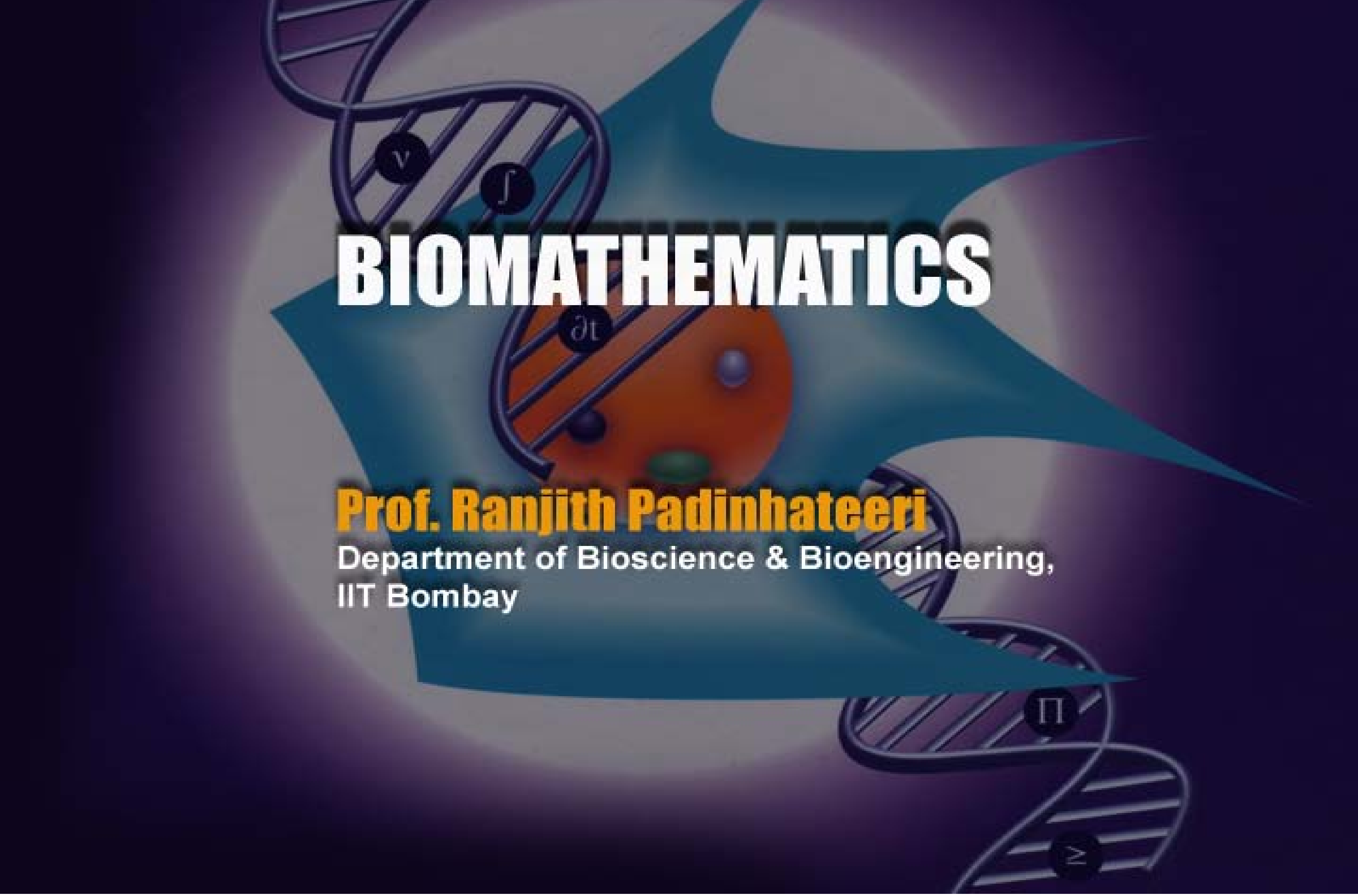
BIOMATHEMATICS
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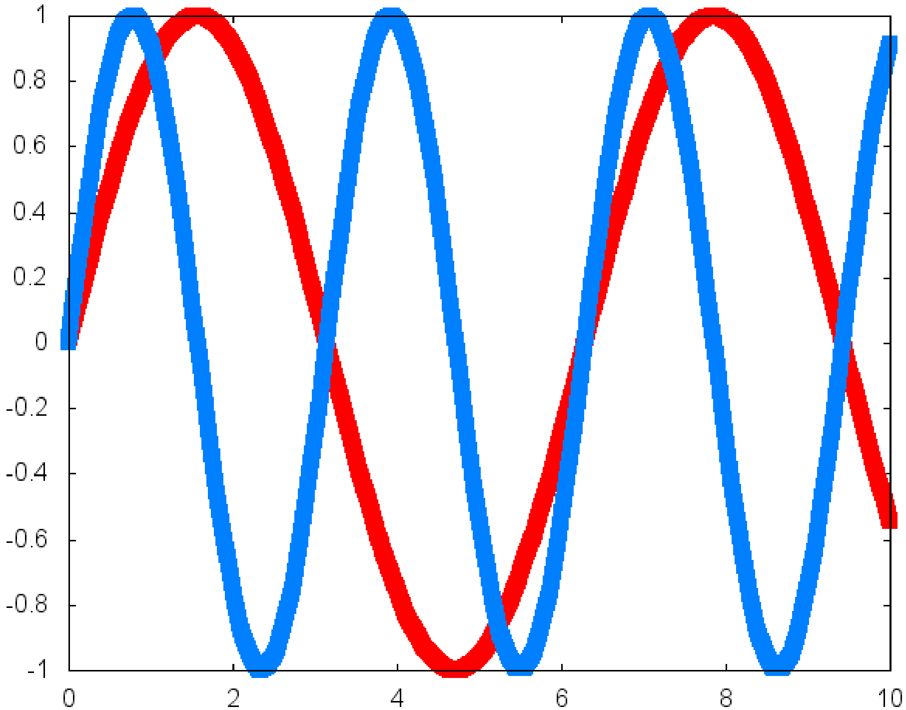
Lecture 29

Fourier Transform

To specify a simple sine or cosine wave, one can either draw it in a paper, or just specify its frequency



$\sin(x)$ $\sin(2x)$

Mathematically,

Imagine a function, which represents the wave in space : $f(x)$

Imagine a different function, which represents the wave, given its frequency : $g(k)$

The way we transform a function in the real space (paper), to an equivalent function in the frequency domain is known as Fourier transform

$$f(x) \leftrightarrow g(k)$$

Definition

$$g(k) = \int_{-\infty}^{\infty} f(x) e^{(-2\pi i k x)} dx$$

Inverse Fourier transform

$$g(k) = \int_{-\infty}^{\infty} f(x) e^{(-2\pi i k x)} dx$$

$$f(x) = \int_{-\infty}^{\infty} g(k) e^{(2\pi i k x)} dk$$

Dirac Delta Function

$$\int_{-\infty}^{\infty} e^{2\pi i k x} dx = \delta(k)$$

$$\int_{-\infty}^{\infty} e^{2\pi i (k-b) x} dx = \delta(k-b)$$

In scattering and diffraction experiments, the output one gets is $g(k)$

Eg. X-ray scattering to find Crystal structure of proteins

One can use Fourier transform (and other similar transforms) to solve differential equations

Trigonometric form

$$e^{-2\pi ikx} = cos(2\pi kx) - isin(2\pi kx)$$

$$g(k) = \int_{-\infty}^{\infty} f(x)\left(cos(2\pi kx) - isin(2\pi kx)\right) dx$$

For even function

$$g(k) = 2\int_{0}^{\infty} f(x)cos(2\pi kx)dx$$

For odd function

$$g(k) = 2\int_{0}^{\infty} f(x)sin(2\pi kx)dx$$