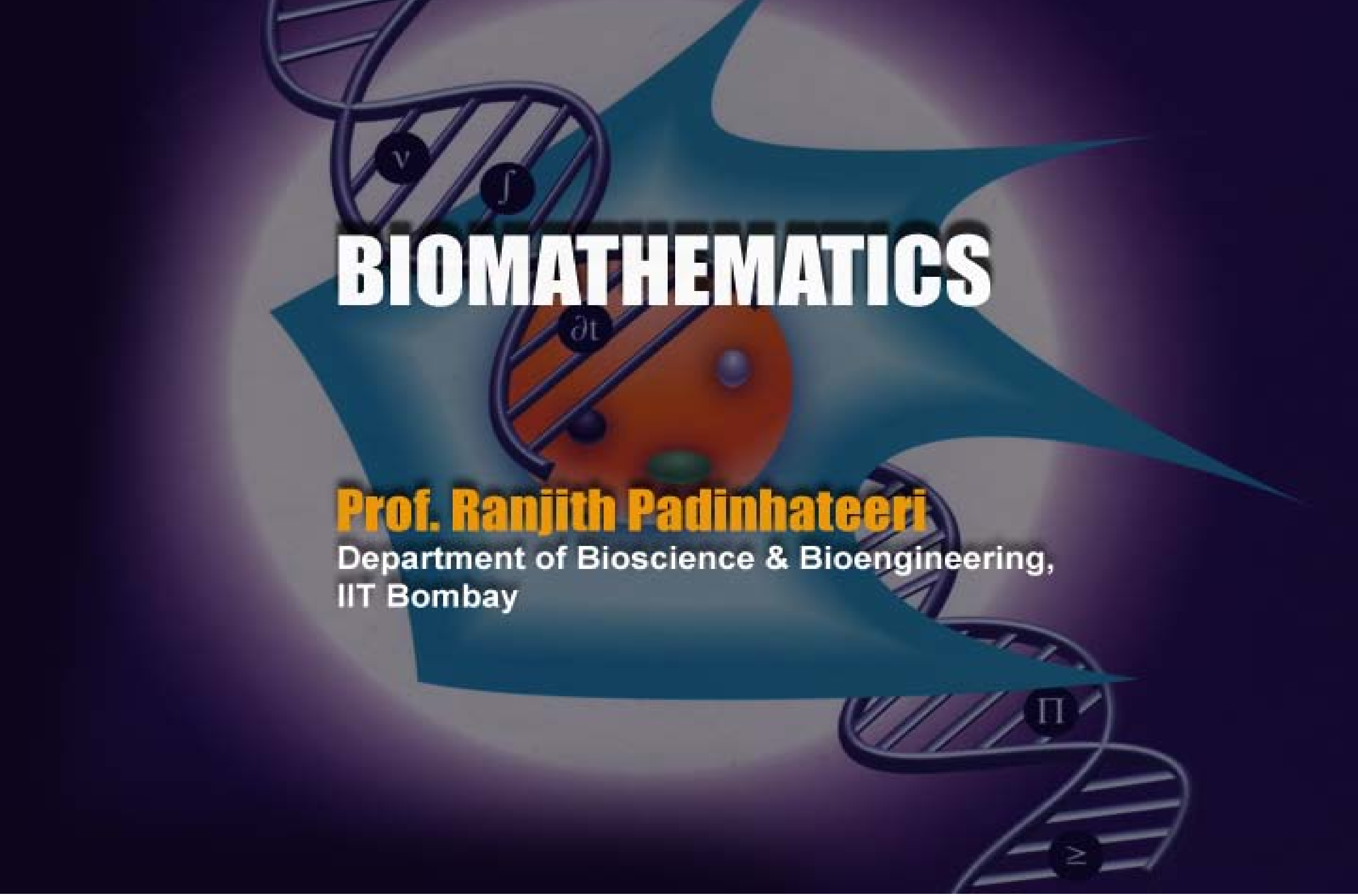
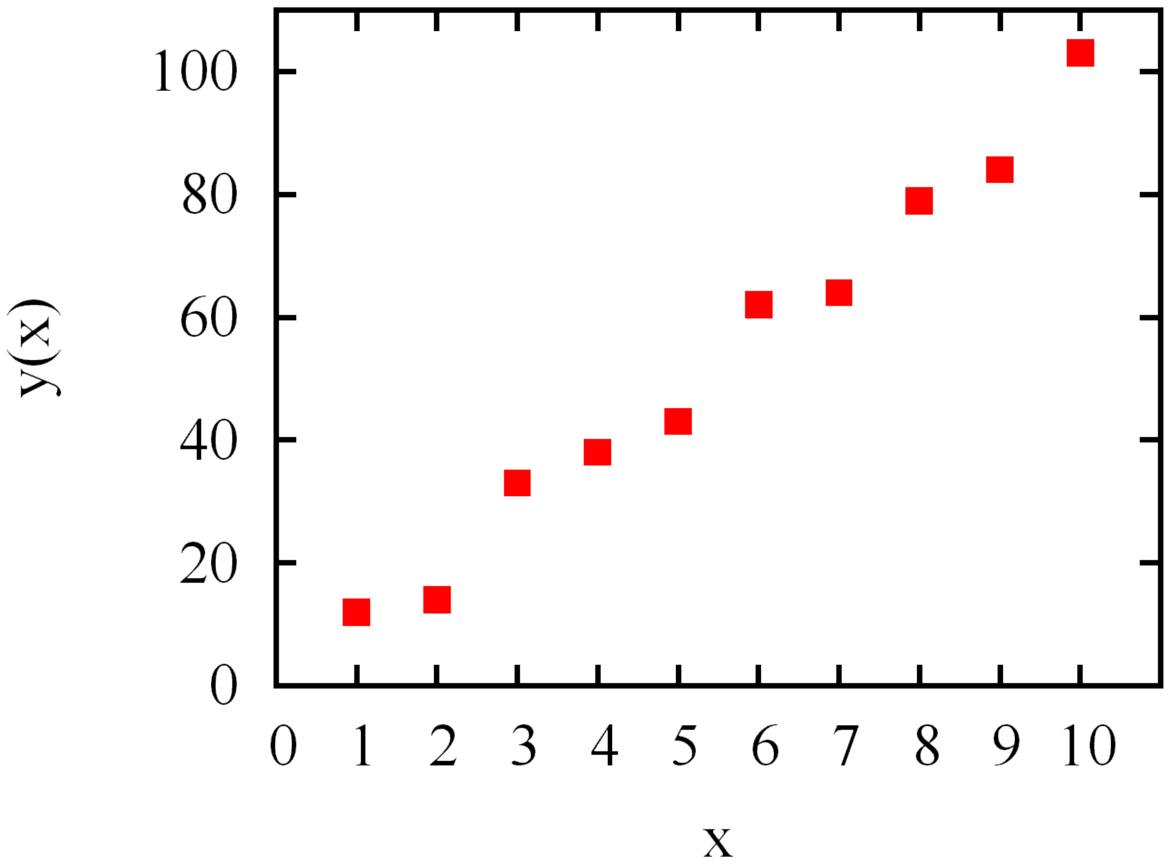

BIOMATHEMATICS
Prof. Ranjith Padinhateeri
Department of Bioscience & Bioengineering,
IIT Bombay

Lecture 24

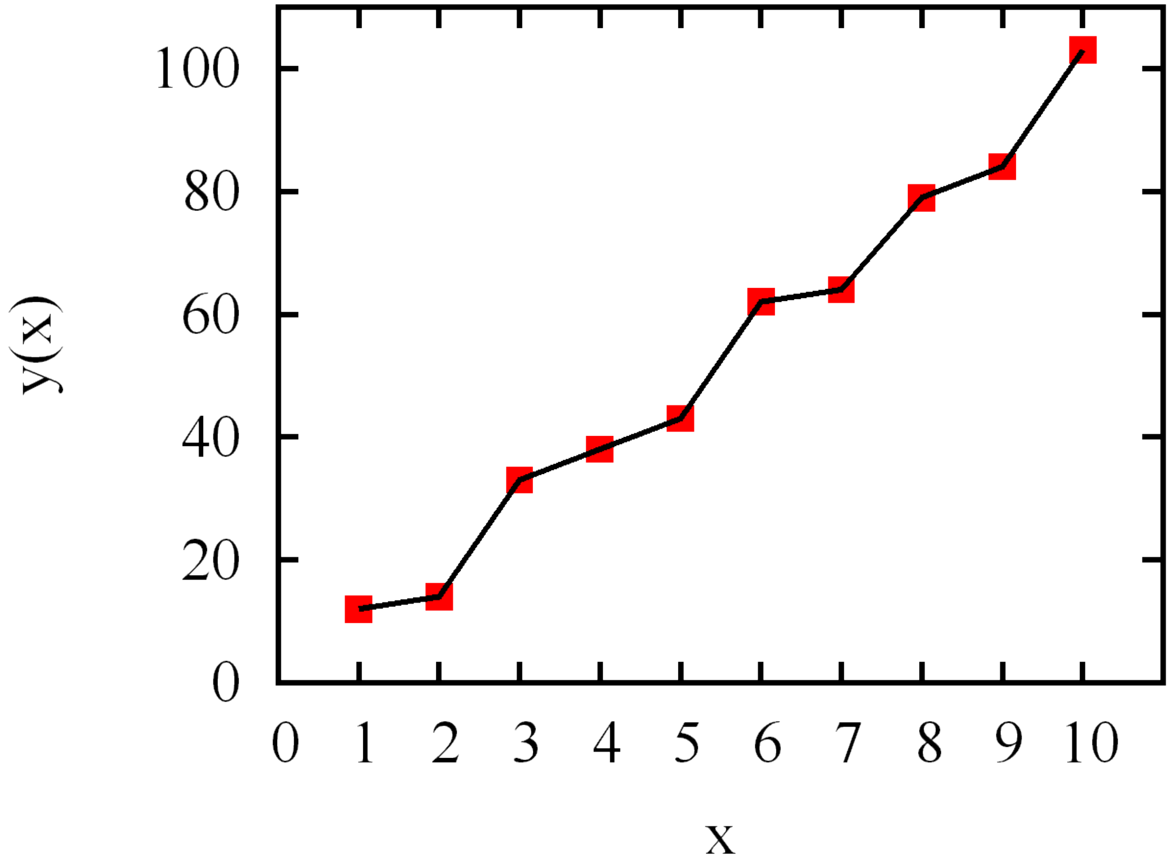
Statistics

Fitting a function to experimental data

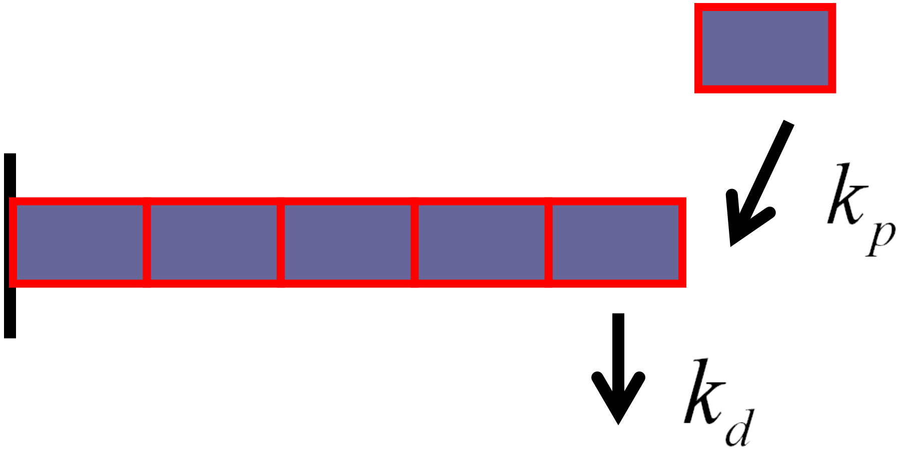
What do you do with this data ?



Is it fine to join data points ?



Polymerization of actin

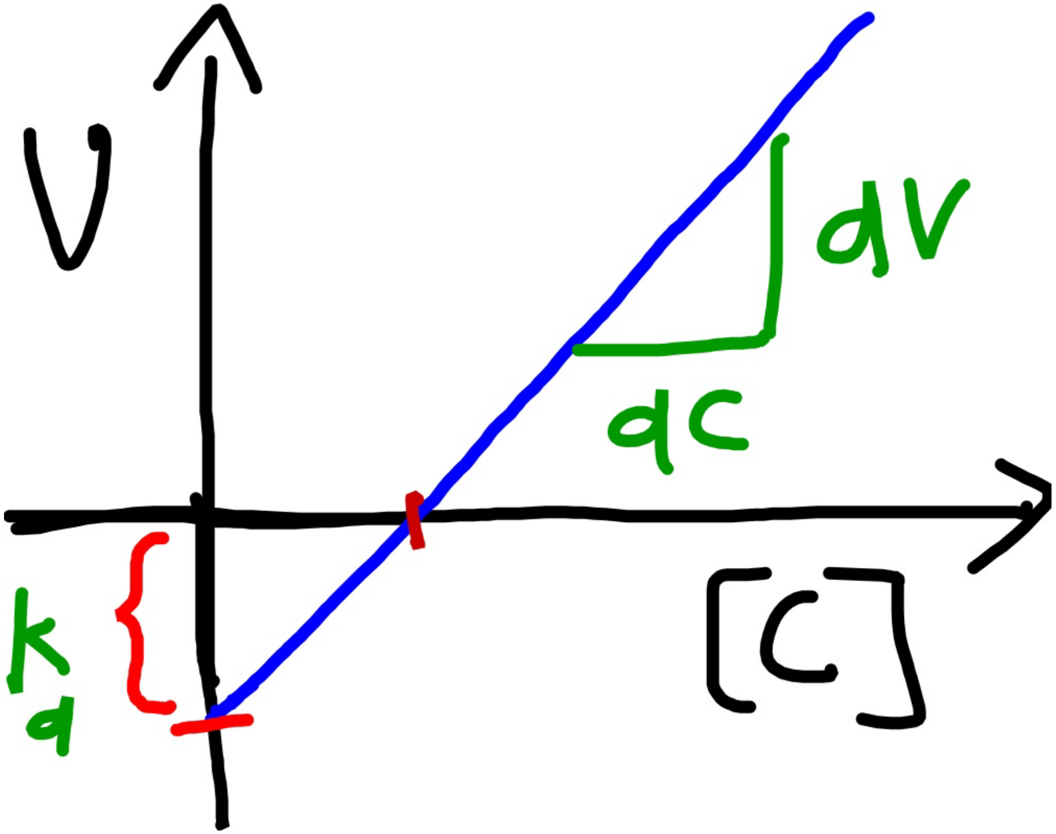


$$k_p = k_0 C$$

Growth speed $$V = k_p - k_d$$

$$V = k_0 C - k_d$$

$$V = k_0[C] - k_d$$



Slope $\dfrac{dV}{dc} = k_0$

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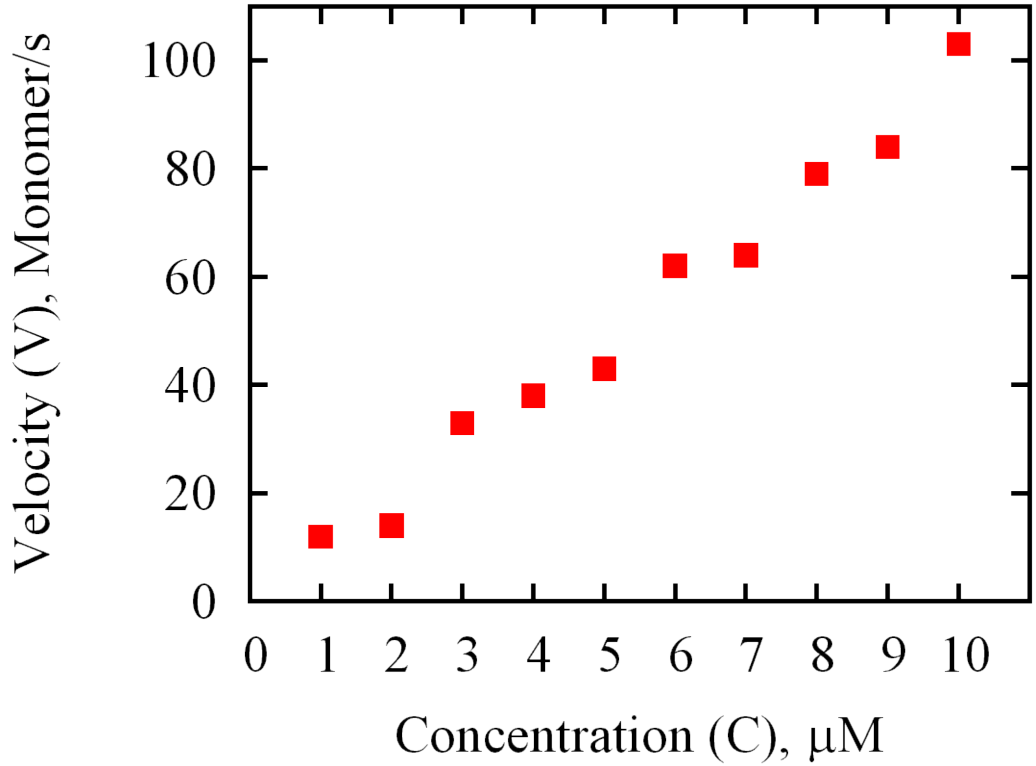
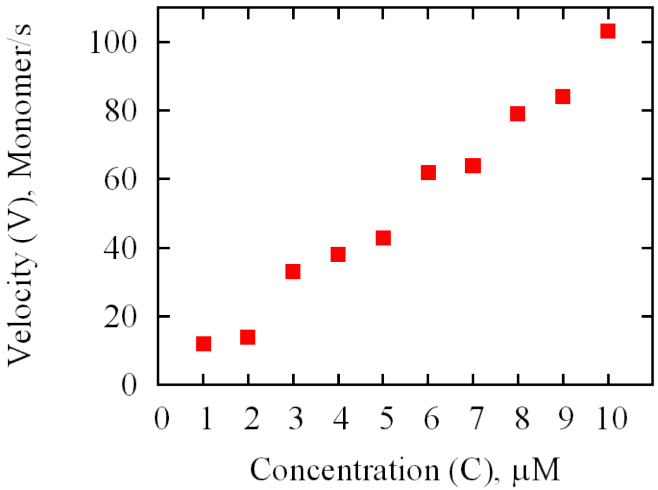
Theoretically we know this should be a straight line



How do you get a straight line from this data ?

How do you get a straight line from this data ?



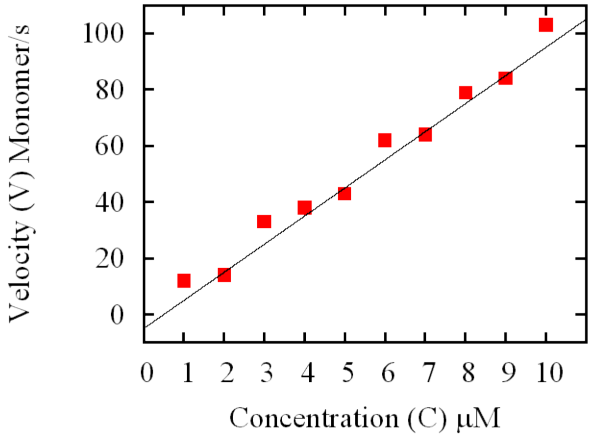
$$V = k_0 C - k_d$$

We need to find a k_0 and k_d that will best "fit" this data points

We want a line where

$$(V_{measured} - V_{theory})$$

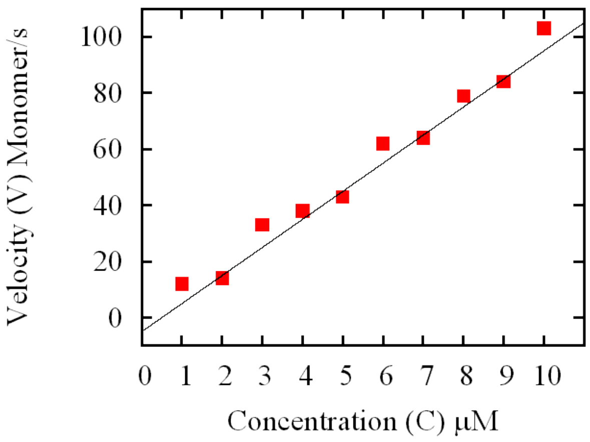
…is minimum



C_i	V_i
1.0	12.0
2.0	14.0
3.0	33.0
4.0	38.0
5.0	43.0
6.0	62.0
7.0	64.0
8.0	79.0
9.0	84.0
10.0	103

Minimum of

$$R\left(k_0, k_d\right) = \sum_i \left(V_i - (k_0 C_i - k_d)\right)^2$$



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Minimum of

$$R\left(k_0, k_d\right) = \sum_i \left(V_i - \left(k_0 C_i - k_d\right)\right)^2$$

$$\frac{\partial R\left(k_0, k_d\right)}{\partial k_0} = 0$$

$$\frac{\partial R\left(k_0, k_d\right)}{\partial k_d} = 0$$

$$\frac{\partial R\left(k_0, k_d\right)}{\partial k_0} = -2\sum_i (V_i - (k_0 C_i - k_d))C_i = 0$$

$$\frac{\partial R\left(k_0, k_d\right)}{\partial k_d} = 2\sum_i (V_i - (k_0 C_i - k_d)) = 0$$

Two equations and two unknowns, k_0 and k_d

$$nk_d + k_0 \sum_i^n C_i = \sum_i^n (V_i)$$

$$k_d \sum_i^n C_i + k_0 \sum_i^n C_i^2 = \sum_i^n (C_i V_i)$$

Solution of these two equations leads to the values of k_0 and k_d

$$k_d = \frac{\sum_{i=1}^{n} V_i \sum_{i=1}^{n} C_i^2 - \sum_{i=1}^{n} C_i \sum_{i=1}^{n} C_i V_i}{n\sum_{i}^{n} C_i^2 - \left(\sum_{i}^{n} C_i\right)^2}$$

$$k_d = \frac{\overline{V}\left(\sum_{i=1}^{n} C_i^2\right) - \overline{C}\sum_{i=1}^{n} C_i V_i}{n\sum_{i}^{n} C_i^2 - (n\overline{C})^2}$$

$$k_0 = \frac{n\sum_{i=1}^{n} C_i V_i - \sum_{i=1}^{n} C_i \sum_{i=1}^{n} V_i}{n\sum_{i}^{n} C_i^2 - \left(\sum_{i}^{n} C_i\right)^2}$$

$$k_0 = \frac{\left(\sum_{i=1}^{n} C_i V_i\right) - n\overline{C}\,\overline{V}}{n\sum_{i}^{n} C_i^2 - (n\overline{C})^2}$$

Known constants we can draw a line