



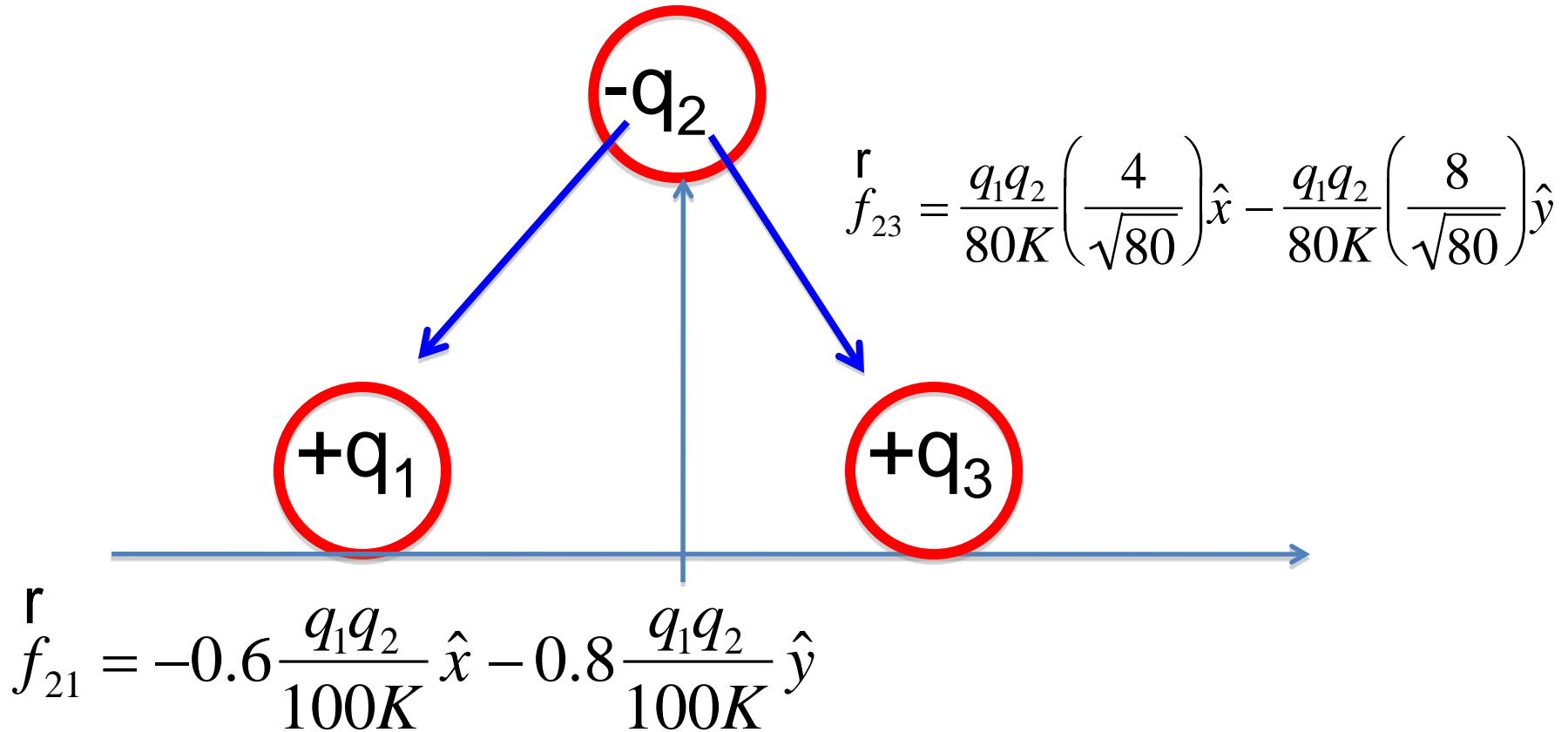
# BIOMATHEMATICS

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## Lecture 16

# Vectors-3



$$\vec{f}_2 = \vec{f}_{21} + \vec{f}_{23}$$

## We learned about:

- Finding out resultant force
- Unit vector
- Magnitude of a vector
- Direction of a vector

## Addition of vectors

Vectors can be added by adding individual components. E.g.

$$\overset{\text{r}}{A} = a_1 \hat{x} + a_2 \hat{y} + a_3 \hat{z}$$

$$\overset{\text{r}}{B} = b_1 \hat{x} + b_2 \hat{y} + b_3 \hat{z}$$

$$\overset{\text{r}}{C} = \overset{\text{r}}{A} + \overset{\text{r}}{B}$$

$$\overset{\text{r}}{C} = (a_1 + b_1) \hat{x} + (a_2 + b_2) \hat{y} + (a_3 + b_3) \hat{z}$$

## Subtraction of vectors

Vectors can be subtracted by subtracting their individual components. E.g.

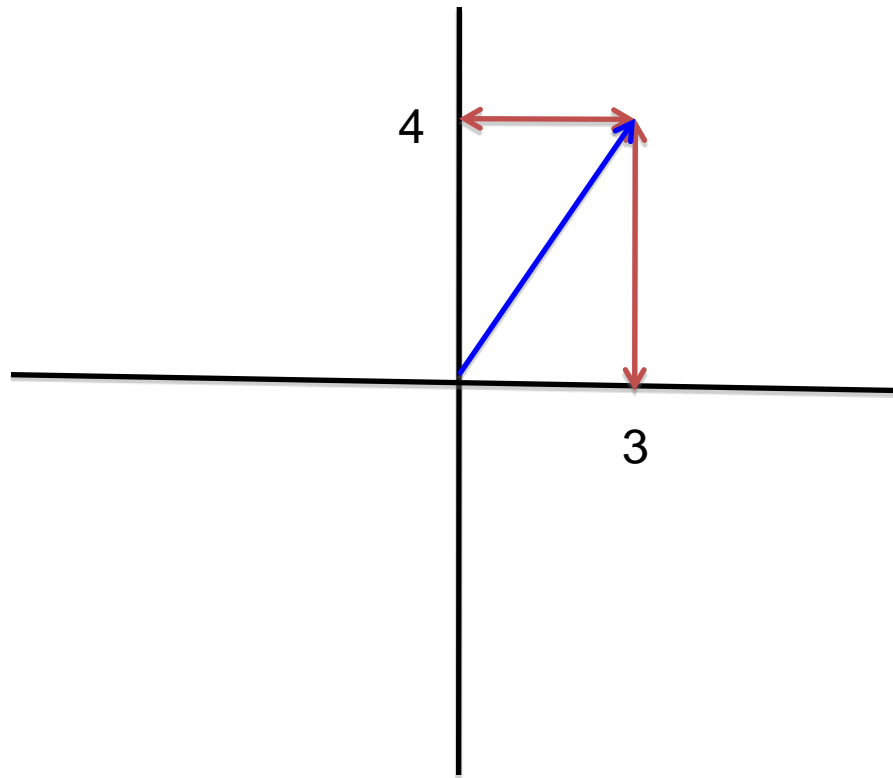
$$\overset{\text{r}}{A} = a_1 \hat{x} + a_2 \hat{y} + a_3 \hat{z}$$

$$\overset{\text{r}}{B} = b_1 \hat{x} + b_2 \hat{y} + b_3 \hat{z}$$

$$\overset{\text{r}}{C} = \overset{\text{r}}{A} - \overset{\text{r}}{B}$$

$$\overset{\text{r}}{C} = (a_1 - b_1) \hat{x} + (a_2 - b_2) \hat{y} + (a_3 - b_3) \hat{z}$$

We saw that two numbers specify a vector in 2D e.g. position of an atom

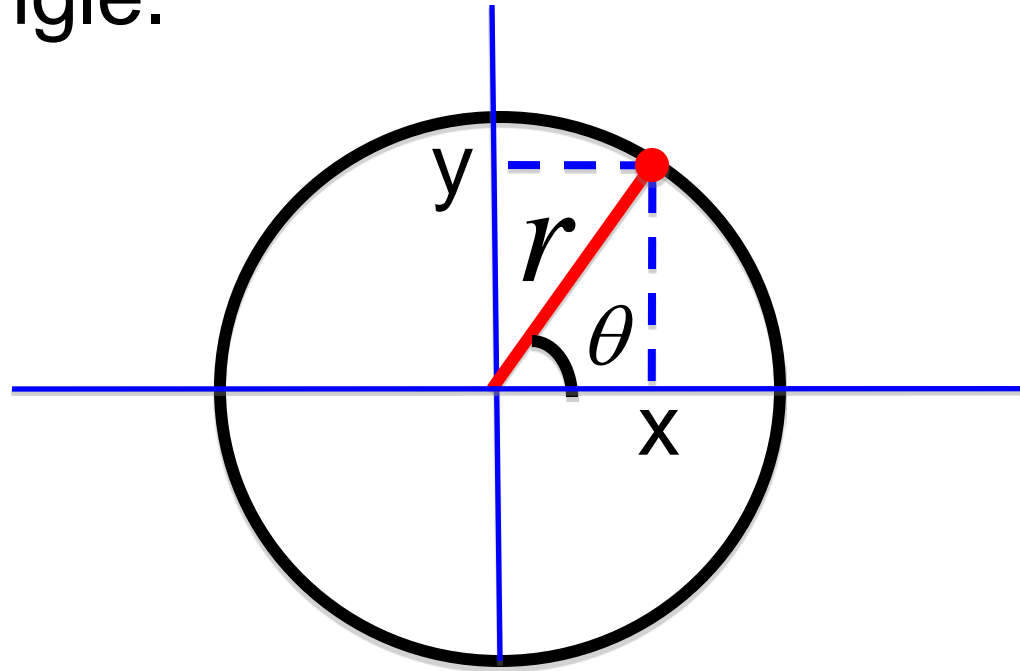


## Plane polar co-ordinate

We can represent the same using a distance and an angle.

$$x = r \cos \theta$$

$$y = r \sin \theta$$





How do we find the product of two vectors ?

## Product of vectors

You can imagine two situations

1) Product of two vectors is a scalar :  
(Scalar product)

Eg. When you apply a force and change the position of an object, the product of force and displacement is work, which is a scalar

2) Product of two vectors is another vector : Vector

## Product of vectors

2) Product of two vectors is another vector  
(Vector product)

Eg. Applying a force to twist DNA

or

Applying a force to rotate an object

# Scalar product of vectors

$$\mathbf{f} = a_1 \hat{x} + a_2 \hat{y} + a_3 \hat{z}$$

$$\mathbf{x} = b_1 \hat{x} + b_2 \hat{y} + b_3 \hat{z}$$

$$C = \mathbf{f} \bullet \mathbf{x}$$

$$C = |\mathbf{f}| |\mathbf{x}| \cos \theta$$

$$C = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Also known as “dot product”

## Vector product

$$\mathbf{r} = r_1 \hat{x} + r_2 \hat{y} + r_3 \hat{z}$$

$$\mathbf{f} = f_1 \hat{x} + f_2 \hat{y} + f_3 \hat{z}$$

$$\mathbf{T} = \mathbf{r} \times \mathbf{f}$$

$$\mathbf{T} = |\mathbf{r}| |\mathbf{f}| \sin \theta \hat{n}$$

Also known as “cross product”

## Gradient of a scalar



$$\mathit{grad}C = \overset{\mathbf{r}}{\nabla} C = \frac{\partial C}{\partial x} \hat{x}$$

## Summary

- Plain polar co-ordinates
- Dot product and cross product
- Gradient