Advanced Mathematical techniques in Chemical Engineering

Module VII : Applications of eigenvalue problems

Exercises

1. Determine possible steady states and examine their stability

$\frac{dx_1}{dt} = x_1^2 - ax_1x_2 - x_1$ and $\frac{dx_2}{dt} = bx_2^2 + x_1x_2 - 2x_2$ where, a, b are real, positive. Find out the steady states. Evaluate the conditions on parameters such that the steady states are stable. Evaluate conditions on parameters such that saddle node and Hopf bifurcation occur in each steady state.

2. For what values of the real parameters *a* and *b*, the following characteristic equations are stable?

(a) $s^2 - (a+2)s + (a-3) = 0$

(b) $s^3 + s^2 + (a+2)s + b = 0$

3. A system is mathematically represented as follows,

$\frac{dx}{dt} = 3(x-y)$ and $\frac{dy}{dt} = -x^2y + \mu x$, where the parameter μ is real positive. Obtain the steady states and check the stability of the steady states.

4. The hydrodynamic flow results when a liquid layer is confined between two flat plates and the lower plate is at higher temperature than the upper plate. The flow field is characterized by three simple ODEs, known as Lorenz equation:

$$\frac{dx}{dt} = -\sigma(x-y);\ \frac{dy}{dt} = rx - y - xz;\ \frac{dz}{dt} = xy - bz$$

Where, σ, *r* and *b* are all real positive.

Find steady state of the system. Check stability of each steady state and find saddle node or Hopf bifurcation points, if exist.

5. A chemical engineering system is mathematically represented as follows,

$\frac{dx_1}{dt} = A - Bx_1 + x_1^2x_2 - x_1$ and $\frac{dx_2}{dt} = Bx_1 - x_1^2x_2$; The parameters A and B are positive real. Find out the steady states and obtain a condition so that the steady state is stable. If A=4, show that for B=17, one may get the Hopf bifurcation.