Module 4 – (L12 - L18): "Watershed Modeling"

Standard modeling approaches and classifications, system concept for watershed modeling, overall description of different hydrologic processes, modeling of rainfall, runoff process, subsurface flows and groundwater flow

WATERSHED MANAGEMENT

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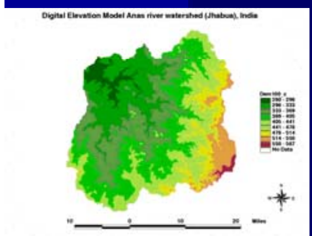
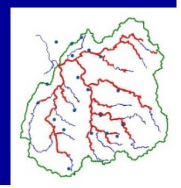
Lecture No - 17

Numerical Watershed Modeling

L17– Numerical Watershed Modeling

- **Topics Covered**
- Physically based watershed modeling, Numerical modeling, Finite difference method; Finite element method, Computer models
- Keywords: Physically based watershed modeling, Numerical modeling, FDM, FEM.





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Watershed Modeling

- Transformation of rainfall into runoff over a watershed
- Generation of flow hydrograph for the outlet
- Use of the hydrograph at the upstream end to route to the downstream end
- Hydrologic simulation models use mathematical equations to calculate results like runoff volume or peak flow
- Computer models allows parameter variation in space and time – with use of numerical methods
- Ease in simulation of complex rainfall patterns and heterogeneous watersheds

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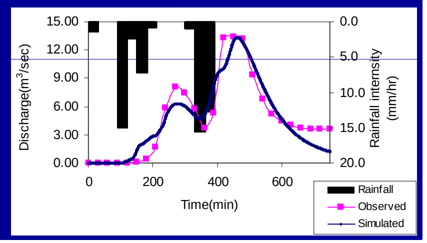
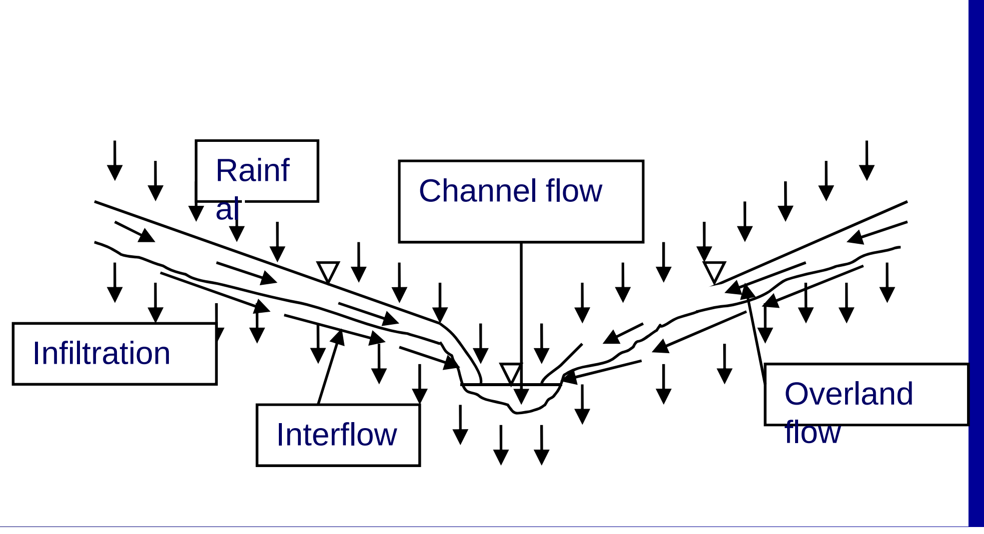


Hydrologic Models

Model Type	Example of Model
Lumped Parameter	Synder Unit Hydrograph
Distributed	Kinematic wave
Event	HEC-1, SWMM
Continuous	Stanford Watershed Model, SWMM, HSPF,
Physically based	HEC-1, SWMM, HSPF
Stochastic	Synthetic stream flows
Numerical	Explicit kinematic wave
Analytical	Nash IUH

Necessity of Distributed models

- Flow of water in a watershed is a distributed process
- Models should be physically based
- Governing equations – St. Venant equations
- Computer models- based on the St. Venant equations
- Allows computation of flow rate and water level as functions of space and time
- Model more closely approximates the actual unsteady non-uniform nature of flow propagation in channels

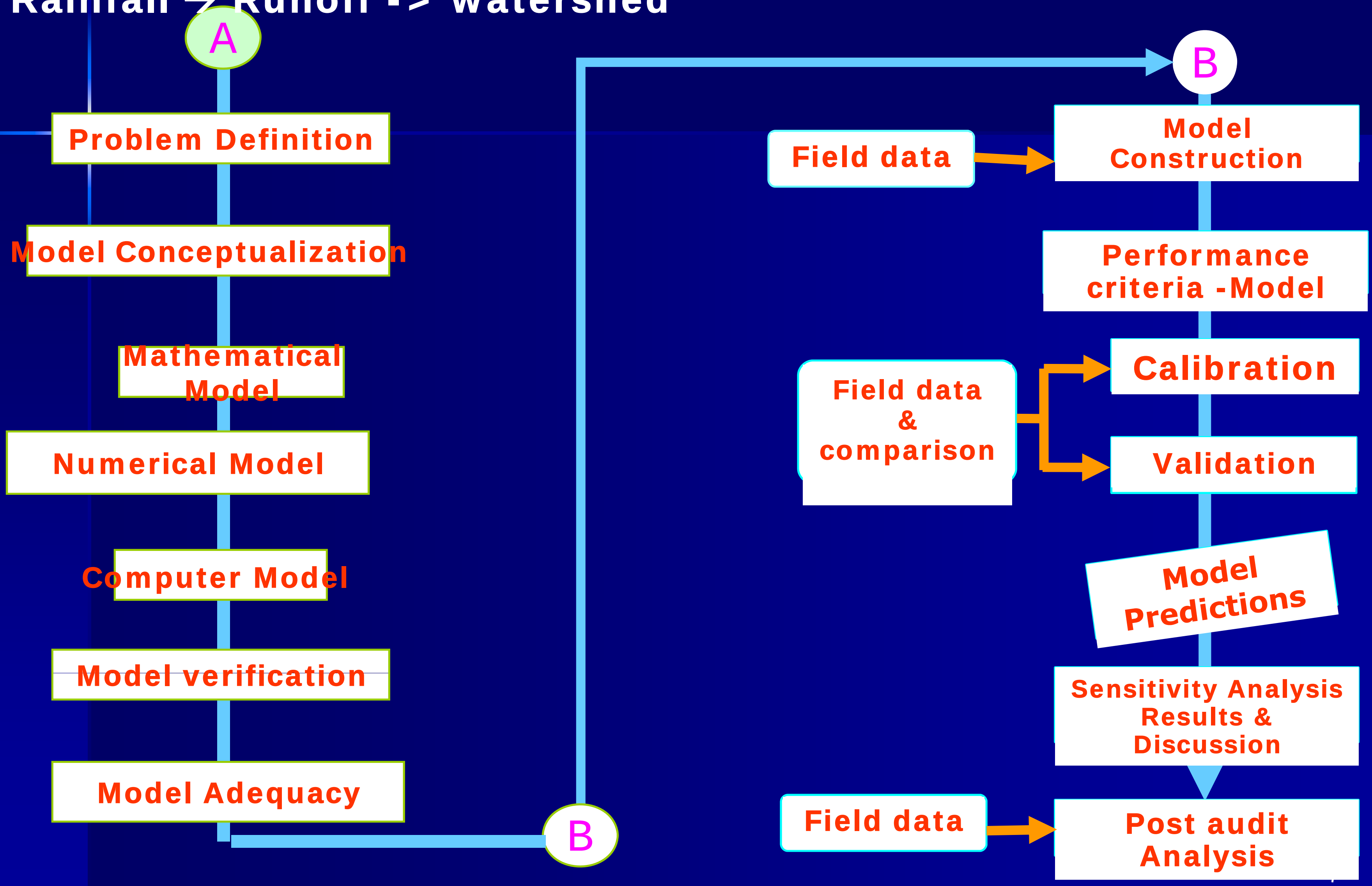


Hydrologic/ Hydraulic Modeling

- Hydrological / Hydraulic model- conceptual or physically based procedure- numerically solving hydrological processes - diagnose or forecast processes.
- Physical based: description of natural system using basic mathematical representation of flows of mass, momentum and various forms of energy.
- Distributed: consider spatial variation of variables & parameters.
- Applications: Rainfall to runoff , Surface water/ groundwater assessment, Flood/ drought predictions, Evaluation of watershed / catchment management strategies, River basin / Agricultural water management etc.

Hydrologic/ Hydraulic Modeling

Rainfall → Runoff -> watershed



Physically based distributed models:

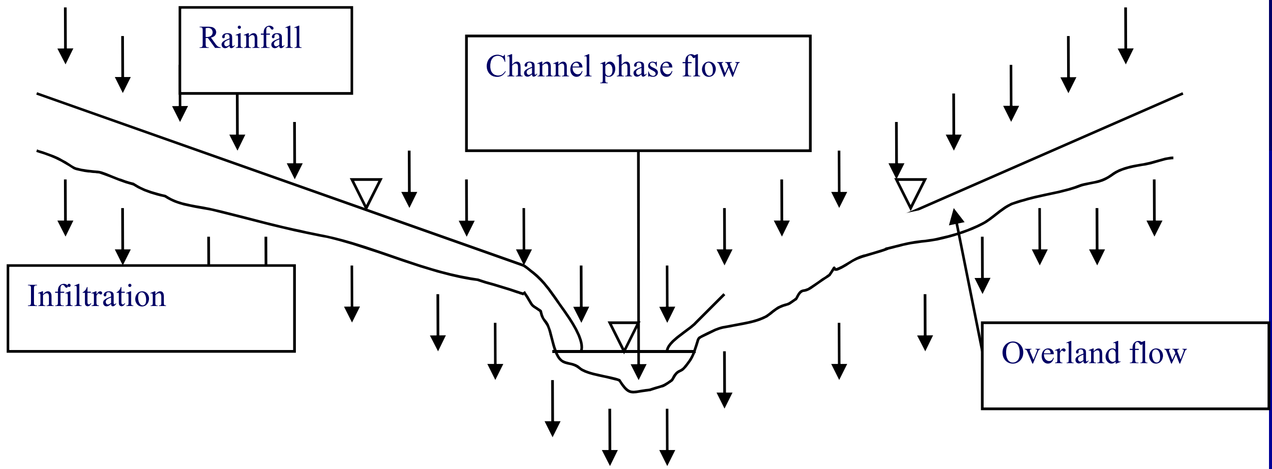


Fig. Flow in a watershed – Typical flow pattern

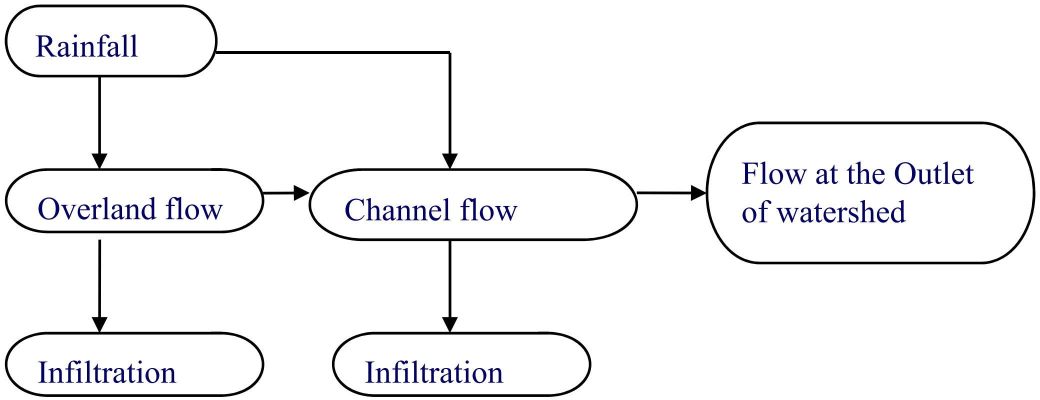


Fig. General concept of flow modeling

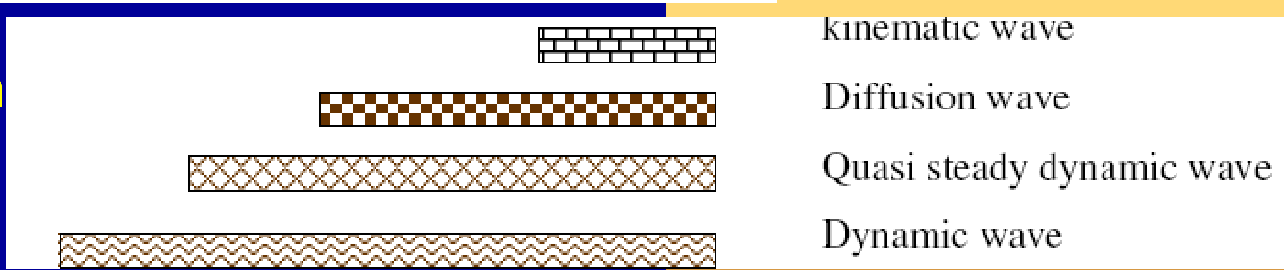
Physically Based Model – Overland Flow Equations

$$\frac{\partial A}{\partial t} + \frac{\partial (vA)}{\partial x} - q = 0$$

Continuity equation

$$\frac{\partial Q}{\partial t} + \frac{\partial (vQ)}{\partial x} + gA\left(\frac{\partial y}{\partial x} - S_0 + S_f\right) = 0$$

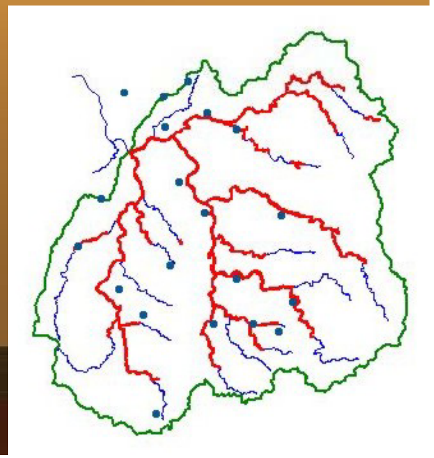
Momentum Equation



Initial and Boundary conditions

IC for overland is usually of dry bed condition. At time t = 0, h =0 and q =0 *at all nodal points*

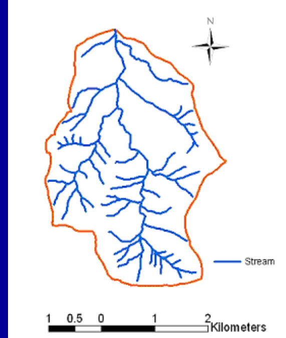
Upstream boundary condition is assumed as zero inflows; h = 0 and q =0 at all times



Gov. Equation for Channel Flow

Equation of continuity $$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0$$

- Momentum equation $$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) = gA\left(S_0 - S_f\right) - gA\frac{\partial h}{\partial x}$$
- Diffusion $$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \qquad \frac{\partial h}{\partial x} = S_o - S_f \qquad Q = \frac{1}{n} R_h^{2/3} S_f^{1/2} A$$
- Kinematic: $$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0$$
  $S_0 = S_f$
- q-lateral inflow; Q-discharge in the channel; A-area of flow in the channel, S_0-bed slope; S_f-friction slope of channel.



Solution Methodologies

- **Analytical method:** For the given mathematical formulation, an analytical expression involving the parameters and the independent variables are obtained using various mathematical procedures.
- Main limitation- only for a small class of mathematical formulations with simplified governing equations, boundary conditions & geometry, analytical solutions can be obtained.
- **Physical method:** As the mathematical model represents a real physical system, although on certain idealized assumptions, variables and parameters of the model can be considered as having physical dimensions and can be analyzed sometimes in the laboratory or in the field itself.
- The physical models are used less frequently since it is expensive, cumbersome and difficult in practice.
- **Computational method**

Computational Method

- In the computational method, the solution is obtained with the help of some approximate methods using a computer. Commonly, numerical methods are used to obtain solution in the computational method.

- Wider class of mathematical formulations & advent of fast computers, computational models have become the most widely used valuable tool for solving the engineering problems.

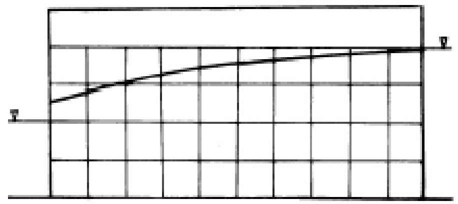
Numerical Modeling

- Variety of numerical methods such as

  - Method of characteristics
  - Finite Difference Method (FDM)
  - Finite Volume Method (FVM)
  - Finite Element Method (FEM)
  - Boundary Element Method (BEM).

Finite Difference Method

- Continuous variation of the function concerned by a set of values at points on a grid of intersecting lines.
- The gradient of the function are then represented by differences in the values at neighboring points and a finite difference version of the equation is formed.
- At points in the interior of the grid, this equation is used to form a set of simultaneous equations giving the value of the function at a point in terms of values at nearby points.
- At the edges of the grid, the value of the function is fixed, or a special form of finite difference equation is used to give the required gradient of the function.



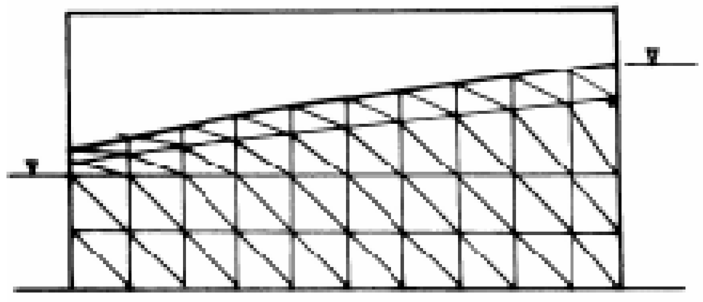
(A) - FDM - DISCRETIZATION

Method of characteristics (MOC)

- MOC - reduce a partial differential equation to a family of ordinary differential equations along which the solution can be integrated from some initial data given on a suitable hyper surface
- For a first-order PDE, MOC discovers curves (called characteristic curves or characteristics) along which PDE becomes an ODE. It is solved along the characteristic curves & transformed into a solution for original PDE.
- Variant of FDM – suitable for solving hyperbolic equations
- MOC to simulate advection dominated transport
- Track idealized particles through flow field
- Efficient & minimize numerical instabilities

Finite Element Method

- The region of interest is divided in a much more flexible way
- The nodes at which the value of the function is found have to lie on a grid system or on a flexible mesh
- The boundary conditions are handled in a more convenient manner.
- Direct approach, variational principle or weighted residual method is used to approximate the governing differential equation

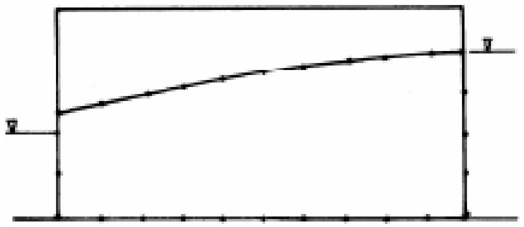


(B)- FEM DISCRETIZATION

WATERSHED MANAGEMENT

Boundary Element Method

- The partial differential equations describing the domain, is transformed in to an integral equation relating only to boundary values.
- The method is based on Green's integral theorem.
- The boundary is discretized instead of the domain.
- A 3-Dimensional problem reduces to a
  2-Dimensional problem and 2-Dimensional problem in to 1-Dimensional problem.
- BEM is ideally suited to the solution of many two and three- dimensional problems in elasticity and potential theory



Analytical Solution–Kinematic wave

$$t_c = \left(\frac{L_w}{\alpha_y r_e^{\beta-1}} \right)^{(1/\beta)}$$

$$q_y = \alpha_y (r_e t)^{\beta} \qquad 0 \le t \le t_c ,$$

$$q_y = \alpha_y (r_e t_c)^{\beta} , t_c \le t \le t_r ,$$

$$q_y = r_e L_w - r_e \beta \alpha^{(1/\beta)} q_y^{(\beta-1/\beta)} (t - t_r) , \; t_r \le t \le t_f$$

$$\alpha = \frac{\sqrt{S_f}}{n_o}$$

$$\beta = \frac{5}{3}$$

- Analytical solution for one-dimensional kinematic wave equations is given by above equations (Jaber and Mohtar, 2003); *t_c is time of concentration (sec); t_r is rainfall duration (sec); t_f is the simulation time* (sec); *Lw is the length of watershed (m) in the direction of main slope. (Jaber, F.H., and Mohtar, R.H. (2003). "Stability and accuracy of two dimensional kinematic wave overland flow modeling." Advances in Water Resources, 26(11), 1189-1198).*

Finite Difference Method (FDM)

- **FDM:** Calculations are performed on a grid placed over the (x, t) plane
- Flow and water surface elevation are obtained for incremental time and distances along the channel

- **Explicit methods:** calculates values of velocity & depth over a grid system based on a previously known data for the river reach
- **Implicit methods:** set up a series of simultaneous numerical equations over a grid system for the entire river & equations are solved at each time step.

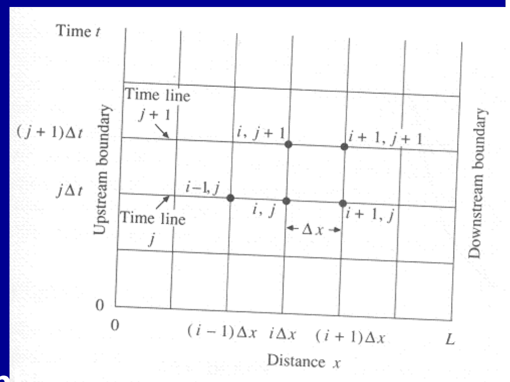


Fig: x-t plane for finite difference scheme

Typical Steps for FDM model

- Governing Partial Differential Equations with Subsidiary conditions
- Divide domain into Grids
- Transformation by Finite Difference Method
- System of difference equations
- Application of Boundary Conditions
- Solve by direct or iterative method
- Solution

		I,J+1	
Δy	I-1,J	I,J	I+1,J
Δx		I,J-1	

Finite Difference Scheme

There are three commonly used finite difference approximations for the solution of PDE

a) Backward difference scheme: We consider the node in the backward direction of the node at which gradient is sought

b) Forward difference scheme

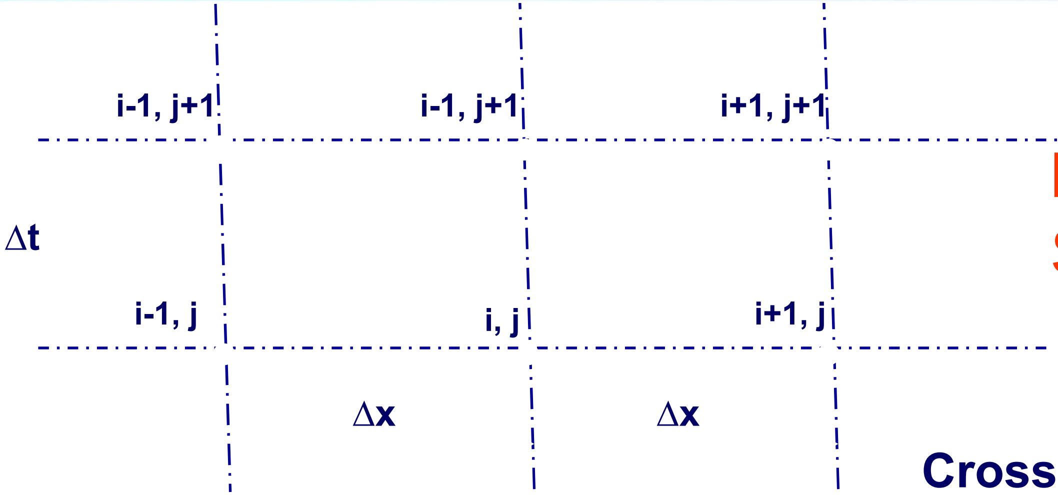
c) Central difference scheme.

$$\left(\frac{\partial h}{\partial x}\right)_I = \frac{h_I - h_{I-1}}{\Delta x}$$

$$\left(\frac{\partial h}{\partial x}\right)_I = \frac{h_{I+1} - h_I}{\Delta x}$$

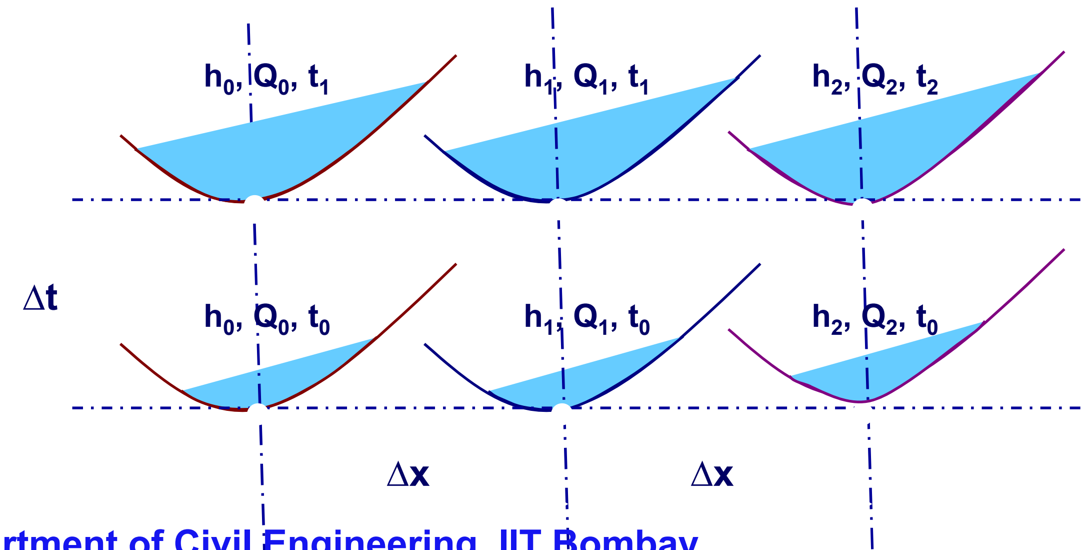
$$\left(\frac{\partial h}{\partial x}\right)_I = \frac{h_{I+\frac{1}{2}} - h_{I-\frac{1}{2}}}{\Delta x}$$

Finite Difference Scheme

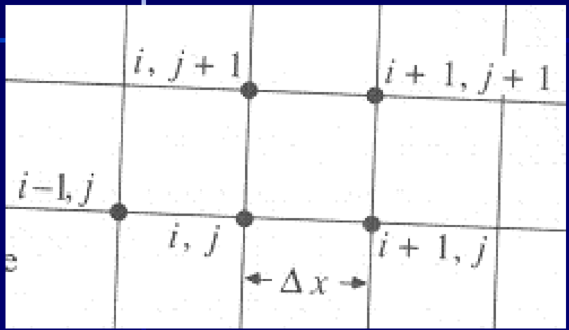


x-t plane

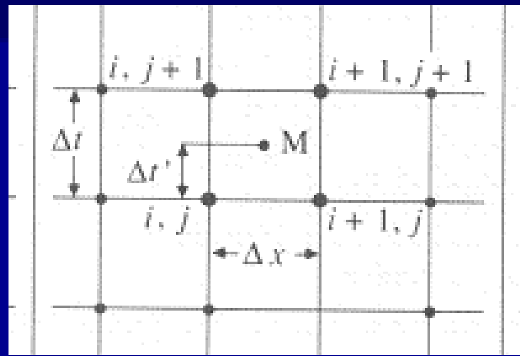
Cross-sectional view in x-t plane



Finite Difference Approximations



- **Explicit**
- **Implicit**



Temporal derivative

$$\frac{\partial u_i^{j+1}}{\partial t} \approx \frac{u_i^{j+1} - u_i^j}{\Delta t}$$

$$\frac{\partial u}{\partial t} \approx \frac{u_i^{j+1} + u_{i+1}^{j+1} - u_i^j - u_{i+1}^j}{2\Delta t}$$

Spatial derivative

$$\frac{\partial u_i^j}{\partial x} \approx \frac{u_{i+1}^j - u_{i-1}^j}{2\Delta x}$$

$$\frac{\partial u}{\partial x} \approx \theta \frac{u_{i+1}^{j+1} - u_i^{j+1}}{\Delta x} + (1-\theta)\frac{u_{i+1}^j - u_i^j}{\Delta x}$$

Spatial derivative is written using terms on known time line

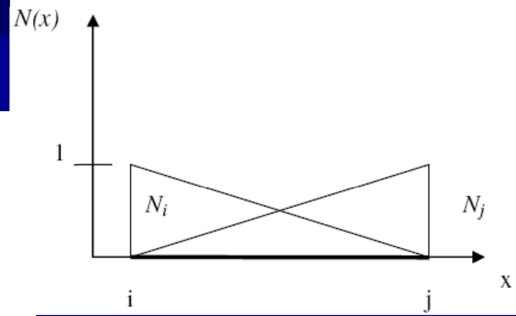
Spatial and temporal derivatives use unknown time lines for computation

Finite Element Method

- 1D-Kinematic & Diffusion Wave Models for Overland Flow
- One-dimensional model with linear line elements
- Apply Galerkin FEM for 1D continuity equation
- $$\int N^T \left(\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} - r_e \right) dx = 0$$ --------- (1)
- $$\int N^T \frac{\partial q}{\partial x} dx + \int N^T \frac{\partial h}{\partial t} dx - \int N^T r_e dx = 0$$ ----- (2)
- Expansion of Eq considering it for one element is given as
- $$\int_0^L N^T \frac{\partial N}{\partial x} \{q\} dx + \int_0^L N^T N \left\{ \frac{\partial h}{\partial t} \right\} dx - \int_0^L N^T r_e dx = 0$$ ------(3)

Finite Element Method

- Shape function N for a linear element can be expressed as [N] = [N1 N2] Where $N_i = 1-(x/L)$ and $N_j = x/L$
- Equation can be written in matrix form as follows:



$$[B]^{(e)}\{q\}+[C]^{(e)}\left\{\frac{\partial h}{\partial t}\right\}-\{f\}^{(e)} r_e = 0$$

where $[B]^{(e)} = \int_0^L N^T \frac{\partial N}{\partial x} dx = \frac{1}{2}\begin{bmatrix}-1 & 1\\ -1 & 1\end{bmatrix}$; $[C]^{(e)} = \int_0^L N^T N dx = \frac{L}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}$;

$$\{f\}^{(e)} = \int_0^L N^T dx = \frac{L}{2}\begin{Bmatrix}1\\ 1\end{Bmatrix};$$

- -----(4)

- Assembling the overland flow line elements and applying implicit finite difference scheme for time domain

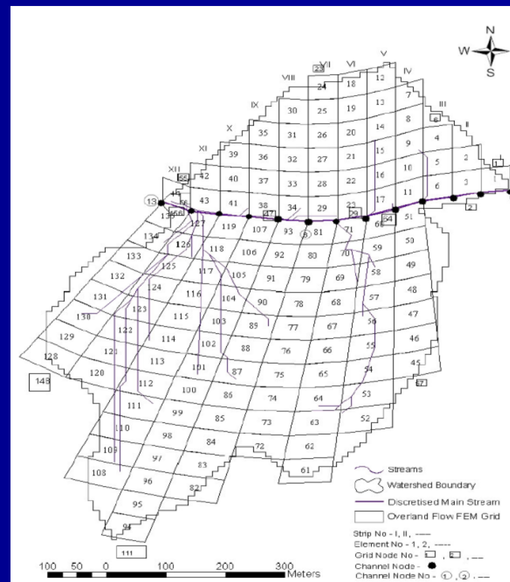
$$[B]\left\{(1-\omega)(q)^t+\omega(q)^{t+\Delta t}\right\}+[C]\left\{\frac{h^{t+\Delta t}-h^t}{\Delta t}\right\}-\{f\}\left\{(1-\omega)(r_e)^t+\omega(r_e)^{t+\Delta t}\right\}=0 \quad \text{--- (5)}$$

Finite Element Method

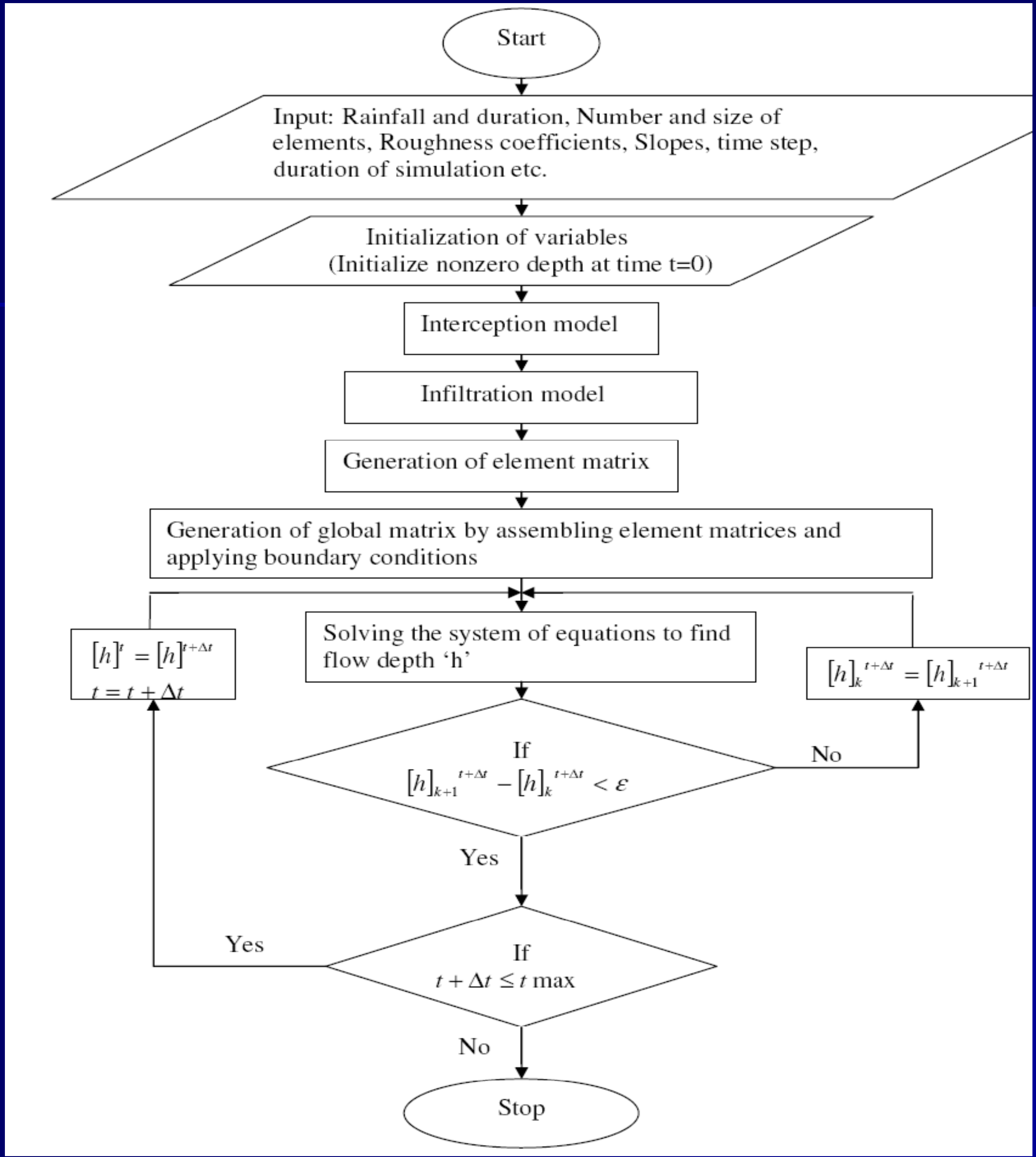
- After rearranging terms, the final form of equation as:

$$[C]\{h\}^{t+\Delta t} = [C]\{h\}^{t} - \Delta t\,[B]\left\{(1-\omega)q^{t} + \omega q^{t+\Delta t}\right\} + \Delta t\,\{f\}\left((1-\omega)(r_e)^{t} + \omega(r_e)^{t+\Delta t}\right)$$

- System of equations will be solved after applying the boundary conditions



Typical Finite element Grid map

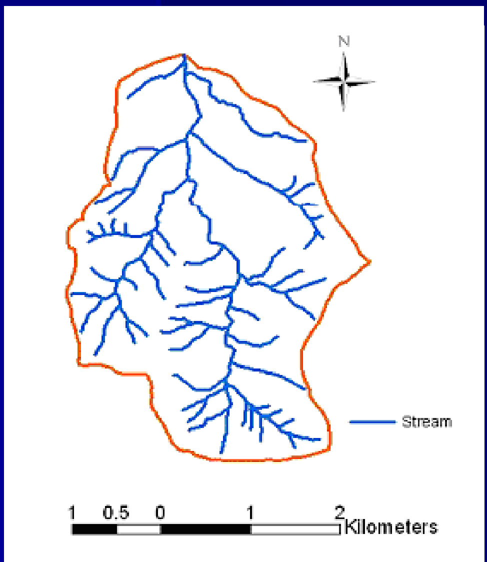


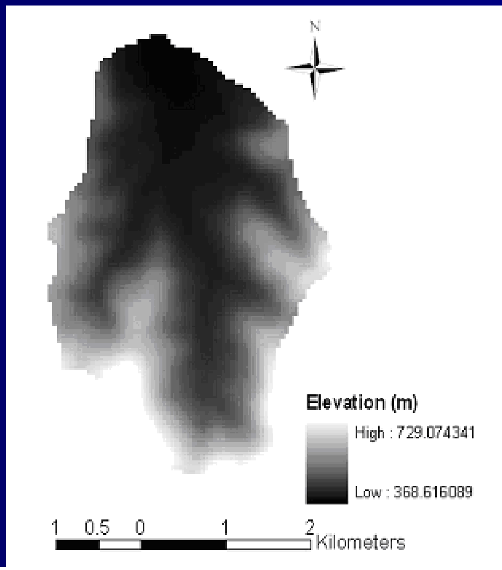
Flow Chart for overland flow FEM model (Reddy et al. (2007))

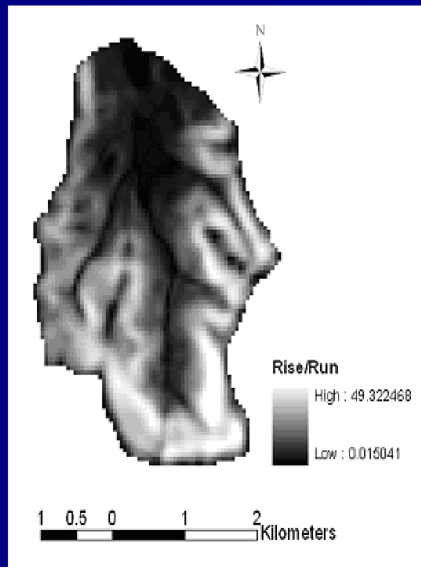
Case study: Harsul Watershed

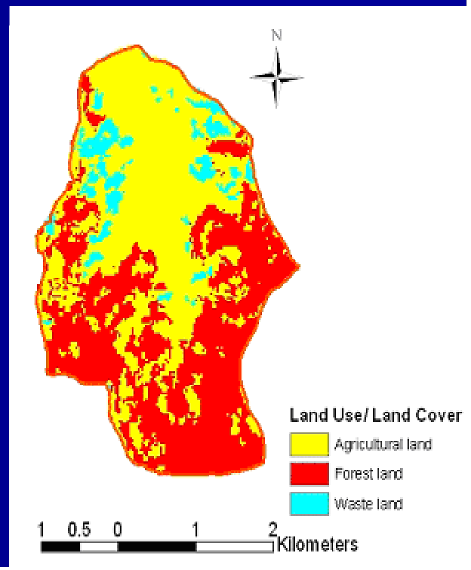
(Venkata Reddy, 2007)

- Location- Nashik district, Maharashtra, India
- Area- 10.929 km^2
- Major Soil class – Gravelly loam
- Remotely Sensed Data- IRS 1D LISS III imagery of January, 1998
- Thematic Maps- Drainage, DEM, Slope and LU/LC

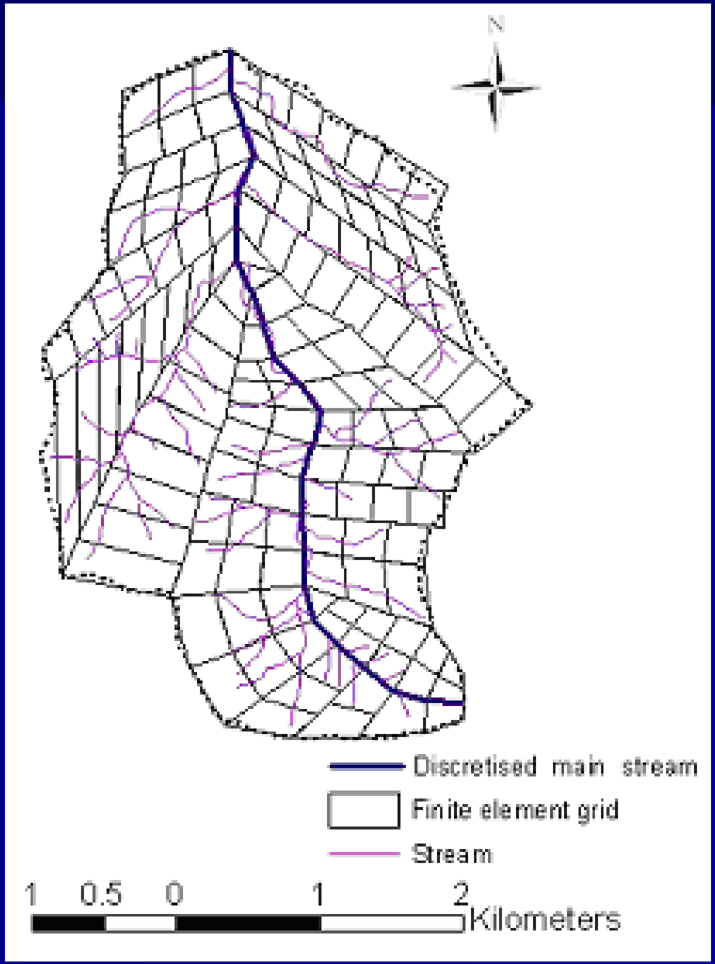








- Overland flow elements - 144
- Overland flow nodes -188
- Channel flow elements - 22
- Channel flow Element length - 0.25 km
- Average bed width - 18 m
- Slope
  - Overland flow
  - Channel flow
- Manning's roughness
  - Overland flow
  - Channel flow



Finite element grid map

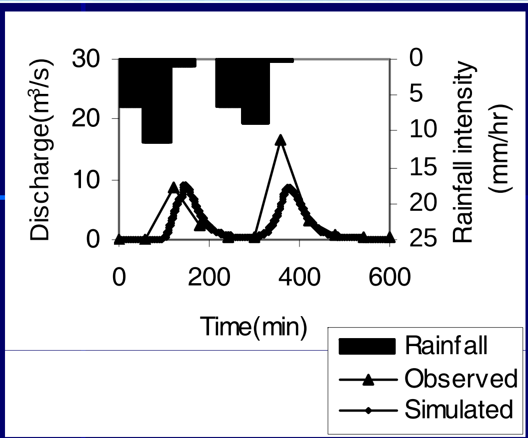
Case study: Harsul Watershed

(Venkata Reddy, 2007)

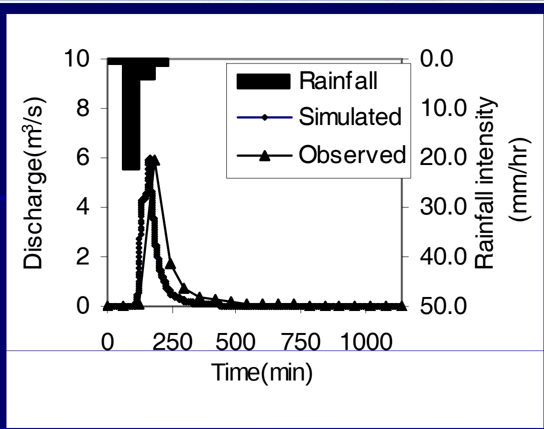
- Diffusion wave- GAML model
- Calibration - 3 Rainfall events
- Validation - 2 Rainfall events

Calibrated parameters for rainfall events (Harsul)

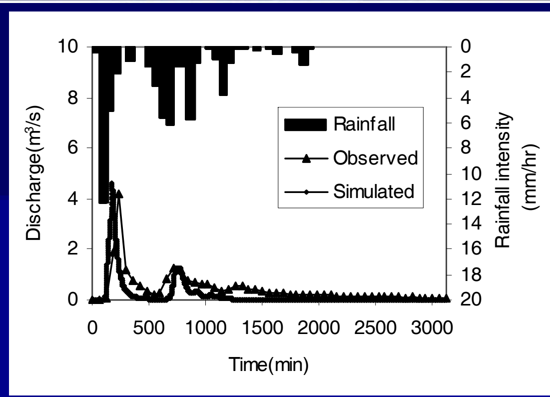
Event date	Saturated Hydraulic Conductivity K_s (cm/hour)	Suction Head (s_{av})(cm)	Saturated Water Content (θ_s)	Initial Water Content (θ_i)
August 22,1997	0.4	4	0.45	0.35
September 23,1997	0.48	10	0.45	0.205
September 26,1997	0.38	5	0.45	0.322



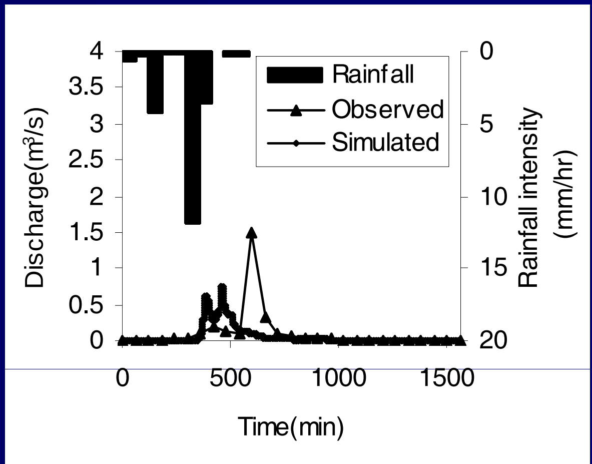
August 22, 1997

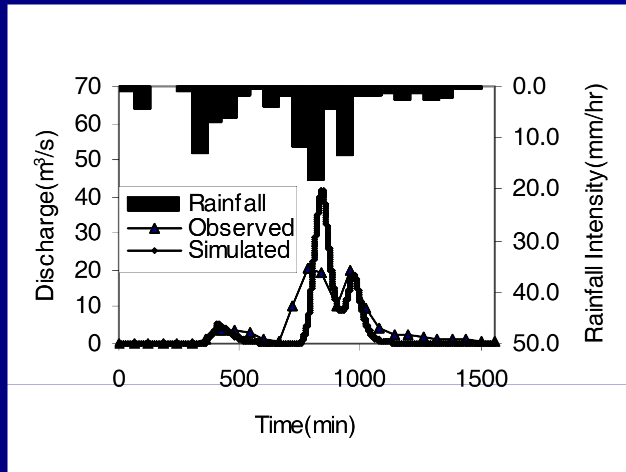


September 23, 1997



September 26, 1997





August 21, 1997

August 23, 1997

Observed & simulated hydrographs of calibration & validation rainfall events

References

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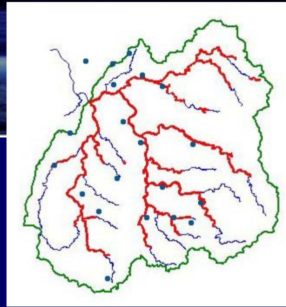
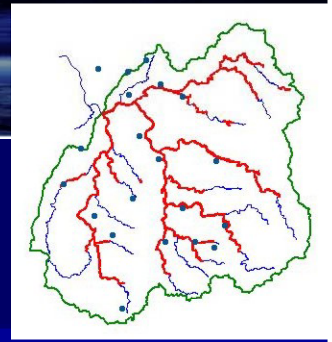


Tutorials - Question!.?.

- Illustrate the necessity of physically based watershed modeling.
- Develop a conceptual model for a typical watershed, for physically based modeling. Describe the merits & demerits of physical modeling.



Self Evaluation - Questions!.

- Why distributed modeling required for watershed modeling?.
- Illustrate various solution methodologies for problem solution.
- Differentiate between explicit & implicit FDM schemes.
- Describe FEM solution methodology with salient features.

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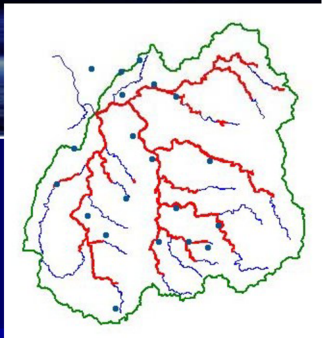
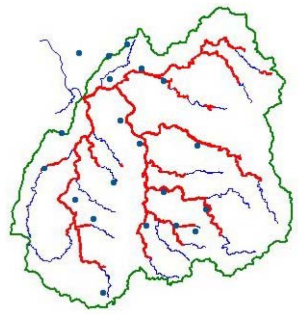


Assignment- Questions?.

- With the help of a flow chart, illustrate hydrologic/ hydraulic modeling.
- Describe FDM solution methodology with salient features.
- Differentiate between FDM & MOC.
- Describe BEM solution methodology with salient features.



Unsolved Problem!.

- Study the salient features & problems of your watershed area. Identify how various physically based models can be used for various problem solutions such as: rainfall-runoff, flooding, drought management, rainwater harvesting, soil erosion etc.

THANK YOU

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