Advanced Mathematical techniques in Chemical Engineering

Module X : Solution of linear, homogeneous PDEs by separation of variables

Exercises

1. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

Solve the above parabolic equation subject to the following conditions:

At t=0, u=0; at x=0, u=1 and at x=1, $\frac{\partial u}{\partial x} = 0$

2. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$

Solve the above parabolic equation subject to the following conditions:

At t=0, u=1; at x=0, u=0 and at x=1, $\frac{\partial u}{\partial x} = 5$

3. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Solve the above parabolic equation subject to the following conditions:

At t=0, u=1; at x=0, u=1 and at x=1, $\frac{\partial u}{\partial x} = 0$; at y=0, $\frac{\partial u}{\partial x} = 0$ and at y=1, u=0

4. Consider the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Solve the above parabolic equation subject to the following conditions:

At t=0, u=1; at x=0, u=0 and at x=1, $\frac{\partial u}{\partial x} = 0$; at y=0, $\frac{\partial u}{\partial y} = 0$ and at y=1, u=0

5. Consider the PDE $0 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$

Solve the above parabolic equation subject to the following conditions:

at x=0, u=1 and at x=1, $\frac{\partial u}{\partial x} = 0$; at y=0, $\frac{\partial u}{\partial y} = 0$ and at y=1, u=0