## Advanced Mathematical techniques in Chemical Engineering

# Module IV : Contraction Mapping

## Exercises

1. A chemical engineering system is mathematically represented as follows, $\frac{dx}{dt} = -\alpha_1 x + \beta_1 (1-x) e^{my}$ and $\frac{dy}{dt} = -\alpha_2 y + \beta_3 (1-x) e^{my}$ ,where the parameters $\alpha_1, \alpha_2, \beta_1, \beta_3, m$ are real positive. The domain of x is (0,1). Prove that there exists a unique steady state if we select the parameters as $m\beta_3\alpha_1 < 4\beta_1\alpha_2$ .

2. Consider an enzyme catalyzed fermentation process is governed by Monod kinetics is occurring in a CSTR with volume V, flow rate q. Feed concentration of the reactant S is $C_f$. The reaction occurs isothermally. The reaction can be considered as an irreversible reaction as S $\rightarrow$ P, where rate equation is given as $-r_A = \frac{k_1 c^2}{1 + k_2 c^2}$.
Show that two parameters of this system are $Da = k_1 V / q$ and $\sigma = k_2 c_f$ . Obtain the conditions on parameters so that unique steady state exists.

3. Consider an enzyme catalyzed fermentation process is governed by Monod kinetics is occurring in a CSTR with volume V, flow rate q. Feed concentration of the reactant S is $C_f$. The reaction occurs isothermally. The reaction can be considered as an irreversible reaction as S $\rightarrow$P, where rate equation is given as $-r_A = \frac{k_1 c^2}{k_2 + k_3 c + c^2}$.
Find out the condition so that unique steady state exists for this system.

4. Consider an enzyme catalyzed fermentation process is governed by Monod kinetics is occurring in a CSTR with volume V, flow rate q. Feed concentration of the reactant S is $C_f$. The reaction occurs isothermally. The reaction can be considered as an irreversible reaction as S $\rightarrow$P, where rate equation is given as $-r_A = \frac{k_1 c^2}{k_2 + k_3 c}$.
Find out the condition so that unique steady state exists for this system.

5. Consider an enzyme catalyzed fermentation process is governed by Monod kinetics is occurring in a CSTR with volume V, flow rate q. Feed concentration of the reactant S is $C_f$. The reaction occurs isothermally. The reaction can be considered as an irreversible reaction as S $\rightarrow$P, where rate equation is given as $-r_A = \frac{k_1 c^3}{k_2 + k_3 c + c^2}$.
Find out the condition so that unique steady state exists for this system.