



# BIOMATHEMATICS

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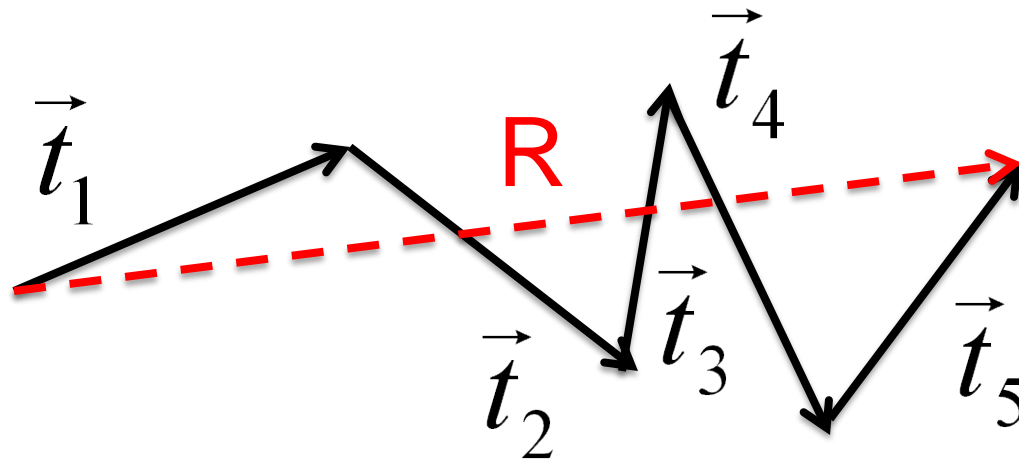
## Lecture 25

# Statistics

# Size of a folded protein: Simplest model

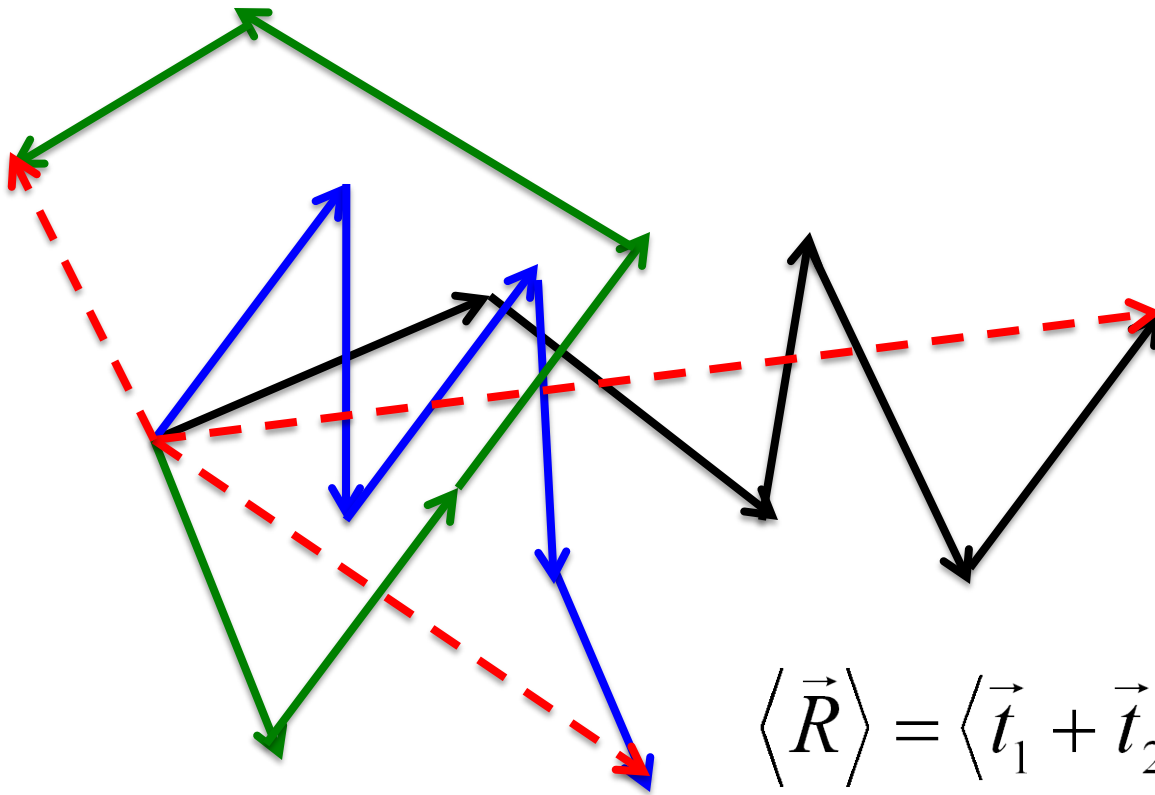
Imagine that a protein of  $N$  amino acids has no preferred Orientation (completely flexible)

# End-to-end distance



$$\vec{R} = \vec{t}_1 + \vec{t}_2 + \vec{t}_3 + \vec{t}_4 + \vec{t}_5$$

## Random orientation



$$\langle \vec{R} \rangle = \langle \vec{t}_1 + \vec{t}_2 + \vec{t}_3 + \vec{t}_4 + \vec{t}_5 \rangle = 0$$

# End-to-end distance

$$\langle \vec{R}^2 \rangle = \left\langle \left( \vec{t}_1 + \vec{t}_2 + \vec{t}_3 + \vec{t}_4 + \vec{t}_5 \right)^2 \right\rangle$$

# Mean-square average

$$\langle \vec{R}^2 \rangle = \left\langle \left( \vec{t}_1 + \vec{t}_2 + \vec{t}_3 + \vec{t}_4 + \vec{t}_5 \right)^2 \right\rangle = 5b^2$$

Where  $b$  is the length of each monomer/amino acid



# Standard deviation

$$\langle \vec{R}^2 \rangle = \left\langle \left( \vec{t}_1 + \vec{t}_2 + \vec{t}_3 + \vec{t}_4 + \dots + \vec{t}_N \right)^2 \right\rangle = Nb^2$$

$$\sigma = \sqrt{\langle \vec{R}^2 \rangle - \langle R \rangle^2} = b\sqrt{N}$$

## Summary

Flexible protein: Average

Flexible protein: Standard deviation

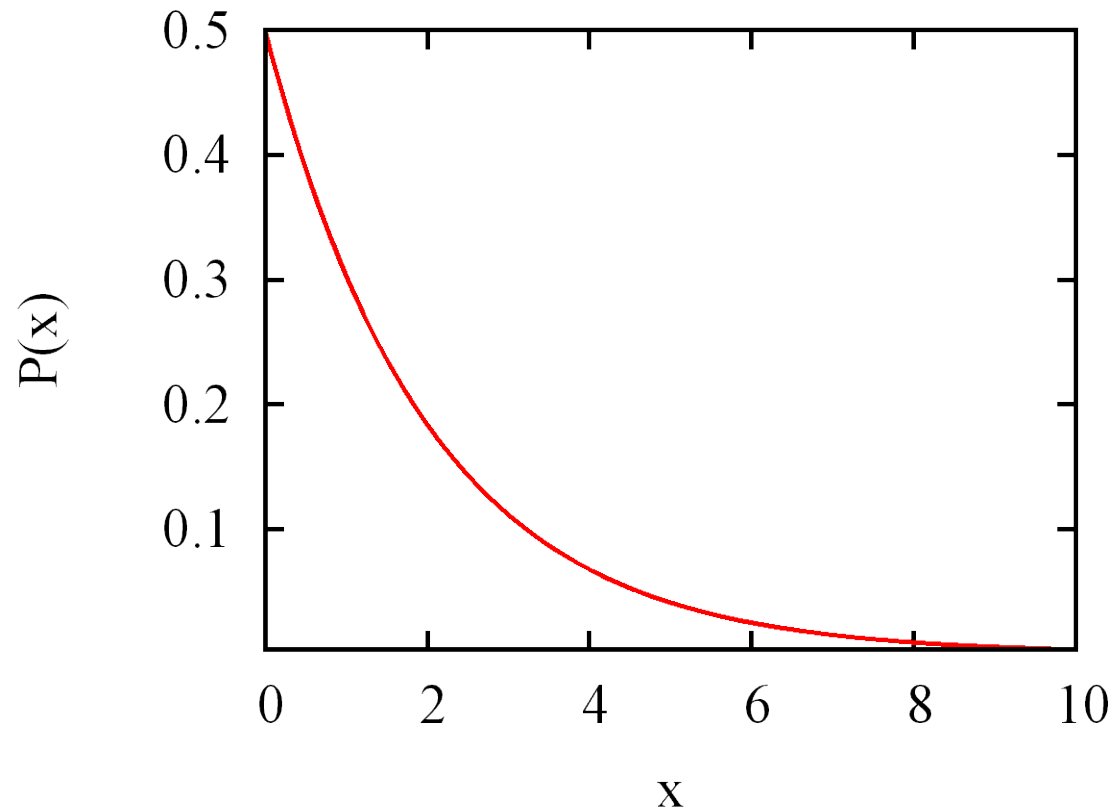
$$\sigma \propto \sqrt{N}$$

## Do you know ... ?

Microtubules rapidly grow and shrink exhibiting the phenomenon known as dynamic instability. What is the length distribution of microtubules ?

Imagine that proteins bind on to the DNA with a rate  $k$ . What is the probability that you have to wait at time 'dt' between consecutive binding events ?

# Exponential distribution



$$P(x) = A \exp(-kx)$$

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$$\Rightarrow A = k$$

# Exponential distribution

$$\langle x \rangle = \int_0^{\infty} xP(x)dx$$

$$\langle x^2 \rangle = \int_0^{\infty} x^2 P(x)dx$$

# Exponential distribution

$$\langle x \rangle = \int_0^{\infty} x k \exp(-kx) dx = \frac{1}{k}$$

$$\langle x^2 \rangle = \int_0^{\infty} x^2 k \exp(-kx) dx = \frac{2}{k^2}$$

$$\sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{1}{k}$$



Exponential distribution  
has standard deviation as big  
as average