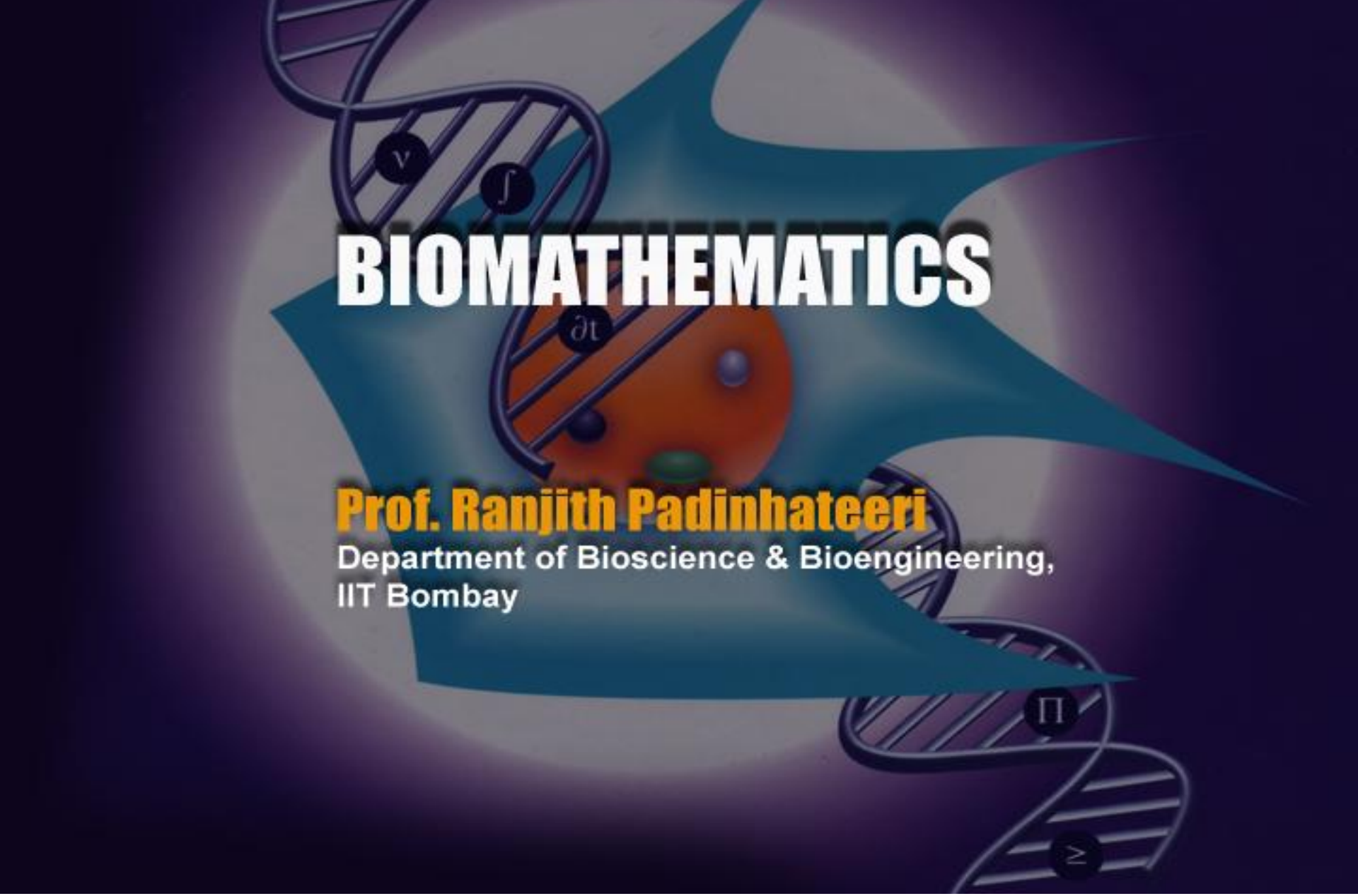
BIOMATHEMATICS
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Lecture 31

Simple mathematical models to understand evolution

Wright-Fisher model

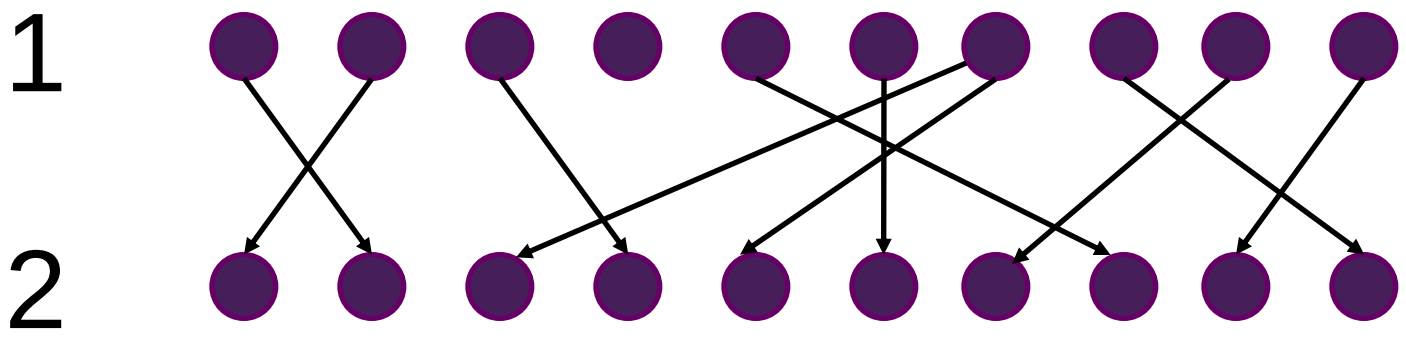
Wright-Fisher model

- Named after Sewall Wright and Ronald Fisher
- Simplest population genetic model

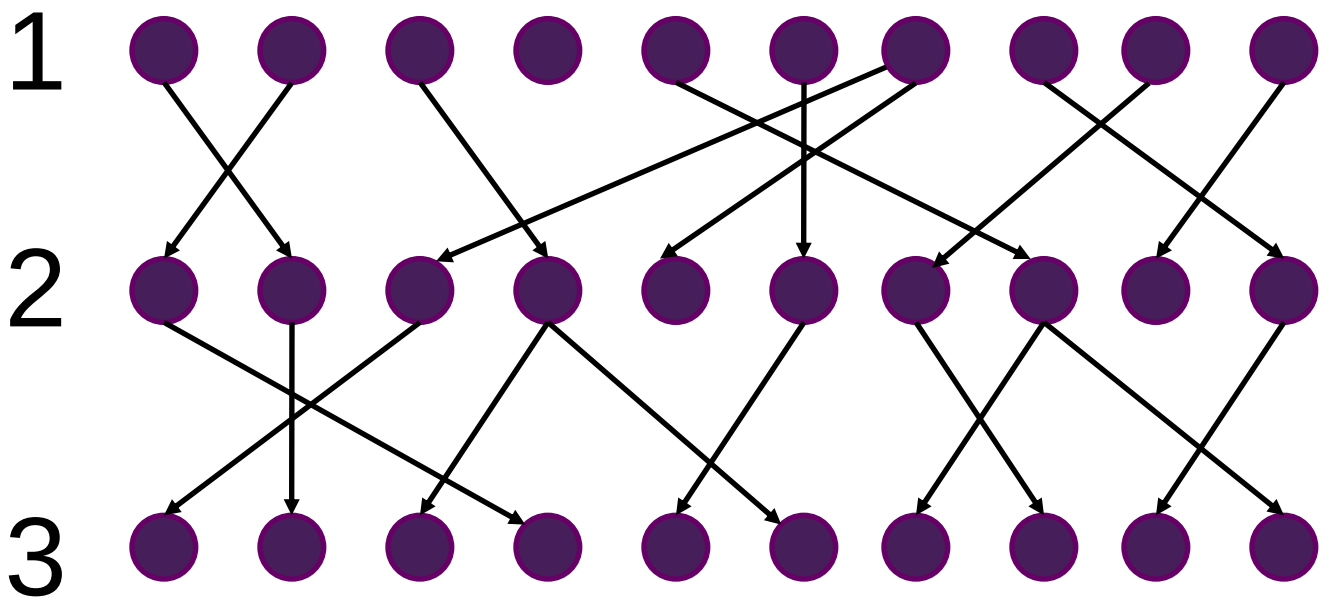
Wright-Fisher model

- Imagine a haploid population of N individuals
- Assume constant population – that is, N offspring individuals
- Each offspring individual picks up parent randomly from the previous generation
- Each offspring inherits chromosome of the parent (no mutation, no change)

Wright-Fisher model



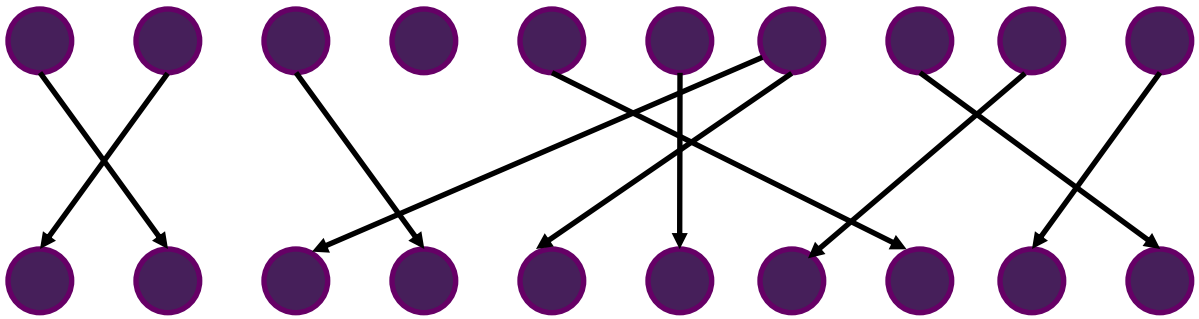
Wright-Fisher model



Allele frequency

Two alleles: a and b

p: probability that of finding allele a in a generation



Probability of finding an allele in the next generation

If ‘p’ is the probability of finding allele ‘a’ in the current generation, what is the probability of finding the same allele in the next generation ?

Probability of finding an allele in the next generation

There are 10 individuals out of 3 have allele ‘a’.

$p=0.3$

In the next generation, what is the probability that 7 of them have allele ‘a’ ?

Probability of finding an allele in the next generation

There are 10 individuals out of 3 have allele ‘a’.

p=0.3

In the next generation, what is the probability that 7 of them have allele ‘a’ ?

$$G = \binom{10}{7} 0.3^{7} \left(1 - 0.3\right)^{(10-7)}$$

Bionomial distrubution

Binomial Distribution

$$G(n,k,p) = \binom{n}{k} p^k (1-p)^{(n-k)}$$

Binomial Distribution

$$G(n,k,p) = \binom{n}{k} p^k (1-p)^{(n-k)}$$

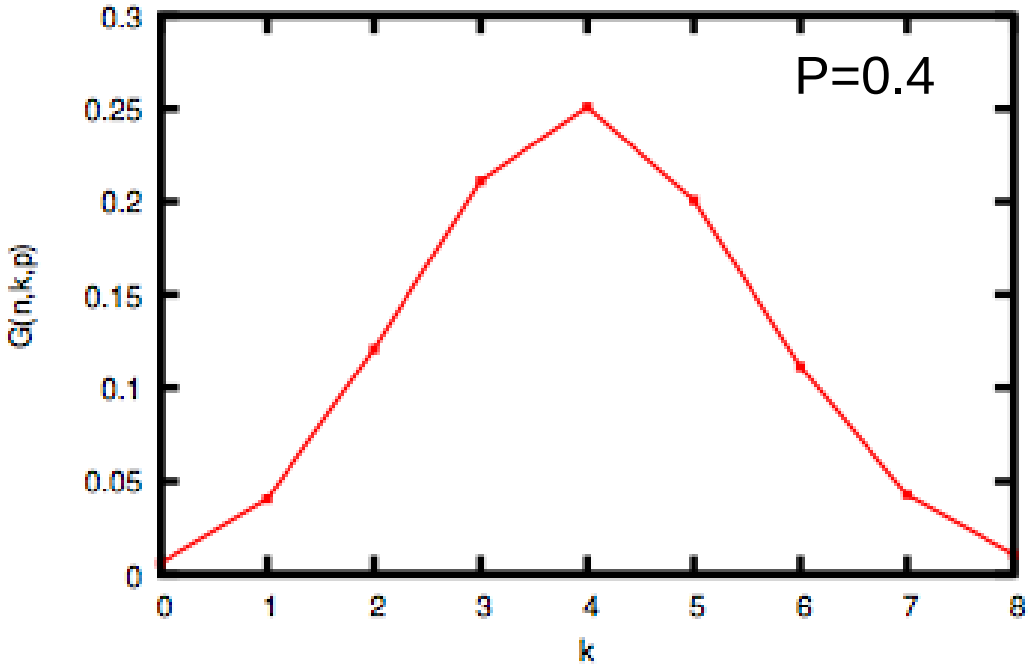
$$< k >= \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{(n-k)} = np$$

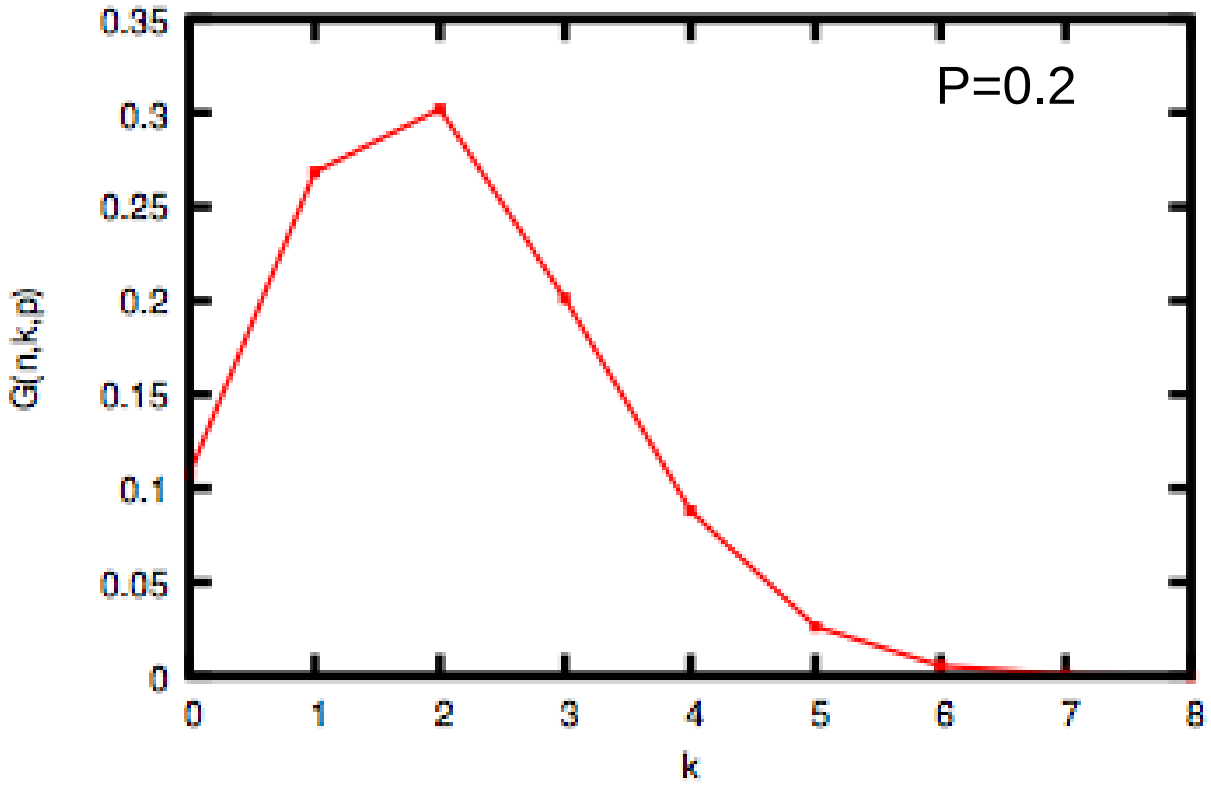
$$< k^2 - k >= \sum_{k=0}^{n} k(k-1) \binom{n}{k} p^k (1-p)^{(n-k)} = n(n-1)p^2$$

Variance

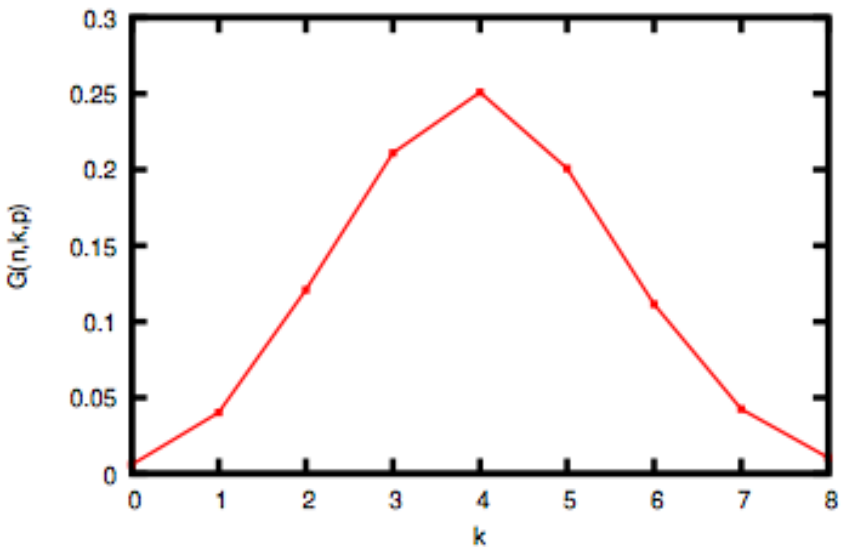
$$= n(n-1)p^2 + np - n^2p^2$$

$$= np(1-p)$$

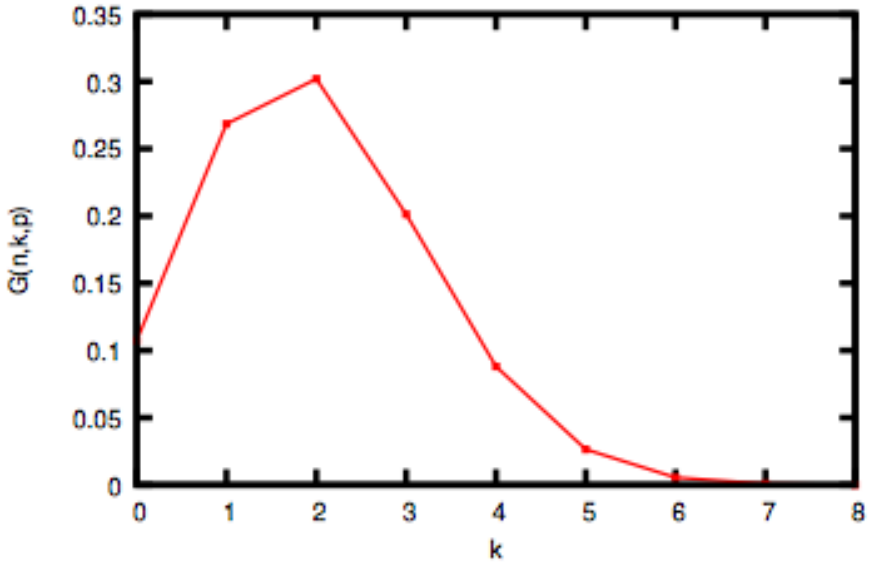

P=0.4
G(n,k,p)
k
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8


P=0.2
G(n,k,p)
k
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
6
7
8

Probability of losing an allele



P=0.4



P=0.2

How do we extend this idea to diploid individuals ?

Ans: assume random mating (panmixia)

Coalescent processes

What is the probability that two Individuals had a common parent t generation ago ?

Incorporating random mutations

- Each offspring inherits chromosome of the parent with a probability (1-m). With probability m, it will change the genotype

Summary

We can use different ideas we learned to make simple models of evolutions