Date of Sub
29-8-11

Advanced Structural Analysis.
TUTORIAL-1

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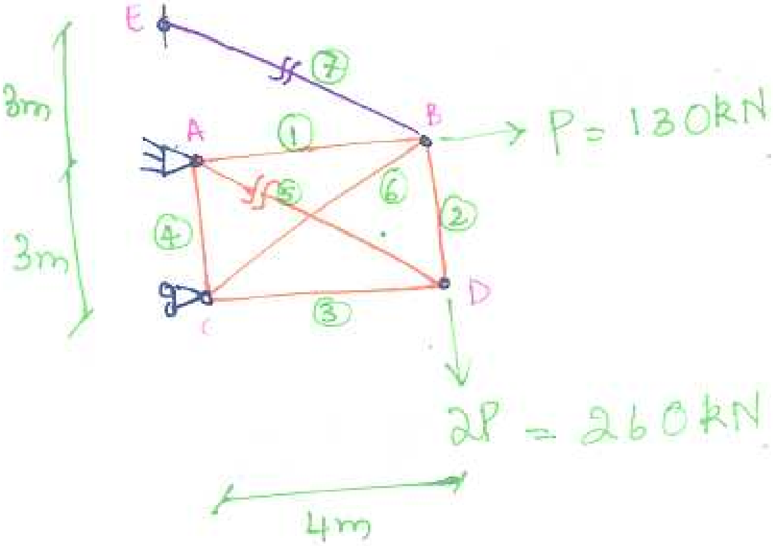
CE09B030.

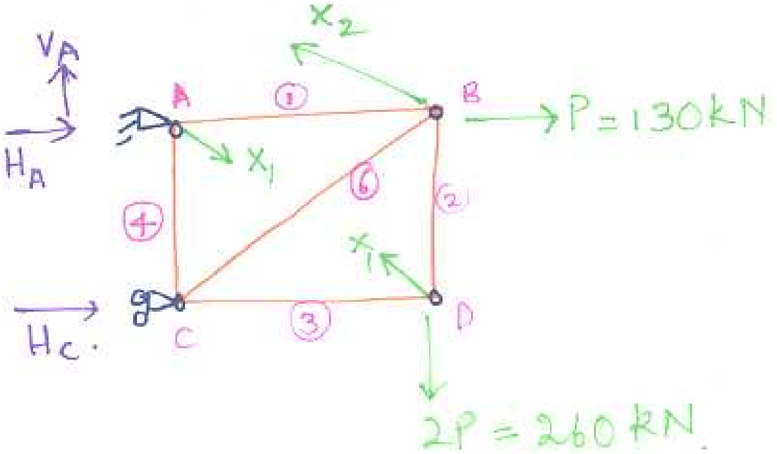
P= 130

②



$n_s = 2$

Choosing bar 5 and cable tension as redundants, $\Rightarrow N_5 = X_1$, $N_7 = X_2$, the primary structure becomes:



$H_A + H_C = P - X_2 \times \frac{4}{5}$

$(130 + \frac{4}{5} x_2)$

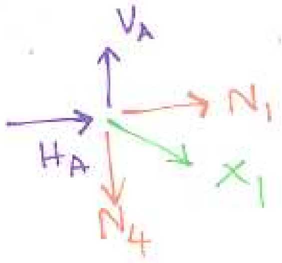
$V_A = (260 kN - \frac{3}{5} X_2)$

$\Sigma M_B = 0$

$\Rightarrow -4V_A + 3H_C = 0$

$H_C = \frac{4}{3}(260 - \frac{3x_2}{5})$

$\Rightarrow H_A = \frac{8}{5} x_2 - \frac{1430}{3}$



At A :

$H_A + N_1 + x_1 \times \frac{4}{5} = 0$

$$\boxed{N_1 = \frac{1430}{3} - \frac{8}{5} x_2 - \frac{4}{5} x_1}$$

$V_A - N_4 - x_1 \times \frac{3}{5} = 0$

$$\boxed{N_4 = 260 - \frac{3}{5} x_2 - \frac{3x}{5}}$$

$$\boxed{V_A = 260 - \frac{3}{5} x_2} ;$$

$$\boxed{H_A = \frac{8}{5} x_2 - \frac{1430}{3}}$$

$$\boxed{H_C = \frac{1040}{3} - \frac{4}{5} x_2}$$

$\sum\left(\frac{\partial N_i}{\partial X_2}\right) N_i f_i EA = 0$

$\Rightarrow \left(\frac{128}{25}+\frac{27}{25}+5\right)X_1 + \left(\frac{256}{25}+\frac{27}{25}+7.5\right)X_2 = \frac{9152}{3}+468+\frac{6500}{3}$

~~$10.48 X_1 + 18.1 X_2 = 5373.33$~~.

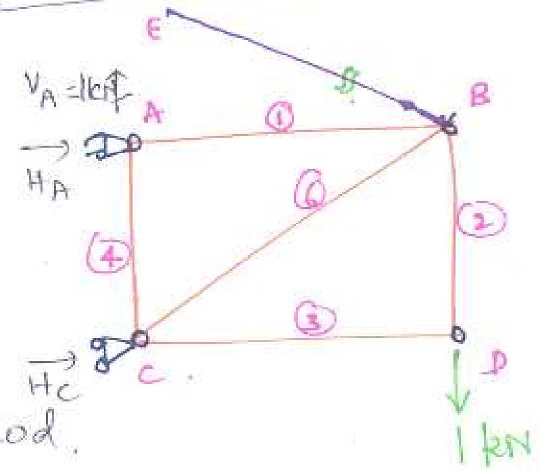
~~$X_1 = 123.3$~~

$X_2 = 225.48$

$\Rightarrow \frac{278}{25}X_1 + \frac{941}{50}X_2 = \cancel{8}\,\frac{17056}{3}$ — ②

$$\boxed{X_1 = 116.73 \text{ kN} \quad X_2 = 233.12 \text{ kN}}$$

In the primary structure, using Unit Load Method.



$\Rightarrow$ At D, $\boxed{n_2 = 1}$; $\boxed{n_3 = 0}$

At C :

$H_C + n_6 \times \frac{4}{5} = 0$

$n_4 + \frac{3}{5} n_6 = 0$

$n_4 = -\frac{3}{5} n_6$

$\boxed{n_4 = 1}$

At . B.

$n_1 + n_6\left(\frac{4}{5}\right) = 0$

$n_6 \times \frac{3}{5} + 1 = 0$

$\boxed{n_6 = -\frac{5}{3}}$

$\boxed{n_1 = +\frac{4}{3}}$

Bar No. i	$f_i = \frac{L_i}{EA_i}$ m/kN	N_i kN	n_i kN/kN	$n_i N_i f_i = n_i N_i f_i$ (m)
1	$\frac{4}{EA}$	10.29	$\frac{4}{3}$	$\frac{54.88}{EA}$
2	$\frac{3}{EA}$	189.96	1	$\frac{569.88}{EA}$
3	$\frac{4}{EA}$	-93.38	0	0
4	$\frac{3}{EA}$	50.09	1	$\frac{150.27}{EA}$
5	$\frac{5}{EA}$	116.73	0	0
6	$\frac{5}{EA}$	-83.48	$-\frac{5}{3}$	$\frac{695.67}{EA}$
7	$\frac{5}{2EA}$	233.12	0	0

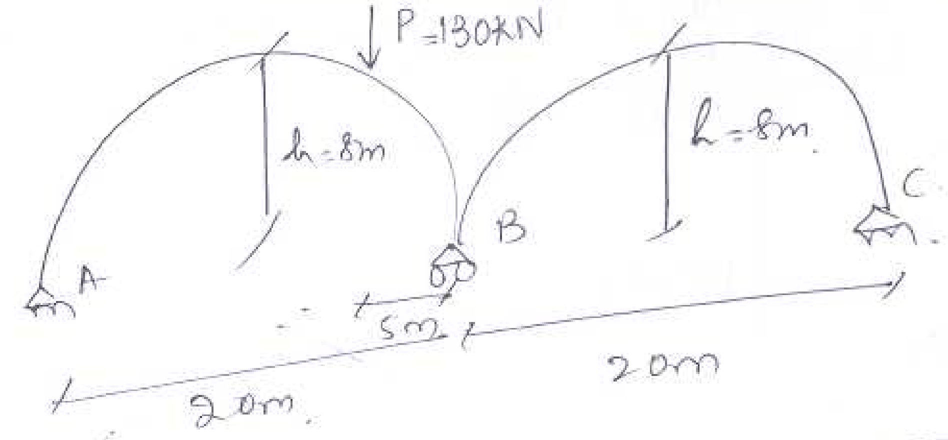
$$D_D = \sum n_i N_i f_i = \frac{1415.82}{EA} \text{ m.}$$

$$\Rightarrow 12 \times 10^{-3} = \frac{1415.82}{EA}$$

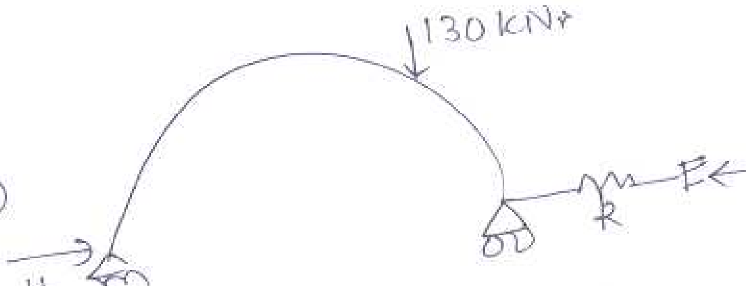
$$\Rightarrow EA = 117.985 \times 10^{3} \text{ kN}$$

$$\Rightarrow \boxed{EA = 1.18 \times 10^{5} \text{ kN}}$$ Ans.

Q①



$\Rightarrow$



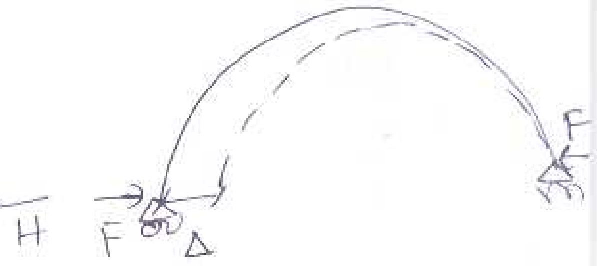


Fig 1

Fig 2.

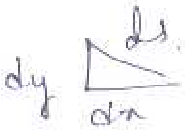
We can separate the arches by placing a translational spring of stiffness k in the place of unloaded arch. k can be found from unloaded arch.

In Fig 2. By Castigliano's Theorem II

$$U^* - F\Delta = 0 \Rightarrow \frac{dU^*}{dF} = \Delta$$

$$\Rightarrow \Delta = \frac{d}{dF}\frac{1}{2}\int_0^{S}\frac{M^2(s)\,ds}{EI(s)} = \frac{d}{dF}\int_0^{S/2}\frac{M^2(s)\,ds}{EI(s)}$$

$$= 2\int_0^{S/2}\frac{M(s)}{EI(s)}\frac{dM}{dF}\,ds$$



But $ds = dx\sqrt{1+(y')^2}$.

$$\Rightarrow \Delta = 2\int_0^{L/2}\frac{M(x)}{EI(x)}\frac{dM(x)}{dF}\,dx\sqrt{1+(y')^2}$$

$I(x) = I_0\sqrt{1+(y')^2}$

0 ∵ there is no loading on free beam

$$\Rightarrow \Delta = \frac{2}{EI_0}\int_0^{L/2}\left(\cancel{M^0(x)} - Hy(x)\right)\left(-y(x)\right)dx$$

$$\Rightarrow \quad \Delta = \frac{2}{EI_0} \int_0^{L/2} F \; y^2(x)\, dx .$$

∵ Parabolic Arch

$$\Delta = \frac{2}{EI_0} F \int_0^{L/2} y^2(x)\, dx = \frac{2F}{EI_0} \left(\frac{4h^2L}{15} \right).$$

$$\Delta = \left(\frac{8h^2L}{15EI_0} \right) F$$

$$\Rightarrow \boxed{k = \frac{15EI_0}{8h^2L}} \quad \text{—} \; ①$$

In fig 1,

$$U^* = U^*_{arch} + U^*_{spring}$$

$$U^* = \frac{1}{2} \times 2 \int_0^{L/2} \frac{M^2(x)\, dx \cancel{\sqrt{1+(y')^2}}}{E\, I_0 \cancel{\sqrt{1+(y')^2}}} + \frac{H^2}{2k} = \frac{1}{EI_0} \int_0^{L/2} (M^0(x) - Hy(x))^2 dx + \frac{H^2}{2k}$$

$$\frac{dU^*}{dH} = \frac{1}{EI_0} \int_0^{L/2} 2(M^0(x) - Hy(x))(-y(x))\, dx + \frac{H}{k} = 0.$$

(By Theorem of Least Work – by choosing H & as redundant.)

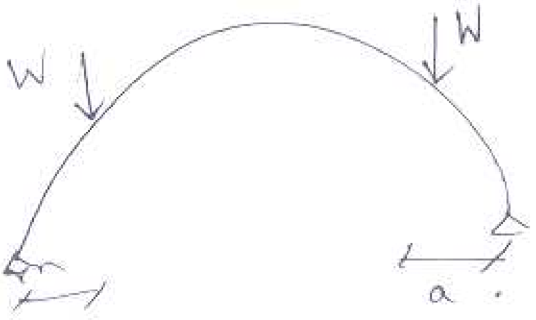
$$\Rightarrow \quad H \left[\frac{EI_0}{k} + 2 \int_0^{L/2} y^2(x)\, dx \right] = 2 \int_0^{L/2} M^0(x)\, y(x)\, dx$$

$$H = \frac{2 \int_0^{L/2} M^0(x)\, y(x)\, dx}{\left[\frac{8h^2L}{15} + \frac{8h^2L}{15} \right]}$$

Substituting from ① $k = \frac{15EI_0}{8h^2L}$

$$\boxed{H = \frac{15}{8h^2L} \int_0^{L/2} M^0(x)\, y(x)\, dx .}$$

H Due to eccentric Load is half of corres. symmetric Load.



W = 130kN
a = 5m
L = 20m.
h = 8m.

$$\Rightarrow \int_0^{42} M^\circ(x)\,y(x)\,dx = \underbrace{\frac{Wha}{3L^2}\left[L^3 - 2La^2 + a^3\right]}_{\text{for symmetric}} \underbrace{\boxed{\times\frac{1}{2}}}_{\text{for eccentric}}$$

$$\Rightarrow H_A = \frac{15}{8}$$

$$\Rightarrow H = \frac{15}{8h^2L} \times \frac{Wha}{3L^2}\left[L^3 - 2La^2 + a^3\right] \times \frac{1}{2}$$

$$= \frac{15 \times 130 \times 5}{8 \times 8 \times 3}\left[1 - 2\left(\frac{a}{L}\right)^2 + \left(\frac{a}{L}\right)^3\right] \times \frac{1}{2}$$

$$= \frac{15 \times 130 \times 5 \times}{64 \times 3}\left(1 - 2 \times \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3\right) \times \frac{1}{2}$$

$$\boxed{= \frac{45.23\text{kN}}{2} \quad \boxed{= 22.614\text{kN}}}\ \text{Ans.}$$

Normal Thrust (Axial compression), Shear force.

$$\begin{Bmatrix} N(x) \\ S(x) \end{Bmatrix} = \begin{pmatrix} 1 & y' \\ -y' & 1 \end{pmatrix} \sqrt{1+(y')^2} \begin{Bmatrix} H \\ V(x) \end{Bmatrix}$$

$$N(x) = \left(H + y'V(x)\right)\sqrt{1+(y')^2}$$

$$S(x) = \left(-y'H + V(x)\right)\sqrt{1+(y')^2}$$

$$M(x) = M^\circ(x) - Hy(x)$$

$$y(x) = \frac{4h}{L^2}x(L-x) = 0.08\ (20x - x^2)$$

$$= 1.6x - 0.08x^2$$

$y'(x) = 0.08(20-2x) = 1.6 - 0.16x$; $H = 22.614$ kN

S.No.	x m	$V(x)$ kN	$y(x)$ $=1.6x$ $-0.08x^2$ m	$y'(x)$ $=1.6-$ $0.16x$	$\sqrt{1+(y')^2}$	$N(x)$ $=\frac{1}{\sqrt{1+(y')^2}} \times (H+y'V(x))$ kN	$S(x)$ $=\frac{1}{\sqrt{1+(y')^2}}(-y'H+V(x))$ kN	$\overset{\circ}{M}(x)$ kNm	$M(x)$ $=\overset{\circ}{M}(x)-Hy(x)$ kN
1.	0	32.5	0	1.6	1.887	39.54	−1.951	0	0
2	2.5	32.5	3.5	1.2	1.562	39.446	3.433	81.25	2.101
3	5	32.5	6	0.8	1.281	37.950	11.248	162.5	26.816
4	7.5	32.5	7.5	0.4	1.077	33.068	21.778	243.75	74.145
5	10	32.5	8	0	1	22.614	32.500	325	144.088
6	12.5	32.5	7.5	−0.4	1.077	8.927	38.575	406.25	236.645
7	15^-	32.5	6	−0.8	1.281	−2.643	39.494	487.5	351.816
8	15^+	−97.5	6	−0.8	1.281	78.500	−61.990	487.5	351.816
9	17.5	−97.5	3.5	−1.2	1.562	89.380	−45.047	243.75	164.601
10	20	−97.5	0	−1.6	1.887	94.655	−32.495	0	0

Max N(x) = 94.655 kN
x=20

Max |S(x)| = 61.990 kN

Max M(x) = 351.81 kNm

→ at x = 15^+

Answers

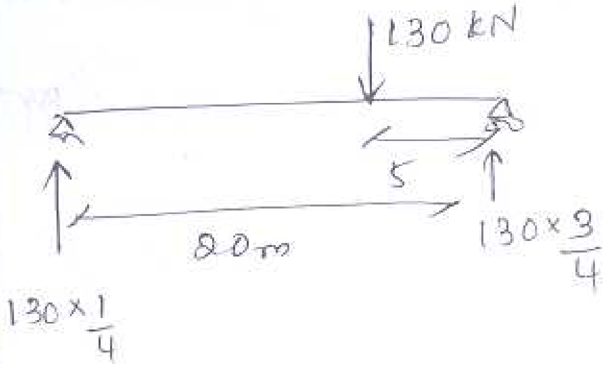
Ans

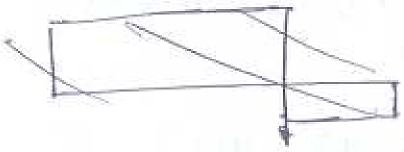
Max N(x) (thrust) = 94.655 kN → x = 20 m

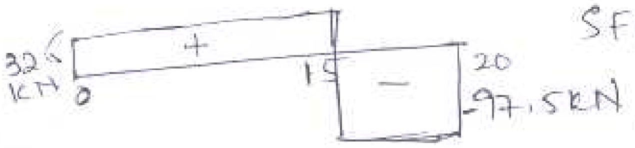
Max |S(x)| Shear force = 61.990 kN.

Max BM → 351.816 kN

61.990 kN and 351.816 kN — at x = 15^+ m







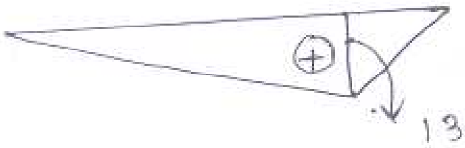
SFD $V(x) = 32.5\,kN \quad 0 < x < 15m$

$= -97.5 kN \quad 15 < x < 20m$

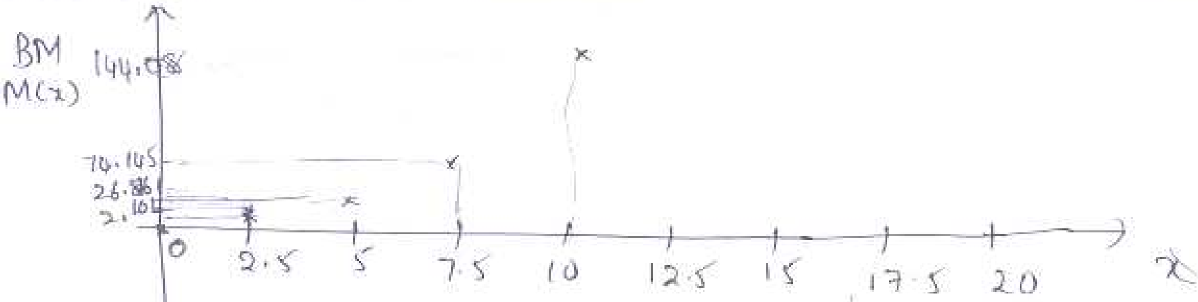
$M^{\circ}(x) = 32.5x \quad 0 < x < 15m$

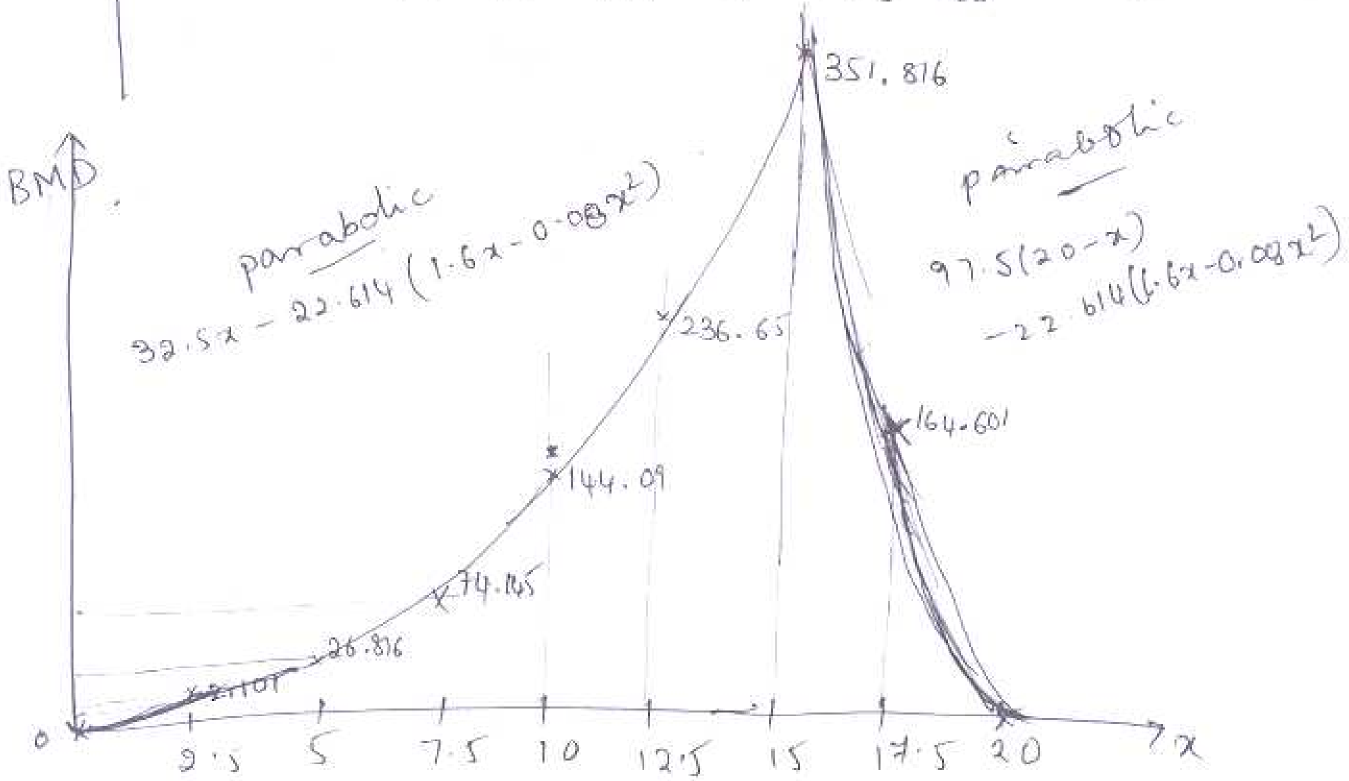
$= 97.5(20-x)$

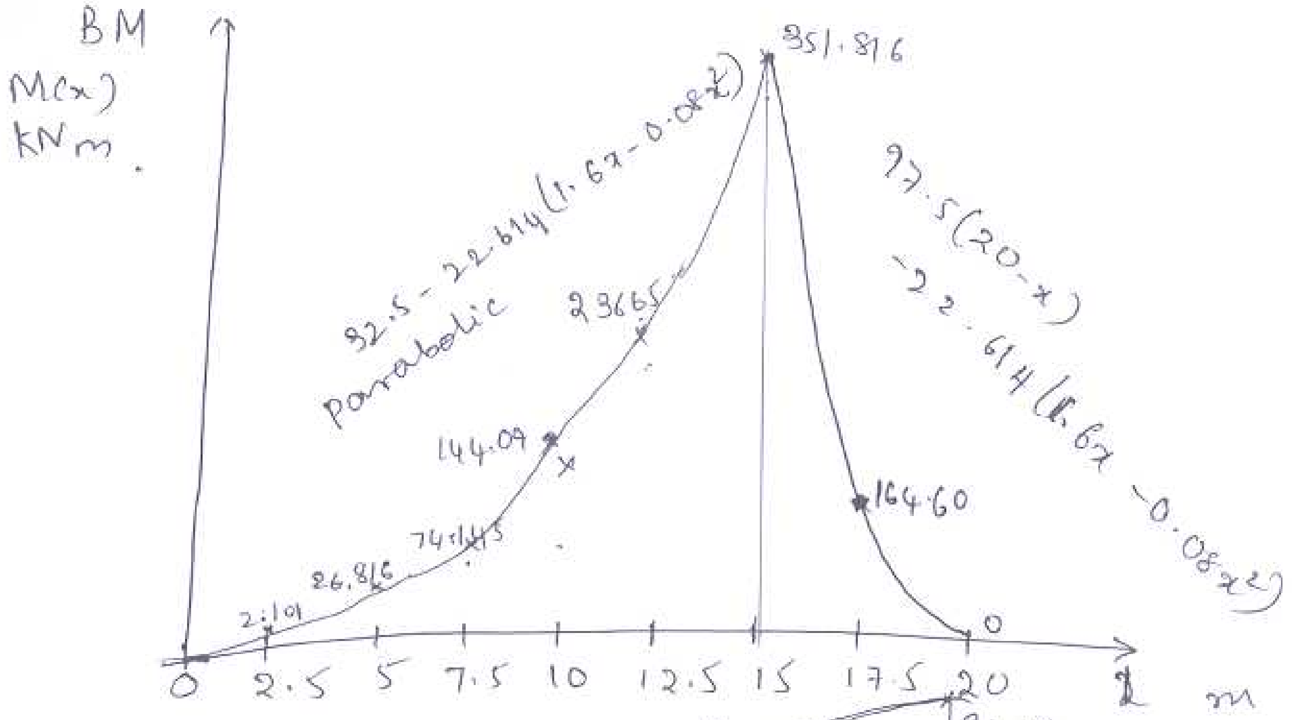
$15 < x < 20m$

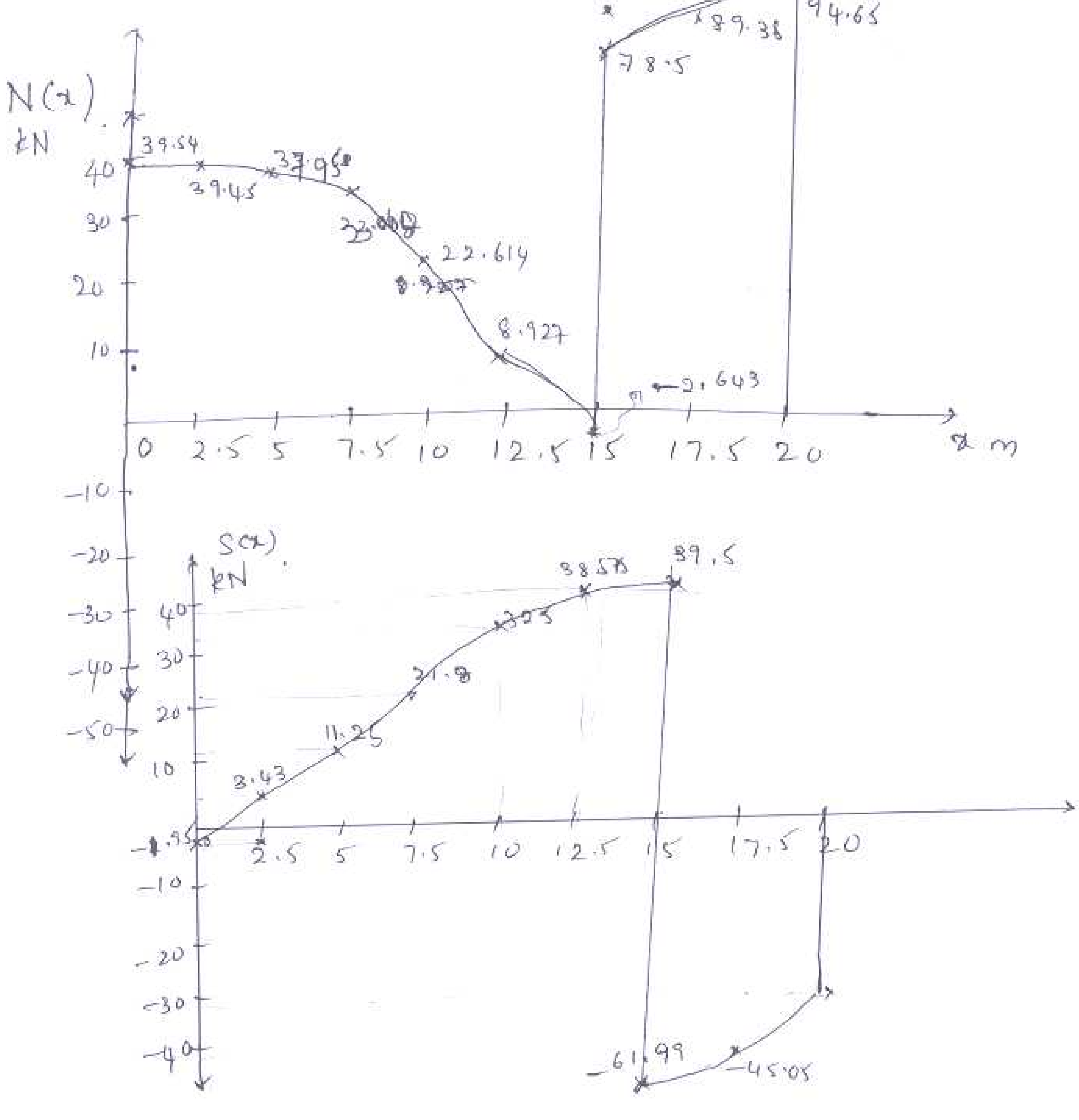


$130 \times \frac{1}{4} \times 15 = 487.5 kNm$






BM
M(x)
kN m.
351.816
32.5 - 22.614(1.6x-0.08x²)
parabolic
296.65
144.09
74.145
26.816
2.10
164.60
0
97.5(20-x)
-22.614(1.6x-0.08x²)
0
2.5
5
7.5
10
12.5
15
17.5
20
x m


N(x)
kN
39.54
39.45
37.958
33.009
22.614
8.927
8.927
0
2.643
78.5
89.38
94.63
40
30
20
10
-10
-20
-30
-40
-50
0
2.5
5
7.5
10
12.5
15
17.5
20
x m
S(x)
kN.
-1.95
3.43
11.25
21.8
30.5
38.578
39.5
-61.99
-45.05
40
30
20
10
-10
-20
-30
-40
0
2.5
5
7.5
10
12.5
15
17.5
20

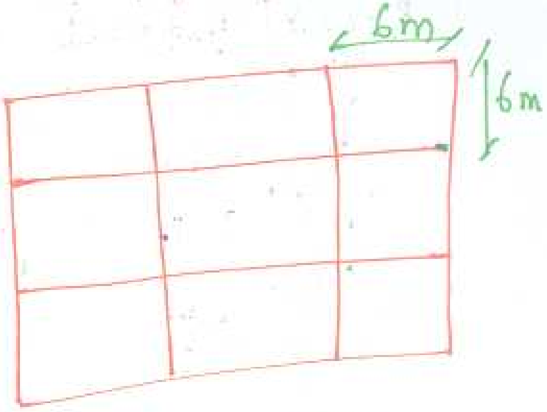
3. No. of frames = 4 = 4

Degree of static indeterminacy

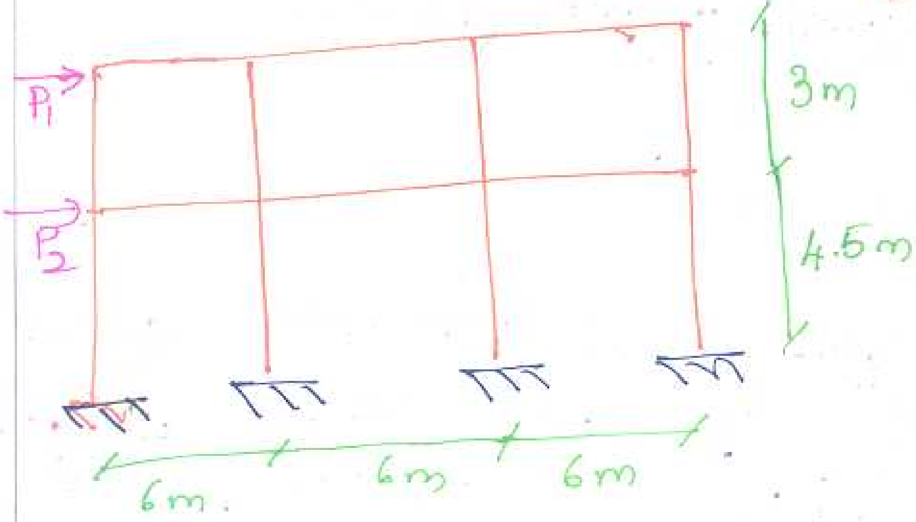
$n_s = 3 \times n_{bay} \times n_{storey}$

$= 3 \times 3 \times 2$

$= 18$



18m × 18m
PLAN



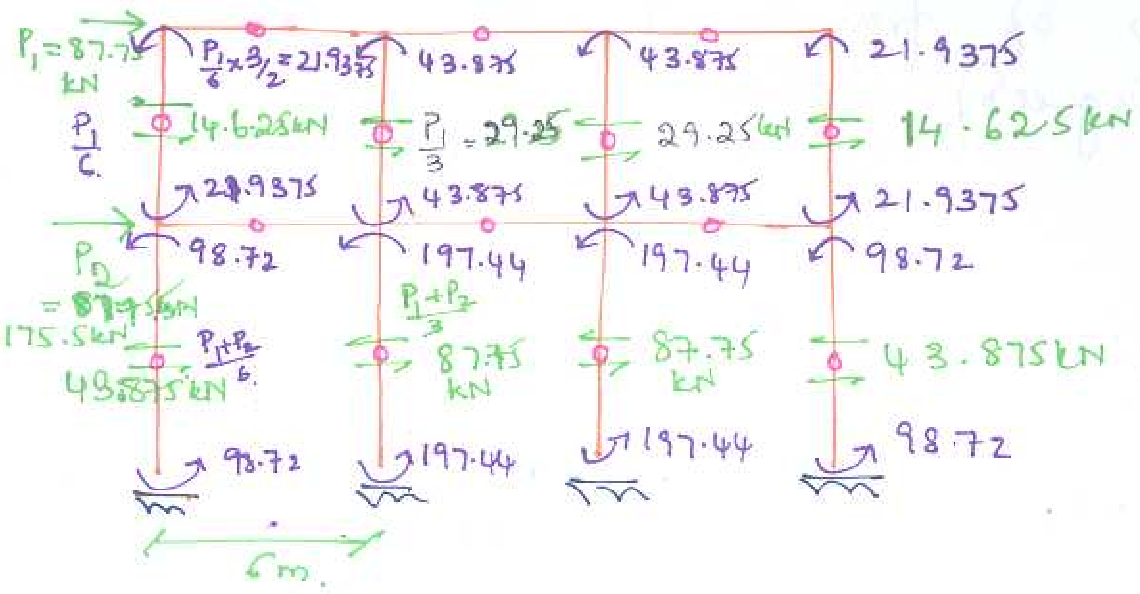
Portal Method.

Load per frame at roof level =

$$P_1 = \frac{0.1 \times 130 \times 18 \times 3.0/2}{4} = 87.75\text{kN}$$

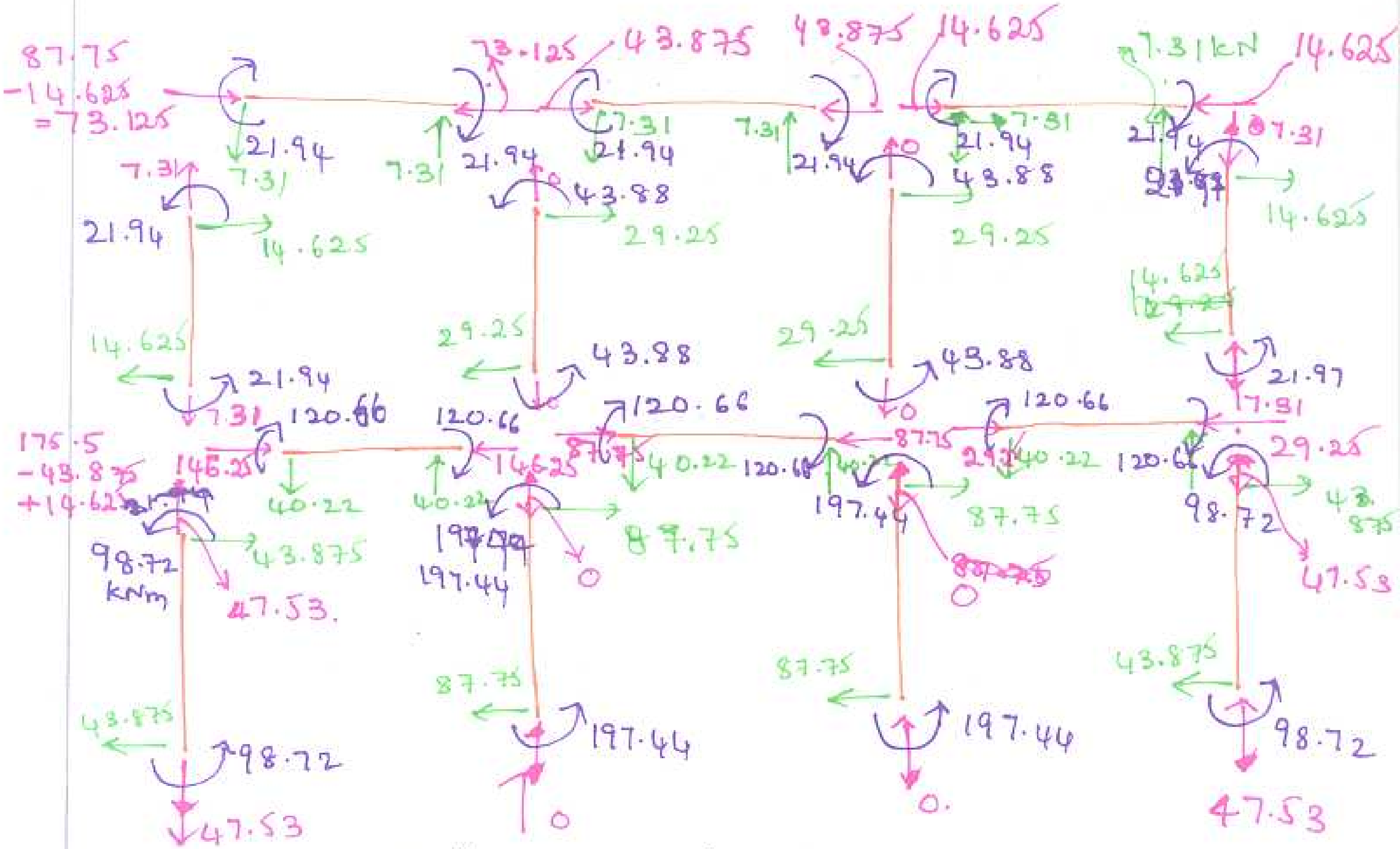
(∵ 0.1P is intensity)

Load per frame at other level = $P_2 = 87.75 \times 2$
$= 175.5$ kN

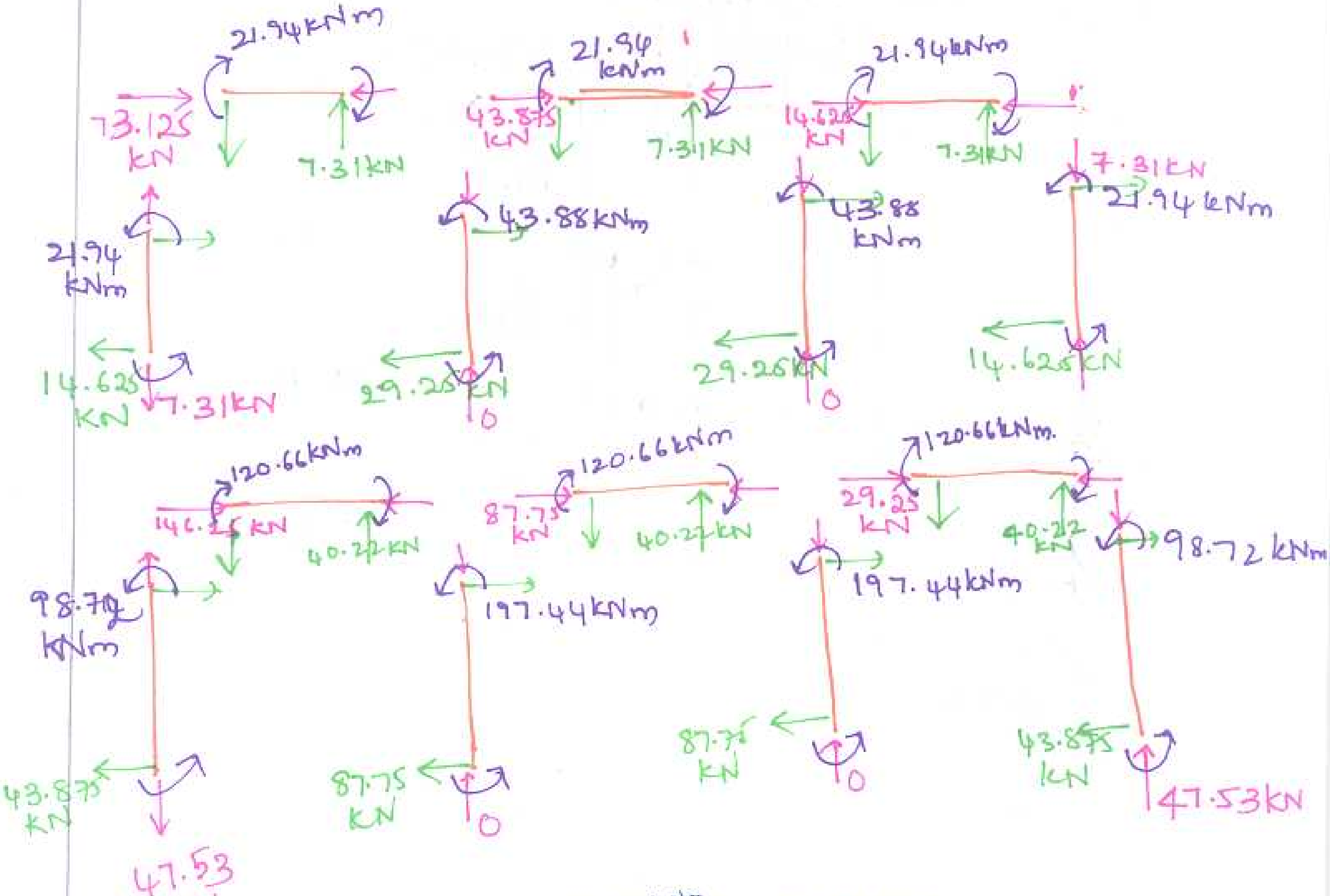


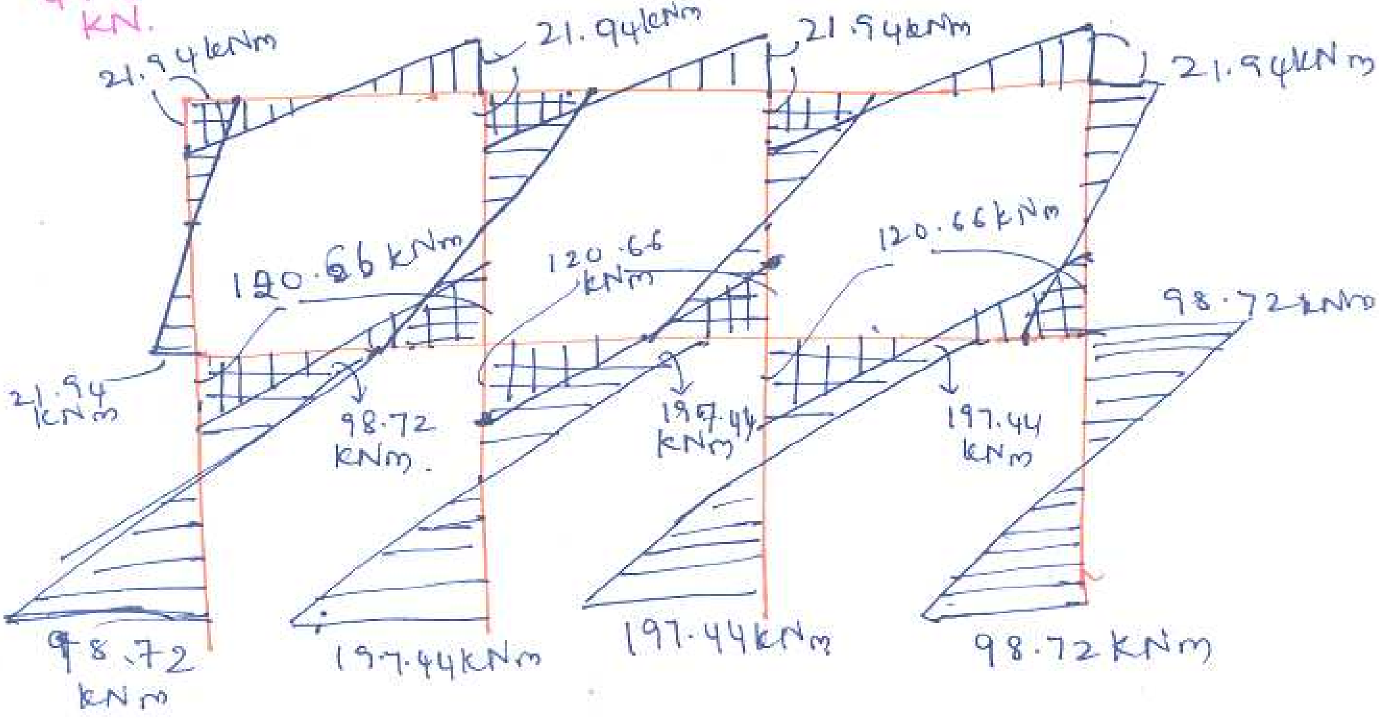
$x_1 \times 2 + 2x_1 \times 2 = 87.75$

$x_1 = 87.75$



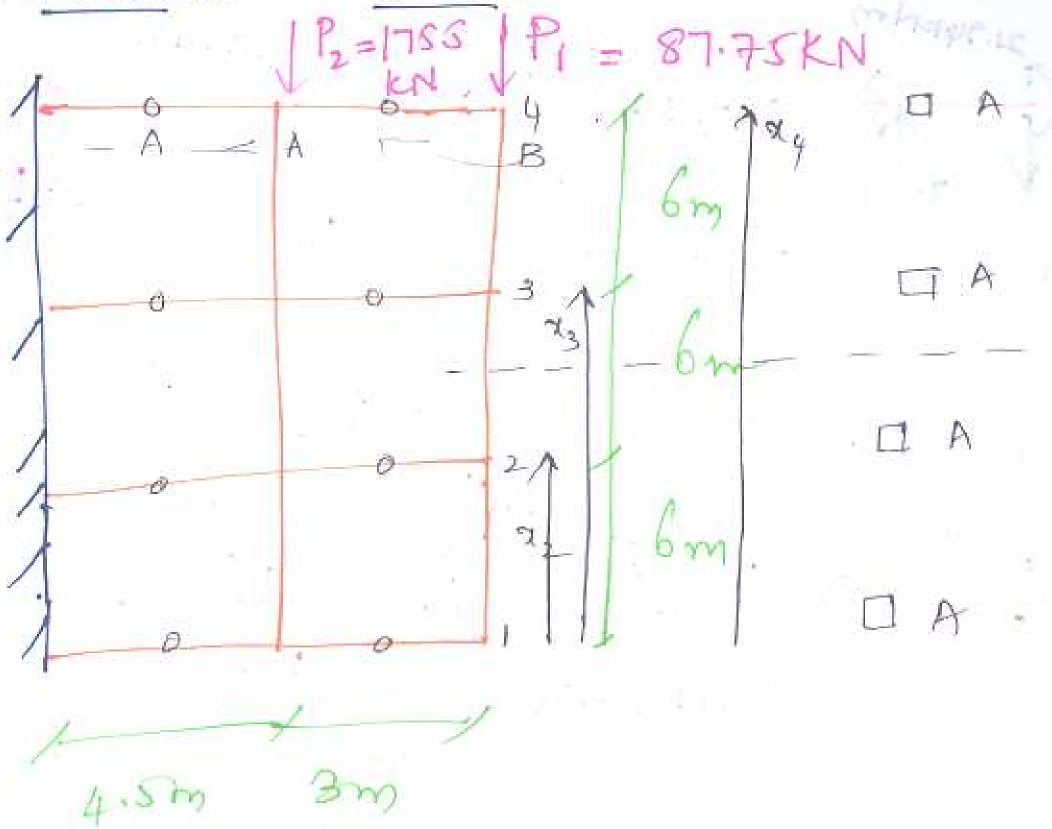
$$N_{1,1} = -P_1\left(1 - \frac{1}{2n_{bay}} - \frac{i-1}{n_{bay}}\right)$$


21.94kNm
73.125 kN
7.31kN
21.94 kNm
43.875 kN
7.31kN
21.94kNm
14.625 kN
7.31kN
7.31kN
21.94kNm
21.94 kNm
43.88kNm
43.88 kNm
14.625 kN
7.31kN
29.25 kN
0
29.26kN
0
14.625kN
120.66kNm
146.25 kN
40.27 kN
120.66kNm
87.75 kN
40.27 kN
120.66kNm
29.25 kN
40.27 kN
98.72kNm
98.72 kNm
197.44kNm
197.44kNm
43.875 kN
87.75 kN
0
87.75 kN
0
43.875 kN
47.53kN
47.53 kN.


21.94kNm
21.94kNm
21.94kNm
21.94kNm
120.66kNm
120.66 kNm
120.66kNm
98.72kNm
21.94 kNm
98.72 kNm.
197.44 kNm
197.44 kNm
98.72 kNm
197.44kNm
197.44kNm
98.72kNm

BMD

CANTILEVER METHOD:



$$\bar{x} = \frac{0\times A + 6\times A + 12\times A + 18A}{4A}$$

$= 9$ m

$$y_1 = \frac{(9-0)^2 + (9-6)^2 + (9-12)^2 + (9-18)^2}{(9-0)}$$

$$= \frac{2\times(81+9)}{9} = \frac{180}{9} = 20 \text{ m}$$

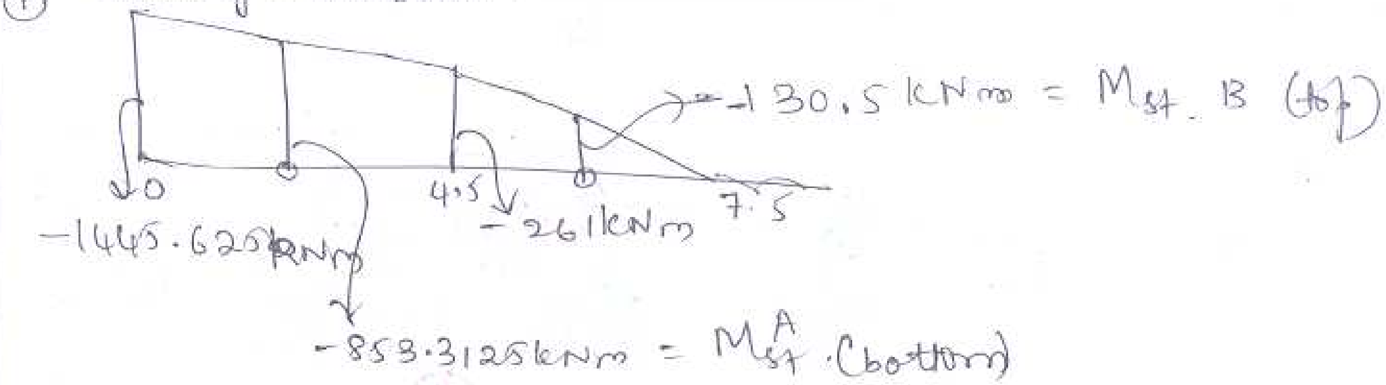
$y_1 = 20$ m ; $y_4 = -20$ m

$$y_2 = \frac{2\times 90}{(9-6)} = \quad = \frac{180}{3} = 60 \text{ m}$$

$= 60$ m

$y_3 = -60$m.

① Storey moments.



Column Axial forces

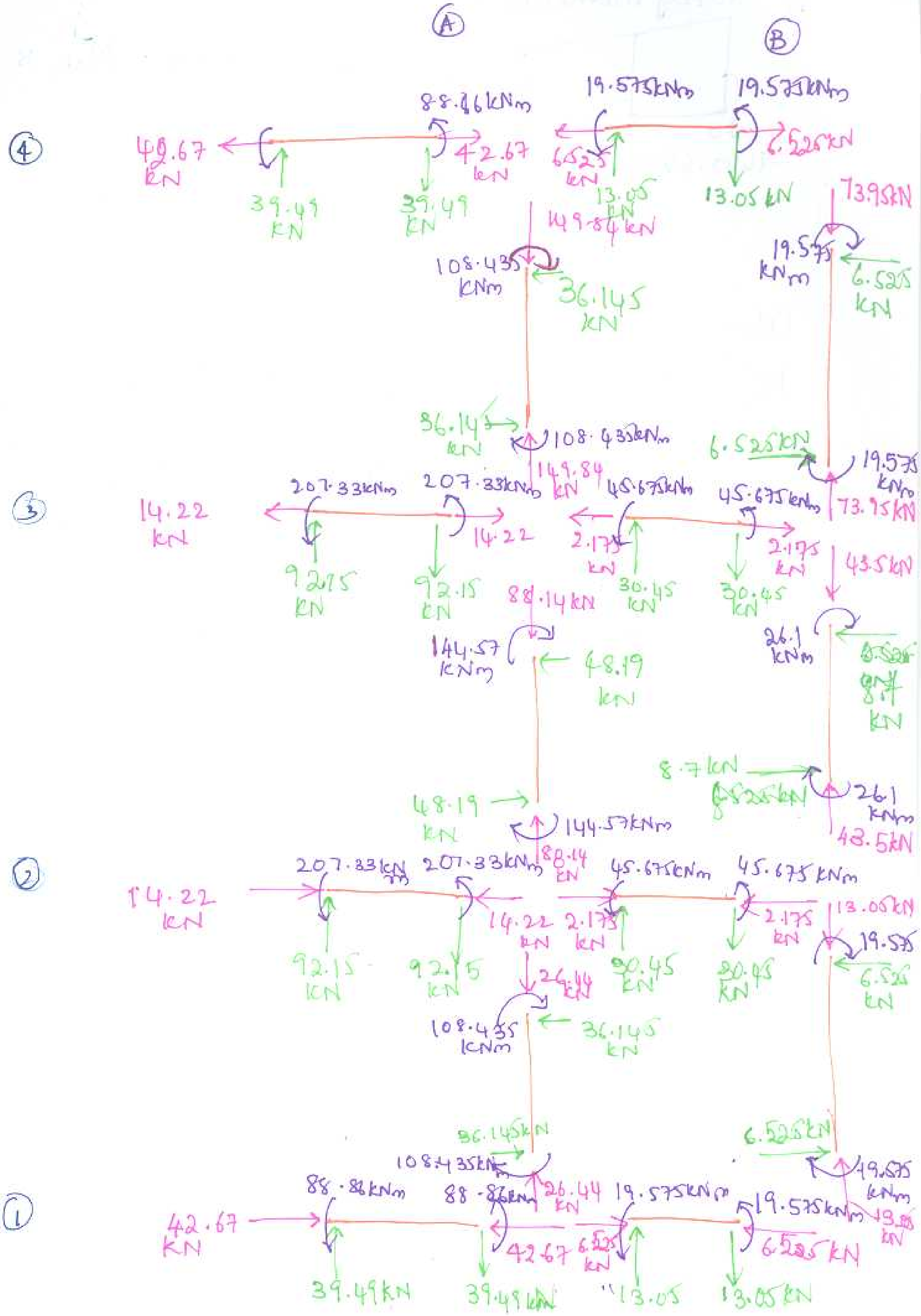
$N^{col}_{B,1} = \frac{M^{st}_{B}}{y1} = \frac{-130.5}{20} = -6.525kN$; $N^{col}_{A,1} = \frac{M^{st}_{A}}{y1} = \frac{-853.31}{20}$

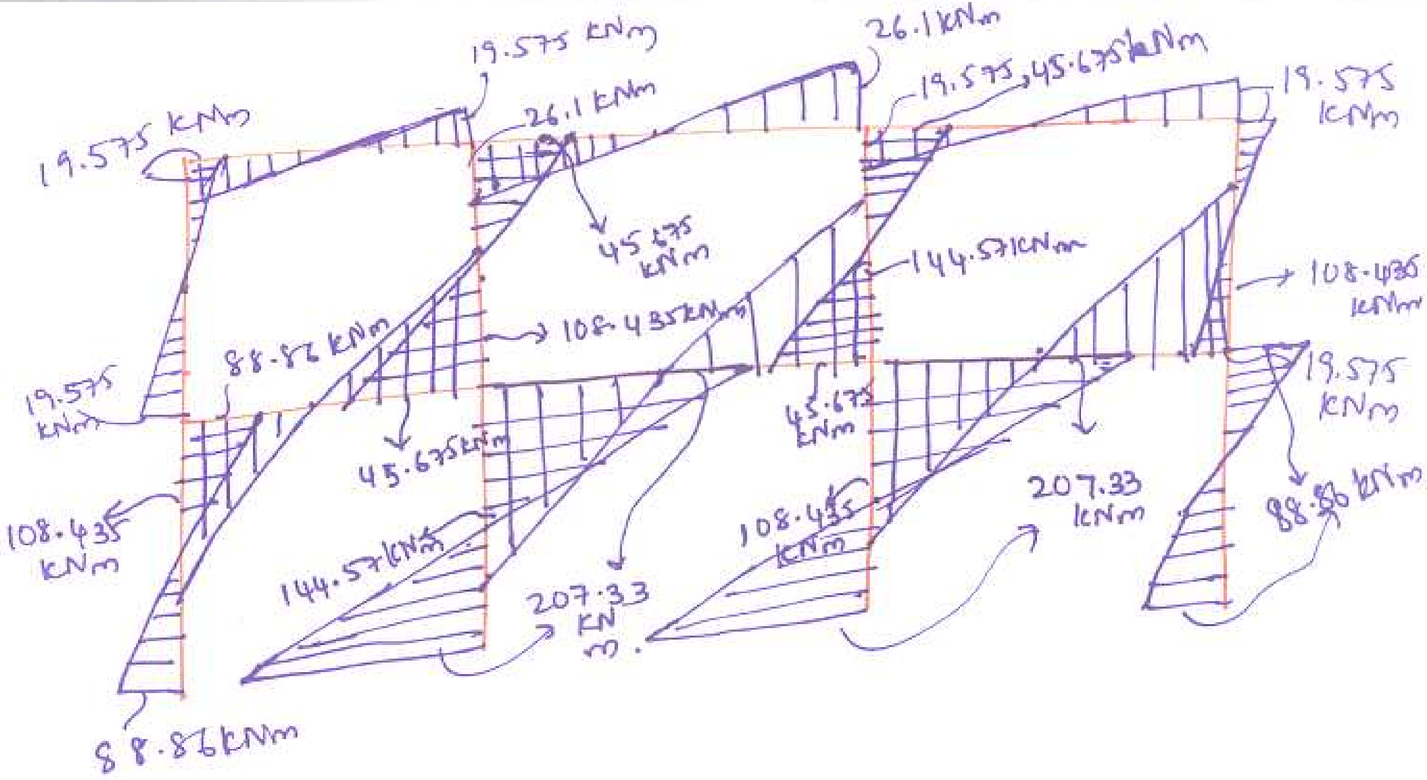
$= -42.67kN$

$N^{col}_{B,2} = \frac{-130.5}{60} = -2.175kN$ $\qquad$ $N^{col}_{A,2} = \frac{-853.31}{60} = -14.22kN$

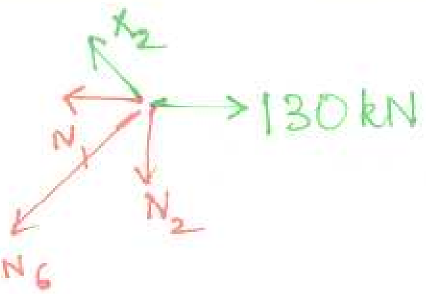
$N^{col}_{B,3} = +2.175kN$ $\qquad$ $N^{col}_{A,3} = +14.22kN$

$N^{col.}_{B,4} = 6.525kN$ $\qquad$ $N^{col}_{A,4} = +42.67kN$


A
B
4
88.86kNm
19.575kNm
19.575kNm
42.67 kN
42.67 kN
6.525 kN
6.525kN
39.49 kN
39.49 kN
13.05 kN
13.05 kN
149.84 kN
73.95kN
108.435 kNm
36.145 kN
19.575 kNm
6.525 kN
36.145 kN
108.435kNm
149.84 kN
6.525kN
19.575 kNm
73.95 kN
3
207.33kNm
207.33kNm
45.675kNm
45.675 kNm
14.22 kN
14.22
2.175 kN
2.175 kN
92.75 kN
92.15 kN
30.45 kN
30.45 kN
43.5kN
88.14kN
144.57 kNm
48.19 kN
26.1 kNm
~~6.525~~ 8.7 kN
8.7 kN
~~6.525kN~~
26.1 kNm
43.5kN
48.19 kN
144.57kNm
88.14 kN
2
207.33kNm
207.33kNm
45.675kNm
45.675 kNm
14.22 kN
14.22 kN
2.175 kN
2.175 kN
13.05kN
19.575
6.525 kN
92.15 kN
92.15 kN
30.45 kN
30.45 kN
26.44 kN
108.435 kNm
36.145 kN
36.145kN
108.435kNm
6.525kN
19.575 kNm
26.44 kN
13.05 kN
1
88.86kNm
88.86kNm
19.575kNm
19.575kNm
42.67 kN
42.67
6.525 kN
6.525 kN
39.49kN
39.49kN
13.05
13.05 kN


19.575 KNm
19.575 KNm
26.1 KNm
26.1 KNm
19.575, 45.675 kNm
19.575 KNm
45.675 KNm
144.57 KNm
108.435 KNm
108.435 KNm
19.575 KNm
88.86 KNm
45.675 KNm
45.675 KNm
19.575 KNm
108.435 KNm
144.57 KNm
207.33 KN m.
108.435 KNm
207.33 KNm
88.86 KNm
88.86 KNm

At B.



$$N_1 + N_6 \frac{4}{5} + x_2 \times \frac{4}{5} = 130. \Rightarrow \frac{4}{5} N_6 = 130 - \frac{1430}{3} + \frac{8}{5} x_2 + \frac{4}{5} x_1 - \frac{4}{5} x_2$$

$$\boxed{N_6 = \frac{5}{4}\left(-\frac{1040}{3} + \frac{4}{5} x_2 + \frac{4}{5} x_1\right)}$$

$$N_6 \times \frac{3}{5} + N_2 \cancel{\times} = x_2 \times \frac{3}{5}$$

$$\Rightarrow N_2 = \frac{3}{5} x_2 - \frac{3}{\cancel{5}}\left(-\frac{1040}{3} + \frac{4}{5} x_2 + \frac{4}{5} x_1\right) \times \frac{5}{4}$$

$$= \frac{3}{5} x_2 + \frac{1040}{4} - \frac{3}{5} x_2 - \frac{3}{5} x_1$$

$$= \frac{1040}{4} - \frac{3}{5} x_1$$

$$N_6 = -\frac{260 \times 5}{3} + x_1 + x_2 = -\frac{1300}{3} + x_1 + x_2$$

$$\Rightarrow \boxed{\begin{aligned} N_6 &= x_1 + x_2 - \frac{1300}{3} \\ N_2 &= 260 - \frac{3}{5} x_1 \end{aligned}}$$

At. D.

$$N_2 + x_1 \frac{3}{5} = 260. \Rightarrow N_2 = 260 - \frac{3 x_1}{5}$$

$$N_3 + x_1 \times \frac{4}{5} = 0$$

$$\boxed{N_3 = -\frac{4}{5} x_1}$$

Bar	Barforces N_i	$f_i = \frac{L_i}{EA_i}$	$\frac{\partial N_i}{\partial x_1}$	$\frac{\partial N_i}{\partial x_2}$	$f_i EA N_i \frac{\partial N_i}{\partial x_1}$	$f_i EA N_i \frac{\partial N_i}{\partial x_2}$
1	$\frac{1430}{3} - \frac{8}{5}x_2 - \frac{4}{5}x_1$	$\frac{4}{EA}$	$-\frac{4}{5}$	$-\frac{8}{5}$	$-\frac{4576}{3} + \frac{128}{25}x_2 + \frac{64}{25}x_1$	$-\frac{9152}{3} + \frac{256x_2}{25} + 128x_1/25$
2	$260 - \frac{3}{5}x_1$	$\frac{3}{EA}$	$-\frac{3}{5}$	0	$-468 + \frac{9\times3}{25}x_1$	0
3	$-\frac{4}{5}x_1$	$\frac{4}{EA}$	$-\frac{4}{5}$	0	$\frac{64}{25}x_1 = \frac{64}{25}x_1$	0
4	$260 - \frac{3}{5}x_2 - \frac{3}{5}x_1$	$\frac{3}{EA}$	$-\frac{3}{5}$	$-\frac{3}{5}$	$-468 + \frac{9\times3x_2}{25} + \frac{27x_1}{25}$	$-468 + \frac{9\times3x_2}{25} + \frac{9\times3x_1}{25}$
5	x_1	$\frac{5}{EA}$	1	0	$5x_1$	0
6	$x_1 + x_2 - \frac{1300}{3}$	$\frac{5}{EA}$	1	1	$5x_1 + 5x_2 - \frac{6500}{3}$	$5x_1 + 5x_2 - \frac{6500}{3}$
7	x_2	$\frac{5}{2EA}$	0	1	0	$\frac{5x_2}{2}$

In trusses

$$U^* = \frac{1}{2}\sum N_i^2 \frac{L_i}{EA_i}$$

$$\frac{dU^*}{dx_1} = \sum N_i\left(\frac{\partial N_i}{\partial x_1}\right)\frac{L_i}{EA_i} = 0$$

$$\Rightarrow \left(\sum N_i\left(\frac{\partial N_i}{\partial x_1}\right)\frac{L_i}{EA_i}\right) \times EA = 0$$

$\Sigma\left(f_i EA N_i \frac{\partial N_i}{\partial x_1}\right)$

$$\Rightarrow -\frac{4576}{3} + \frac{128}{25}x_2 + \frac{64}{25}x_1 - 468 + \frac{27}{25}x_1 + \frac{64}{25}x_1$$

$$-156 + \frac{9\times3}{25}x_2 + \frac{9\times3}{25}x_1 + 5x_1 + 5x_1 + 5x_2 - \frac{6500}{3} = 0$$

$$x_1\left(\frac{64}{25} + \frac{27}{25} + \frac{64}{25} + \frac{27}{25} + 10\right) + x_2\left(\frac{128}{25} + \frac{27}{25} + 5\right)$$

$$-\frac{4576}{3} - 468 - 468 - \frac{6500}{3}$$

$$\frac{432}{25} * x_1 + \overset{56/5}{} x_2 = 4628 \Rightarrow \frac{432}{25}x_1 + \frac{56}{5}x_2 = 4628 \quad ①$$