times 16875}{4} = 12656.25 \text{ kNm/rad}$$

Displacement Transformation Matrix

$$\underset{5\times 1}{D_*} = \underset{(5\times 3)}{T_D}\ \underset{3\times 1}{D}$$

$$T_D = \begin{array}{c} \\ \left[\begin{array}{ccc} D_1 = 1 & D_2 = 1 & D_3 = 1 \\ -1/4 & 1 & 0 \\ -1/4 & 0 & 0 \\ 0 & 1 & 0 \\ -1/4 & 0 & 1 \\ -1/4 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \\ 1* \\ 2* \\ 1* \\ 2* \\ 1* \end{array} \begin{array}{l} \\ \left.\right\} (1) \\ \\ \left.\right\} (2) \\ \\ \to (3) \end{array}$$

$$= \begin{array}{c} \begin{array}{ccc} (1) & (2) & (3) \end{array} \\ \left[\begin{array}{ccc} -0.25 & 1 & 0 \\ -0.25 & 0 & 0 \\ 0 & 1 & 0 \\ -0.25 & 0 & 1 \\ -0.25 & 0 & 1 \end{array}\right] \end{array} \begin{array}{l} \\ 1* \quad (1) \\ 2* \\ 1* \quad (2) \\ 2* \\ 1* \quad (3) \end{array}$$

Unassembled stiffness matrix :

$$\tilde{k}'_* = \left[\begin{array}{ccc} \tilde{k}^1_* & & \\ & \tilde{k}^2_* & \\ & & \tilde{k}^3_* \end{array}\right] = \left[\begin{array}{cc|cc|c} 16875 & 8437.5 & 0 & 0 & 0 \\ 8437.5 & 16875 & 0 & 0 & 0 \\ \hline 0 & 0 & 11250 & 5625 & 0 \\ 0 & 0 & 5625 & 11250 & 0 \\ \hline 0 & 0 & 0 & 0 & k^3_* \end{array}\right]$$

$$= \left[\begin{array}{cc|cc|c} 16875 & 8437.5 & 0 & 0 & 0 \\ 8437.5 & 16875 & 0 & 0 & 0 \\ \hline 0 & 0 & 11250 & 5625 & 0 \\ 0 & 0 & 5625 & 11250 & 0 \\ \hline 0 & 0 & 0 & 0 & 12656.25 \end{array}\right]$$

Structure Stiffness Matrix

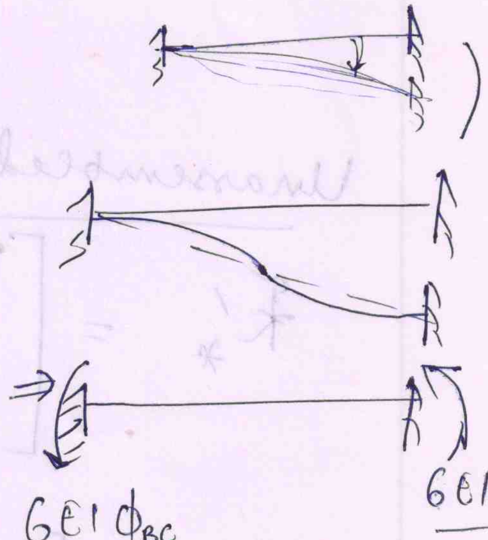
$$\tilde{k}_{AA} = T_D^T \, \tilde{k}_* \, T_D .$$

$$= \begin{bmatrix} 0.4658 & -0.77344 & -0.59786 \\ -0.77344 & 2.8125 & 0.5625 \\ -0.59786 & 0.5625 & 2.3906 \end{bmatrix} \times 10^4$$

* Fixed end forces in BC due to support settlement @ D of 10 mm. $\Rightarrow \Delta_D = \Delta_C = -10\,mm$.

ϕ_{BC} = Chord rotation of BC $= -\left(\frac{0.010}{6}\right)$ m/m

$$F^2_{*f,\,settlement} = \begin{Bmatrix} -\frac{6EI}{L}\phi_{BC} \\ -\frac{6EI}{L}\phi_{BC} \end{Bmatrix}$$



$$= \begin{Bmatrix} -\frac{6EI}{L}\phi_{BC} \\ -\frac{6EI}{L}\phi_{BC} \end{Bmatrix}$$

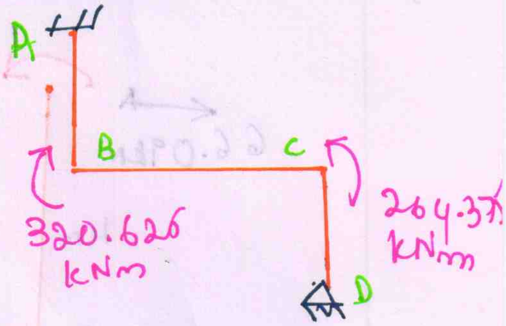
$\frac{6EI}{L}\phi_{BC}$ $\quad$ $\frac{6EI\phi}{L}$

$$-\frac{6EI}{L}\phi_{BC} = -\frac{6 \times 16875}{6} \times \left(-\frac{0.01}{6}\right)$$

$$= 28.125 \text{ kNm}$$

$$F^2_{*f,\,total} = \begin{Bmatrix} 292.5 \\ -292.5 \end{Bmatrix} + \begin{Bmatrix} 28.125 \\ 28.125 \end{Bmatrix} \text{ kNm}$$

$$\Rightarrow F_f = \begin{Bmatrix} 0 \\ 320.625\,kNm \\ -264.375\,kNm \end{Bmatrix}$$



$\therefore$ Net load vector $F_{net} = F - F_f = 0 - F_f$

$$F_{AA} = \begin{Bmatrix} 0 \\ -320.625\,kNm \\ +264.375\,kNm \end{Bmatrix}$$

Equilibrium Condition

$$F_A = k_{AA} D_A.$$

Active displacement vector.

$$D_A = inv(k_{AA})\, F_A$$

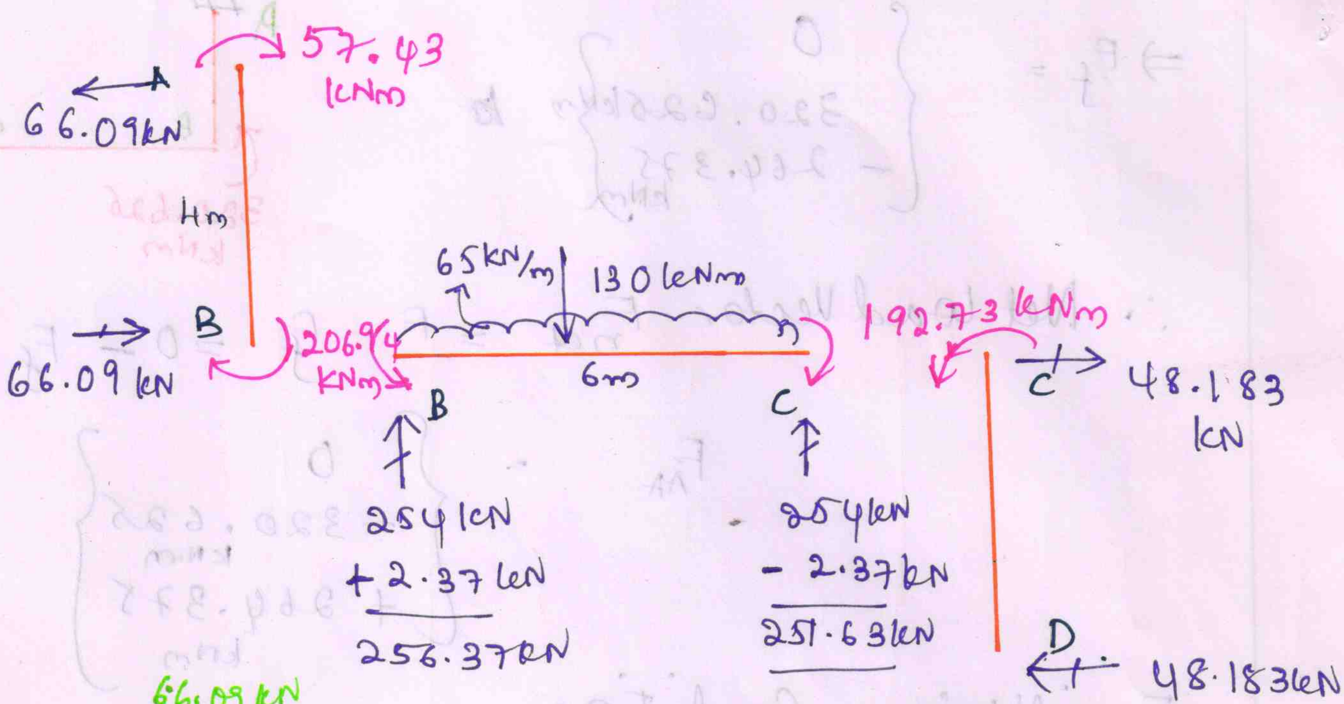
$$= \begin{Bmatrix} -1.455 \\ -1.772 \\ -1.159 \end{Bmatrix} \times 10^{-2} \Rightarrow \text{Sway} = -0.1455\ mm$$

$\Rightarrow$ to the left

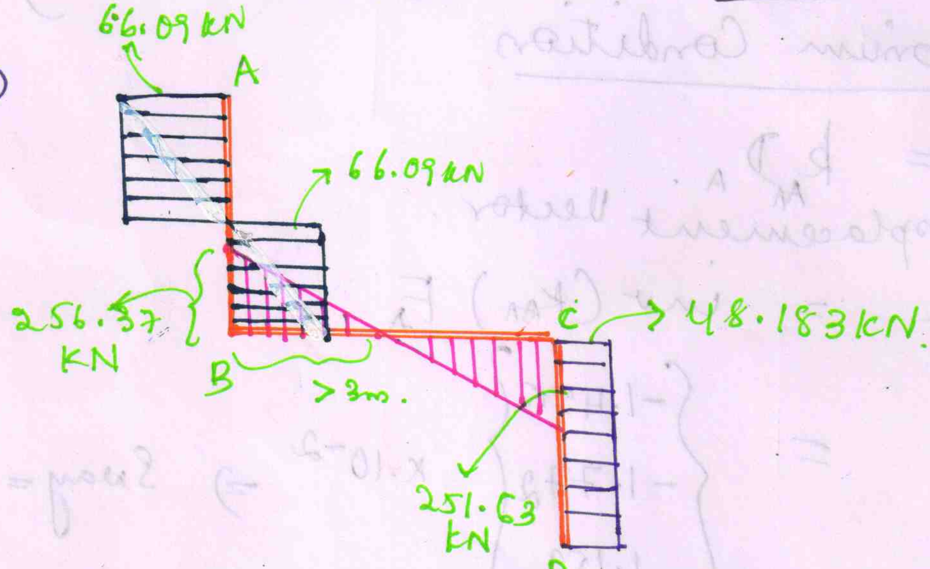
Member forces

$$F_* = F_{*f} + \tilde{k}_* T_{DA} D_A$$

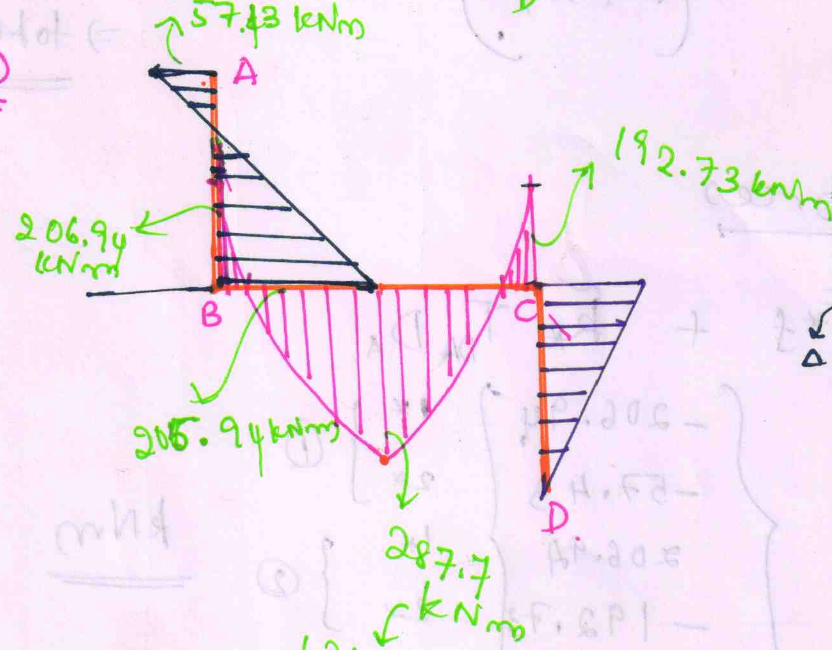
$$= \begin{Bmatrix} -206.94 \\ -57.43 \\ 206.94 \\ -192.73 \\ 192.73 \end{Bmatrix} \begin{matrix} 1* \\ 2* \\ 1* \\ 2* \\ 1* \end{matrix} \begin{matrix} \left.\right\} (1) \\ \\ \left.\right\} (2) \\ \\ (3) \end{matrix} \quad kNm$$

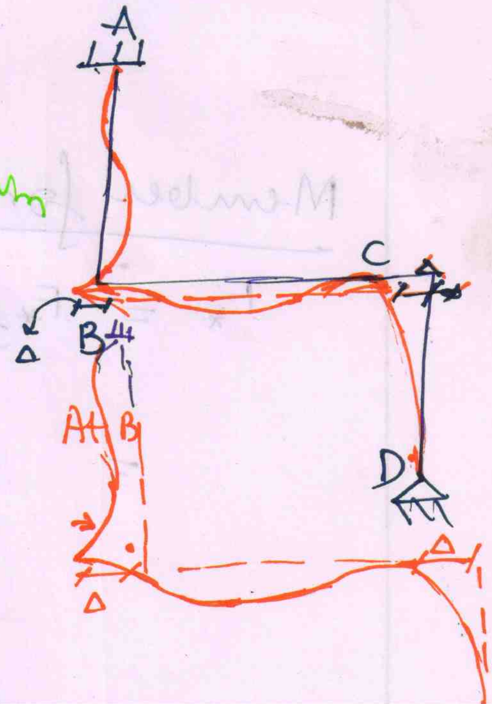


SFD



BMD





W = 130 kN

$\frac{WL}{4}$ + 94.8