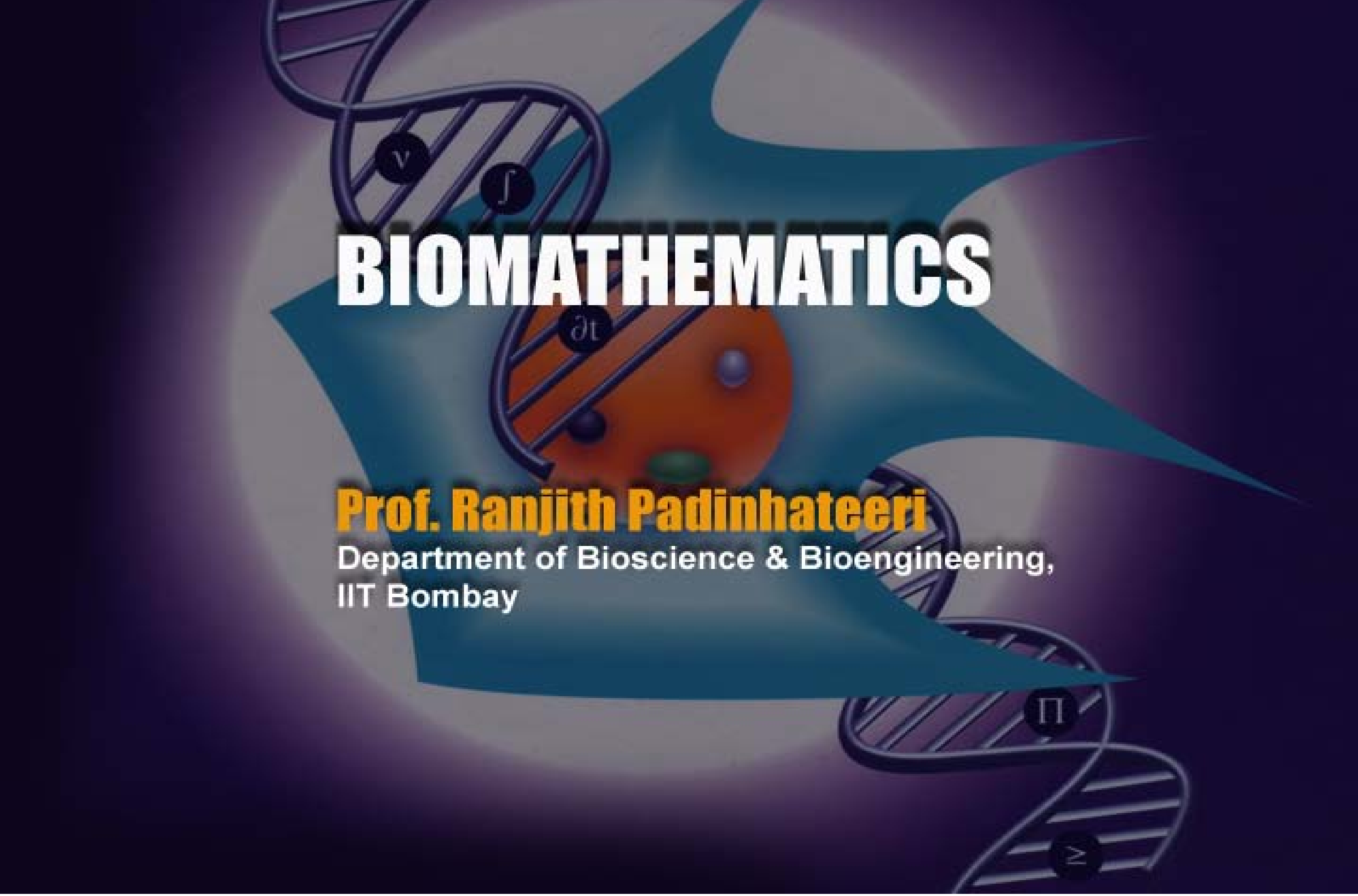
BIOMATHEMATICS
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Lecture 18

Applications of calculus and vector algebra in biology

Diffusion

Current/flow

$$\vec{J}_D = -D\vec{\nabla} C$$

$$\vec{J}_D = -D\frac{\partial C}{\partial x}\hat{x}$$

Constant flow/current

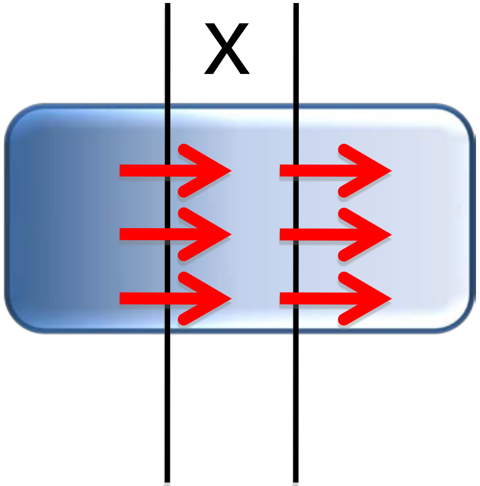
$$C = -x$$

$$\vec{J}_D = -D\frac{\partial C}{\partial x}\hat{x} = D\hat{x}$$

D=diffusion constant

Constant flow/current

$$\dot{J}_D = D\hat{x}$$



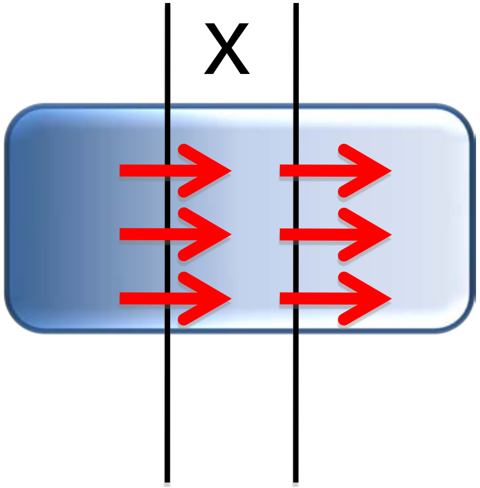
Whatever comes into “x” will go out of “x”

No, net change in concentration

$$\frac{\partial C(x,t)}{\partial t} = 0$$

Constant flow/current

$$\frac{\partial C(x,t)}{\partial t} = 0$$



For change in concentration, J should change along space

$\vec{\nabla}$ operator

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x}$$

Divergence

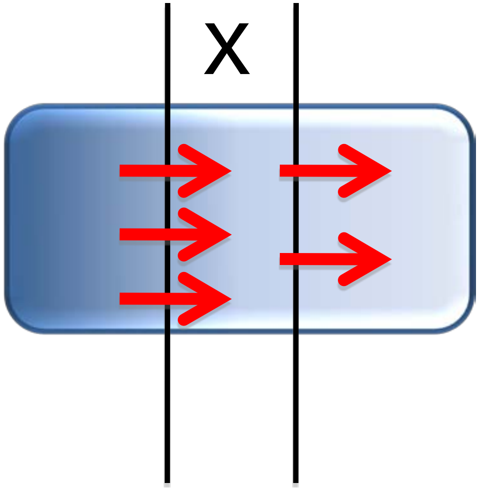
$$\vec{\nabla} \bullet \vec{J} = \left(\frac{\partial}{\partial x}\right)\hat{x} \bullet \vec{J}$$

$$\vec{J}(x) = j(x)\hat{x}$$

$$\vec{\nabla} \bullet \vec{J} = \frac{\partial j(x)}{\partial x}$$

Continuity equation

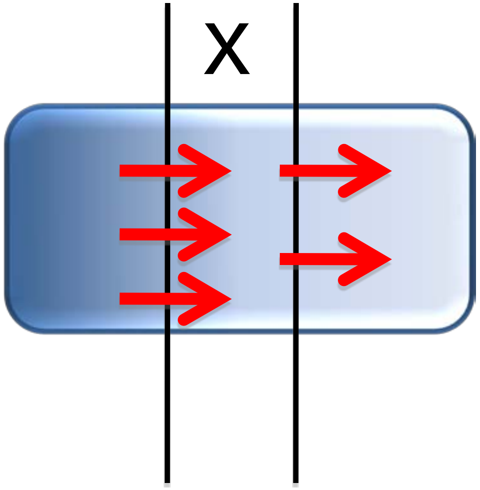
$$\frac{\partial C(x,t)}{\partial t} = -\vec{\nabla} \bullet \vec{J}$$



For change in concentration, J should depend on space variable

Diffusion equation

$$\frac{\partial C(x,t)}{\partial t} = -\vec{\nabla} \bullet \vec{J}$$



$$\vec{J} = -D\frac{\partial C}{\partial x}\hat{x}$$

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial^2 C(x,t)}{\partial x^2}$$

$$\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, c(x)\, dx$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 c(x)\, dx$$

Summary

- Divergence
- Continuity equation
- Diffusion equation
- Mean-square position