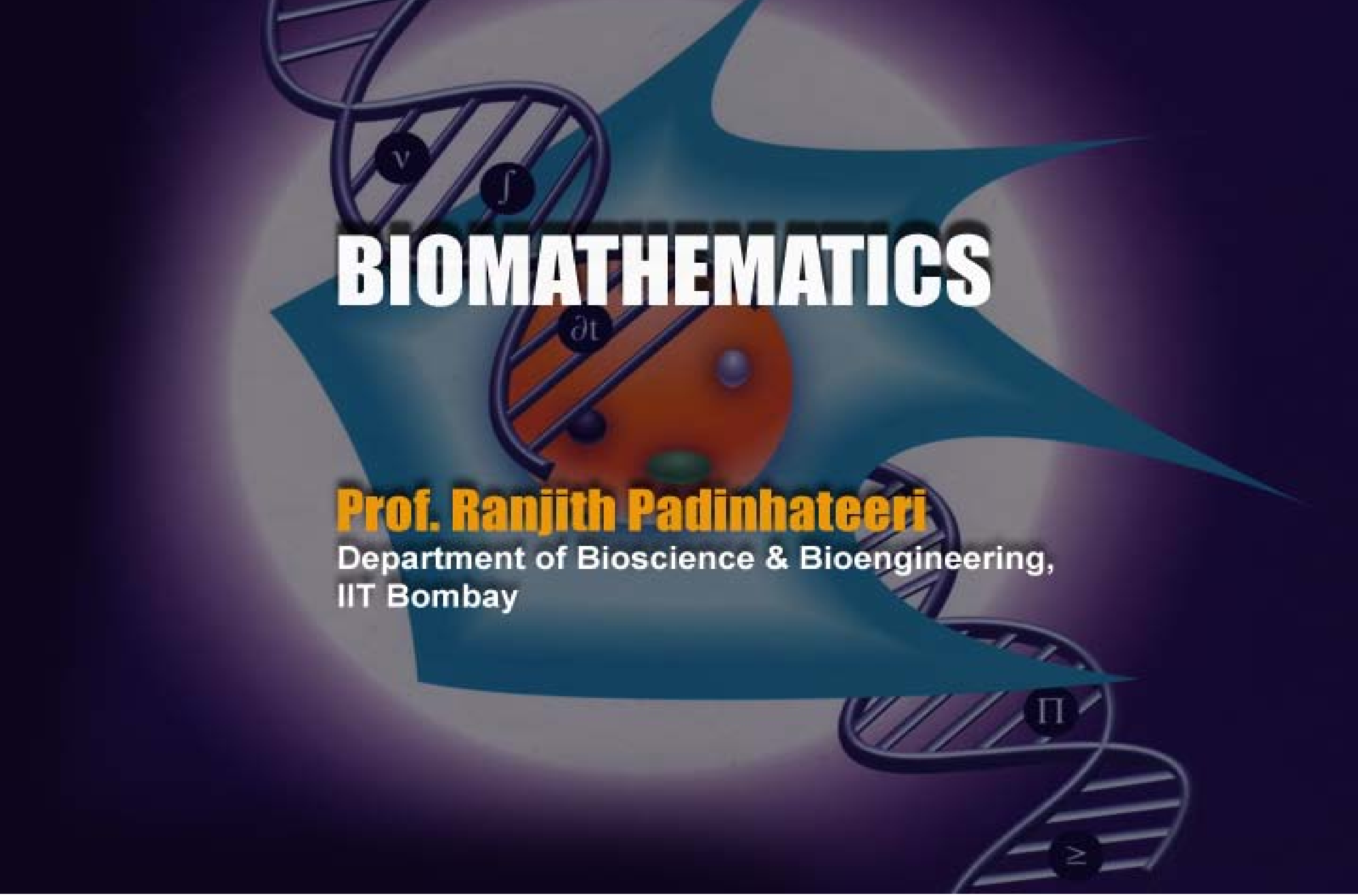
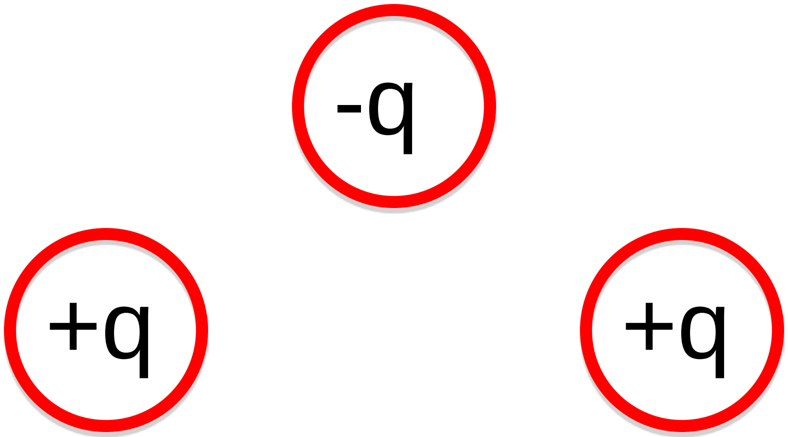

BIOMATHEMATICS
Prof. Ranjith Padinhateeri
Department of Bioscience & Bioengineering,
IIT Bombay

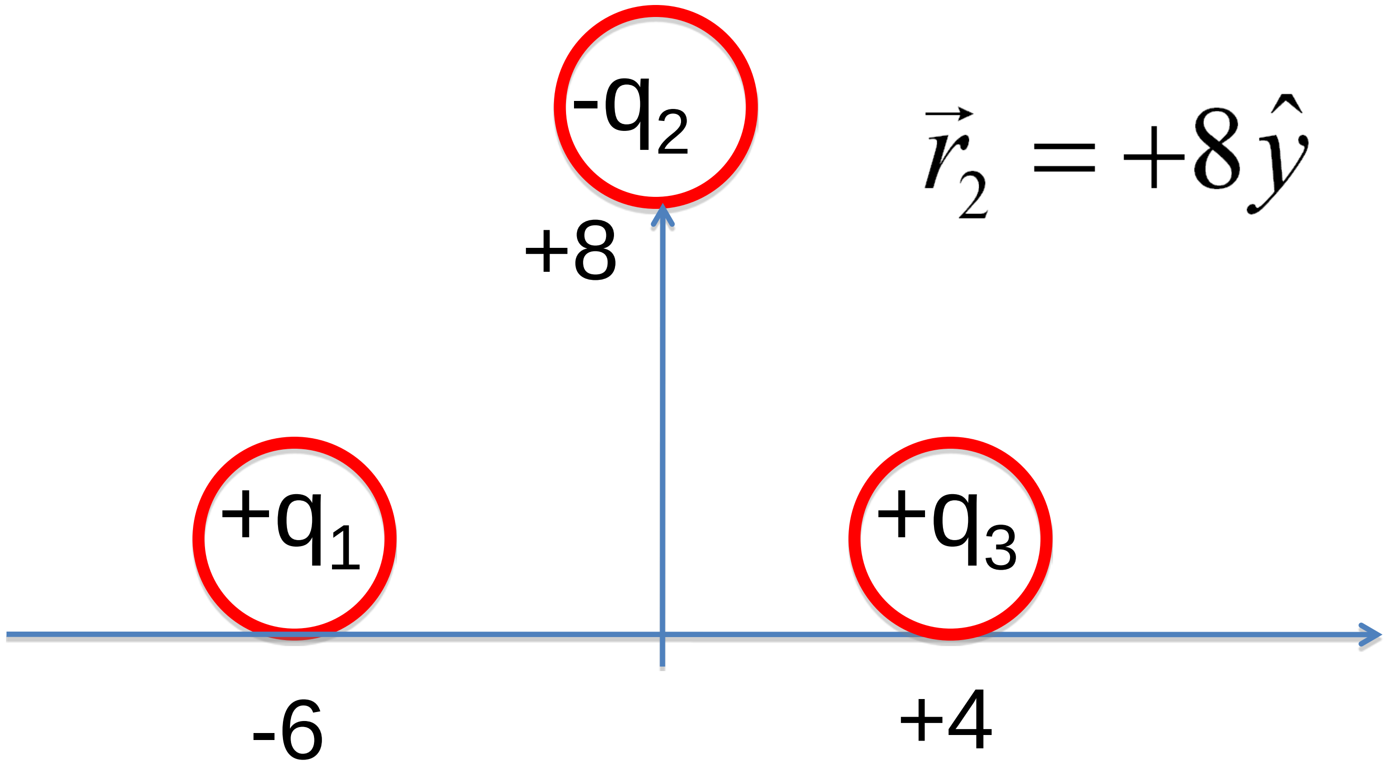
Lecture 15

Vectors-2

Three charges: total force



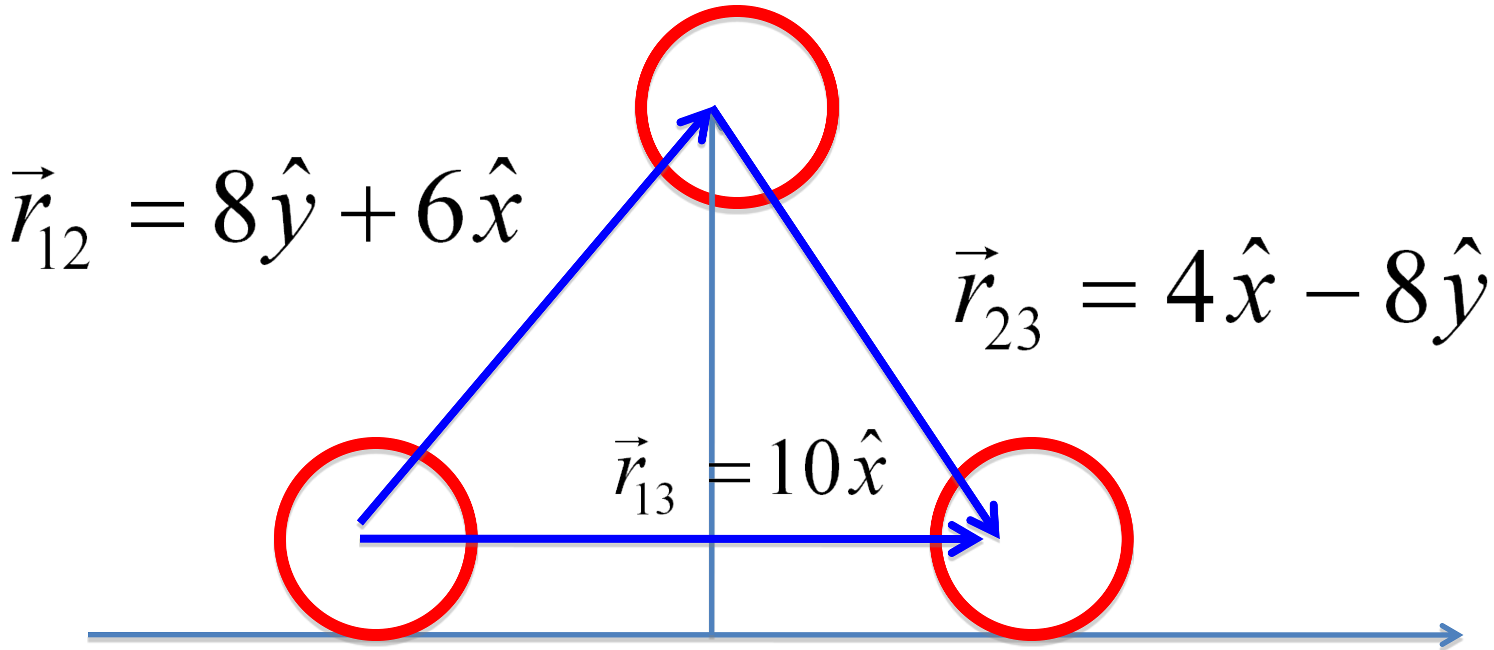
In which direction will the negative charge move ?

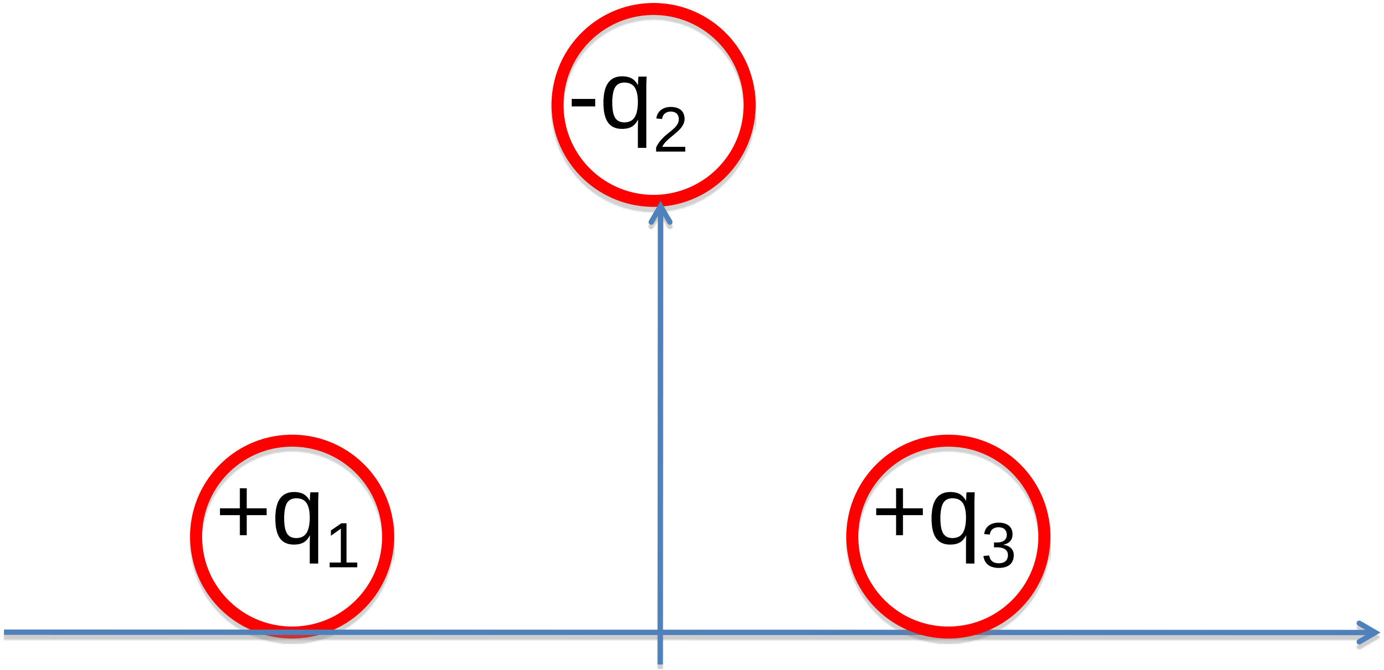


$$\vec{r}_2 = +8\hat{y}$$

$$\vec{r}_1 = -6\hat{x}$$

$$\vec{r}_3 = +4\hat{x}$$

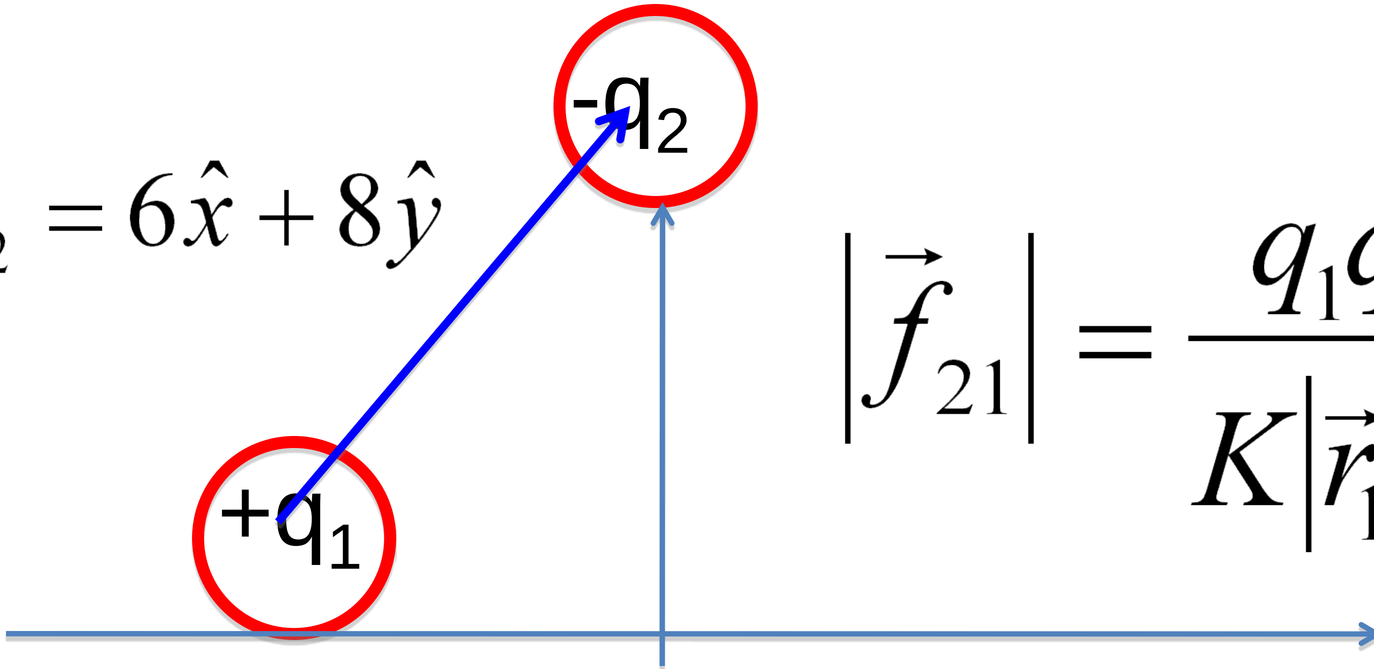

$\vec{r}_{12} = 8\hat{y} + 6\hat{x}$
$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$
$\vec{r}_{13} = 10\hat{x}$



Force on the second charge ?

$$\vec{f}_2 = \vec{f}_{21} + \vec{f}_{23}$$

Force on charge q_2 due to q_1



$$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$$

$$\left|\vec{f}_{21}\right| = \frac{q_1 q_2}{K\left|\vec{r}_{12}\right|^2}$$

$$\left|\vec{r}_{12}\right| = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$

Magnitude of a vector

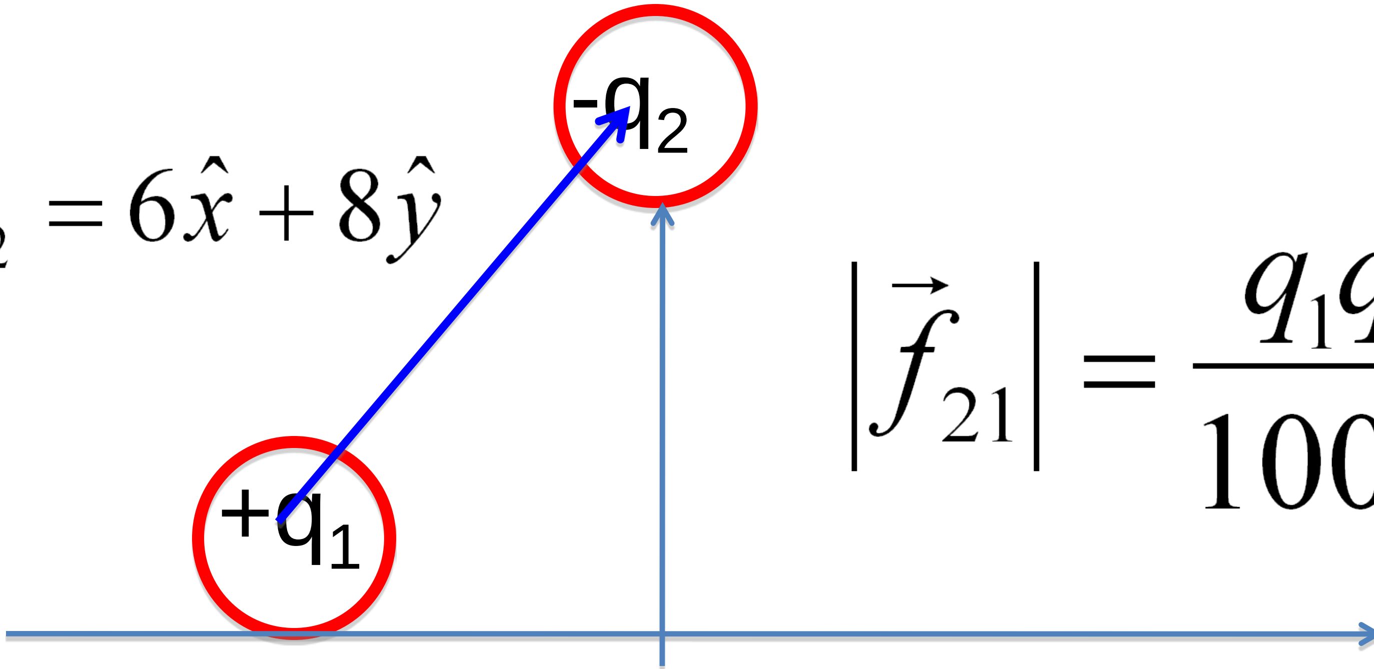
Consider a vector A

$$\vec{A} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$$

Magnitude of the vector = length of the vector

$$\left|\vec{A}\right| = \sqrt{a_1^{\,2} + a_2^{\,2} + a_3^{\,2}}$$

What is the direction of the force ?



$$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$$

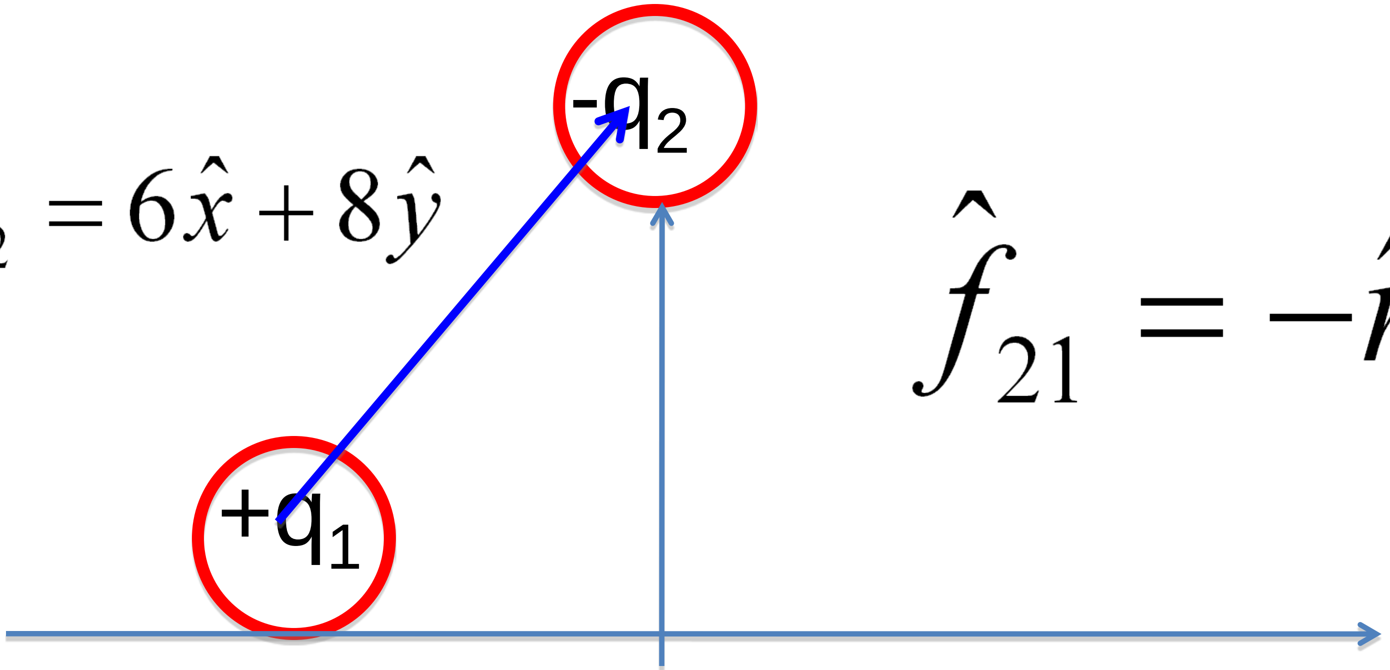
$$\left|\vec{f}_{21}\right| = \frac{q_1 q_2}{100K}$$

q_2 will be attracted towards q_1

Direction is opposite to the direction of $\vec{r}_{12}$

Opposite to the direction of $\vec{r}_{12}$

$$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$$

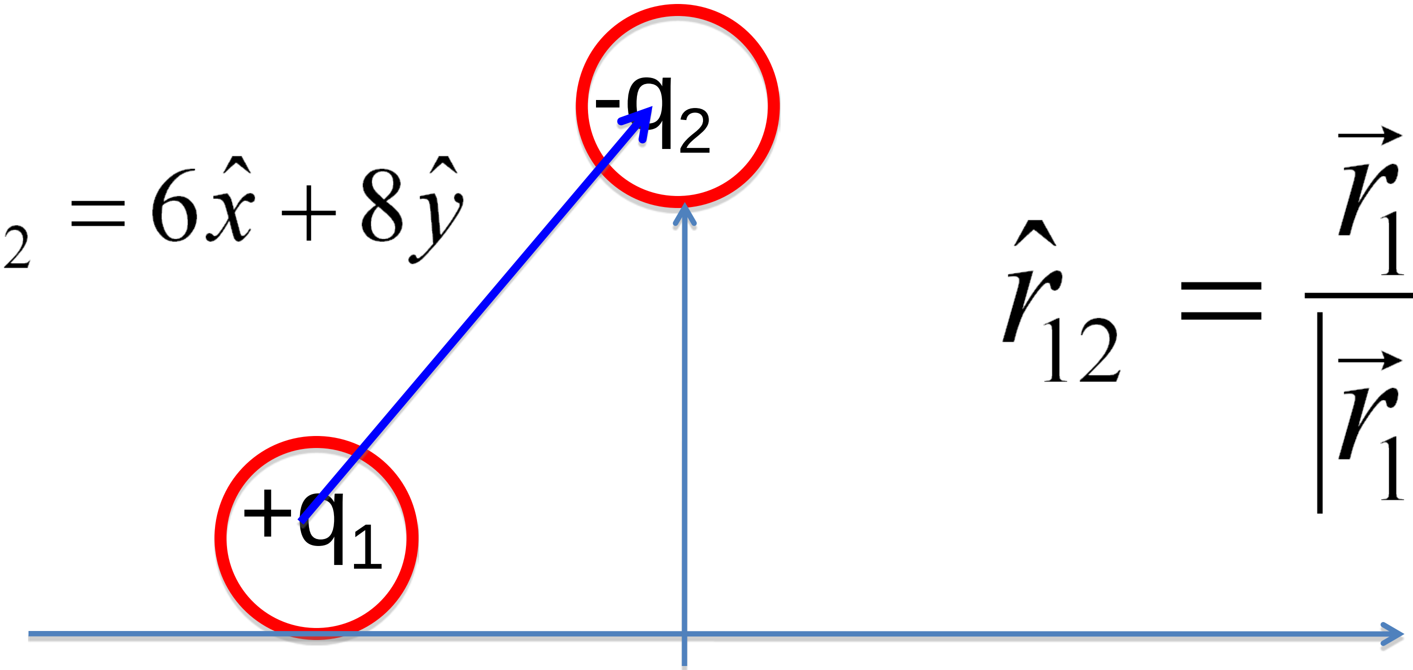


$$\hat{f}_{21} = -\hat{r}_{12}$$

A unit vector represents direction

How do we calculate $\hat{r}_{12}$?

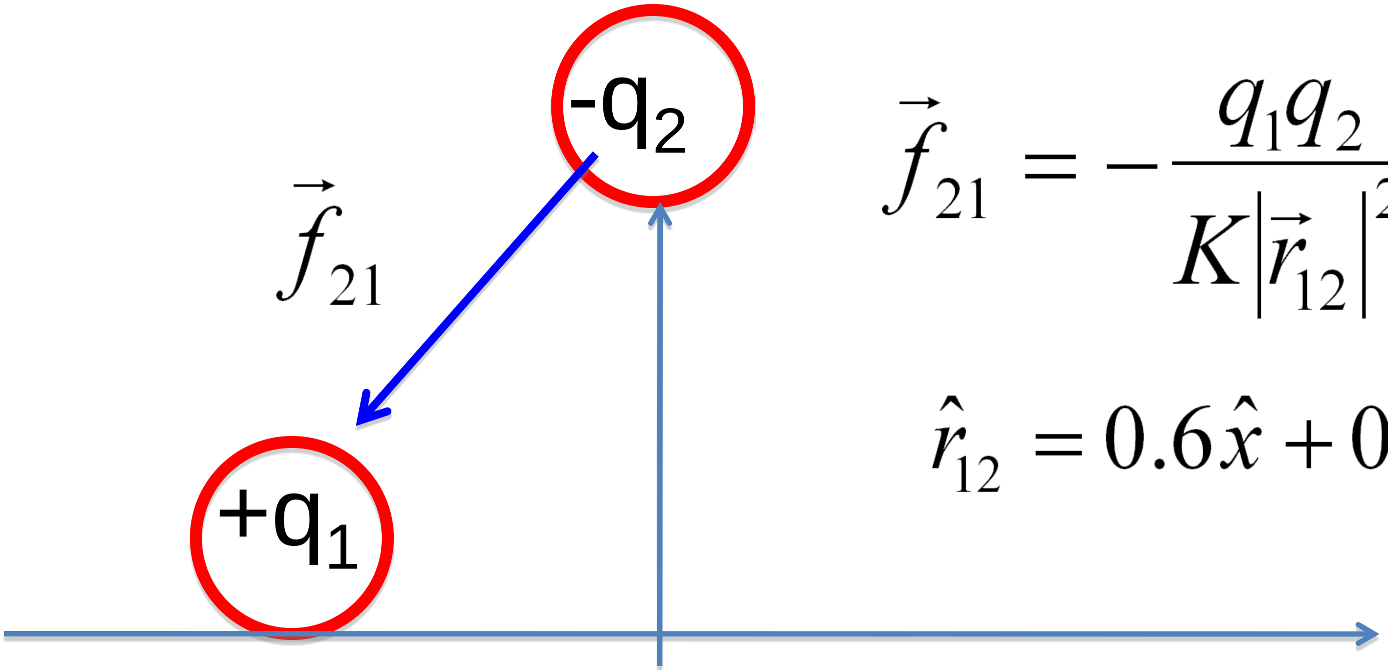
$$\vec{r}_{12} = 6\hat{x} + 8\hat{y}$$



$$\hat{r}_{12} = \frac{\vec{r}_{12}}{\left|\vec{r}_{12}\right|}$$

$$\hat{r}_{12} = \frac{\vec{r}_{12}}{\left|\vec{r}_{12}\right|} = \frac{6\hat{x} + 8\hat{y}}{\sqrt{6^2 + 8^2}} = 0.6\hat{x} + 0.8\hat{y}$$

Force on charge q_2 due to q_1



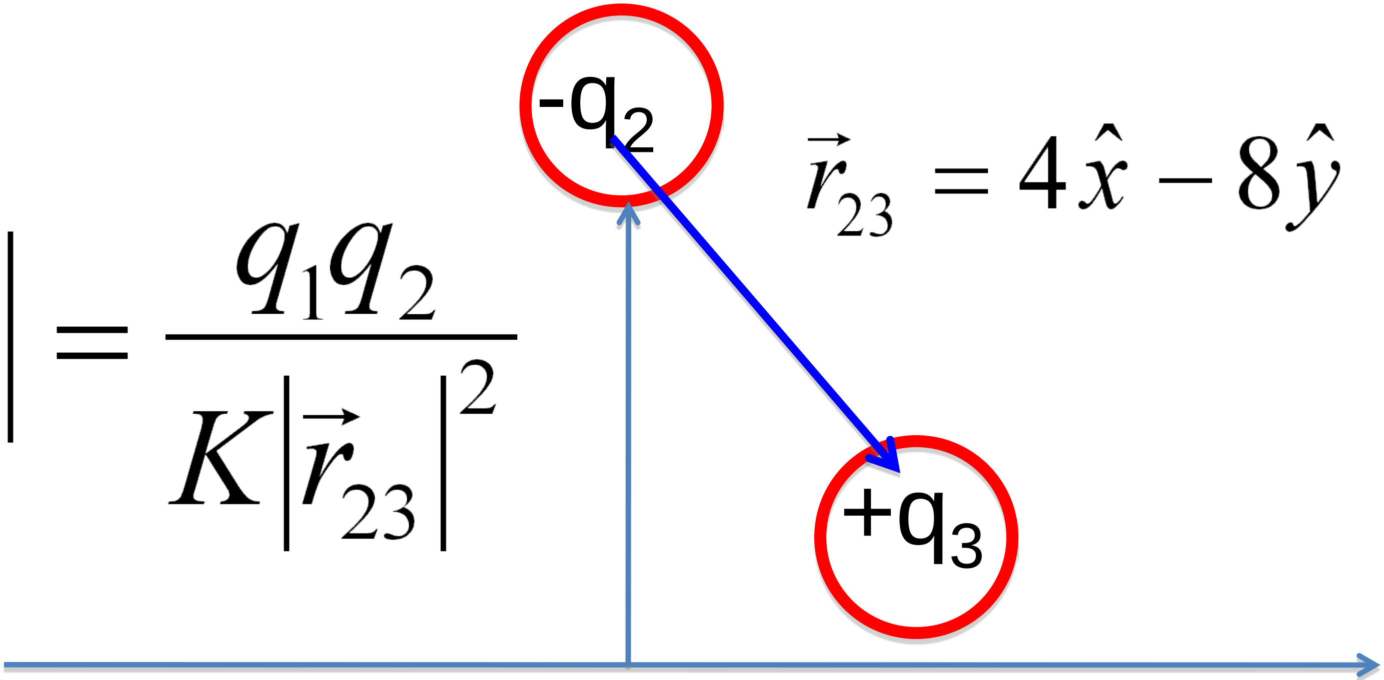
$$\vec{f}_{21} = -\frac{q_1 q_2}{K\left|\vec{r}_{12}\right|^2}\hat{r}_{12}$$

$$\hat{r}_{12} = 0.6\hat{x} + 0.8\hat{y}$$

$$\vec{f}_{21} = -0.6\frac{q_1 q_2}{100K}\hat{x} - 0.8\frac{q_1 q_2}{100K}\hat{y}$$

Force on charge q_2 due to q_3

$$\left|\vec{f}_{23}\right| = \frac{q_1 q_2}{K\left|\vec{r}_{23}\right|^2}$$



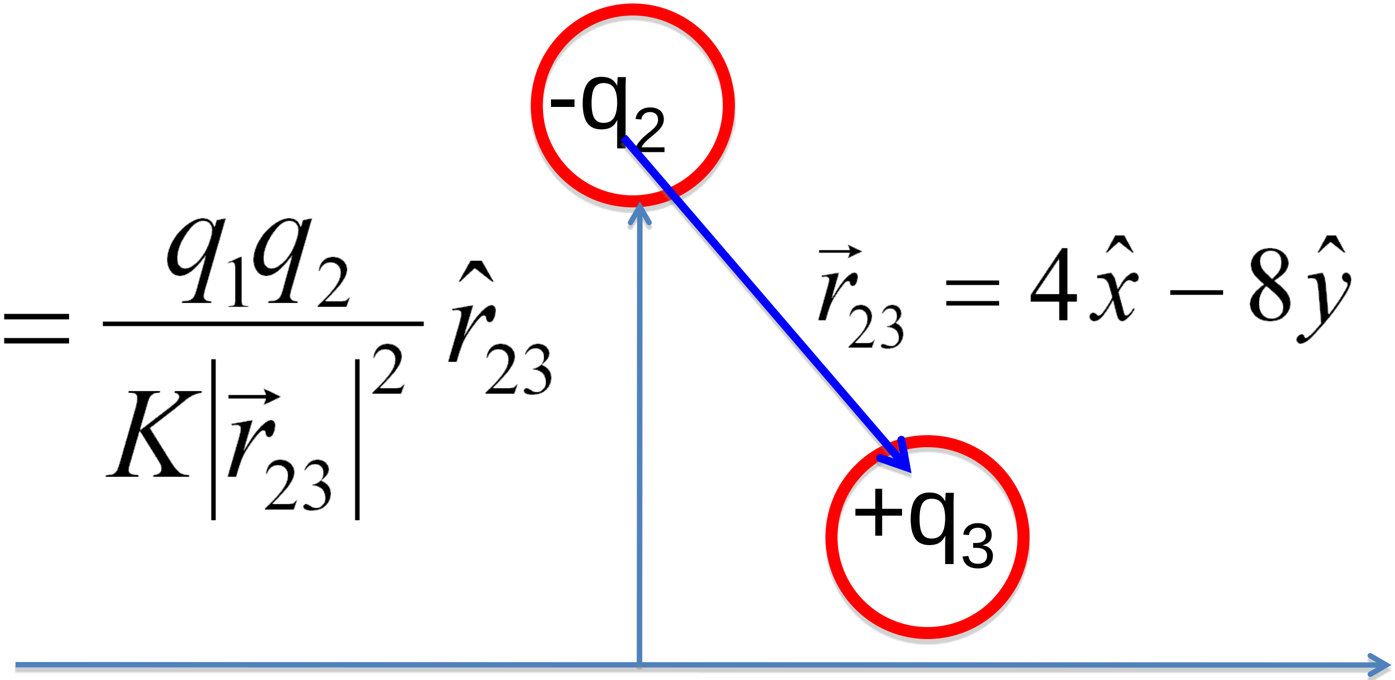
$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

q_2 will be attracted towards q_3

Direction is the same as direction of $\vec{r}_{23}$

Force on charge q_2 due to q_3

$$\vec{f}_{23} = \frac{q_1 q_2}{K\left|\vec{r}_{23}\right|^2}\hat{r}_{23}$$

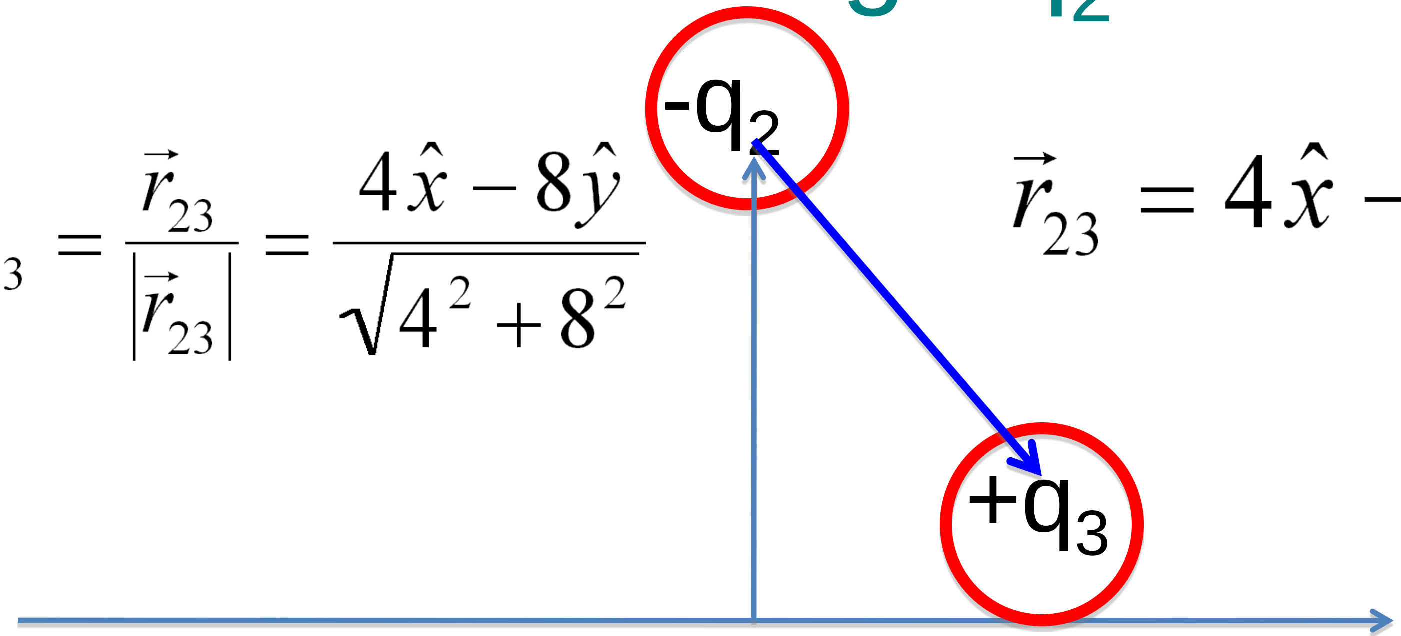


$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{\left|\vec{r}_{23}\right|} = \frac{4\hat{x} - 8\hat{y}}{\sqrt{4^2 + 8^2}}$$

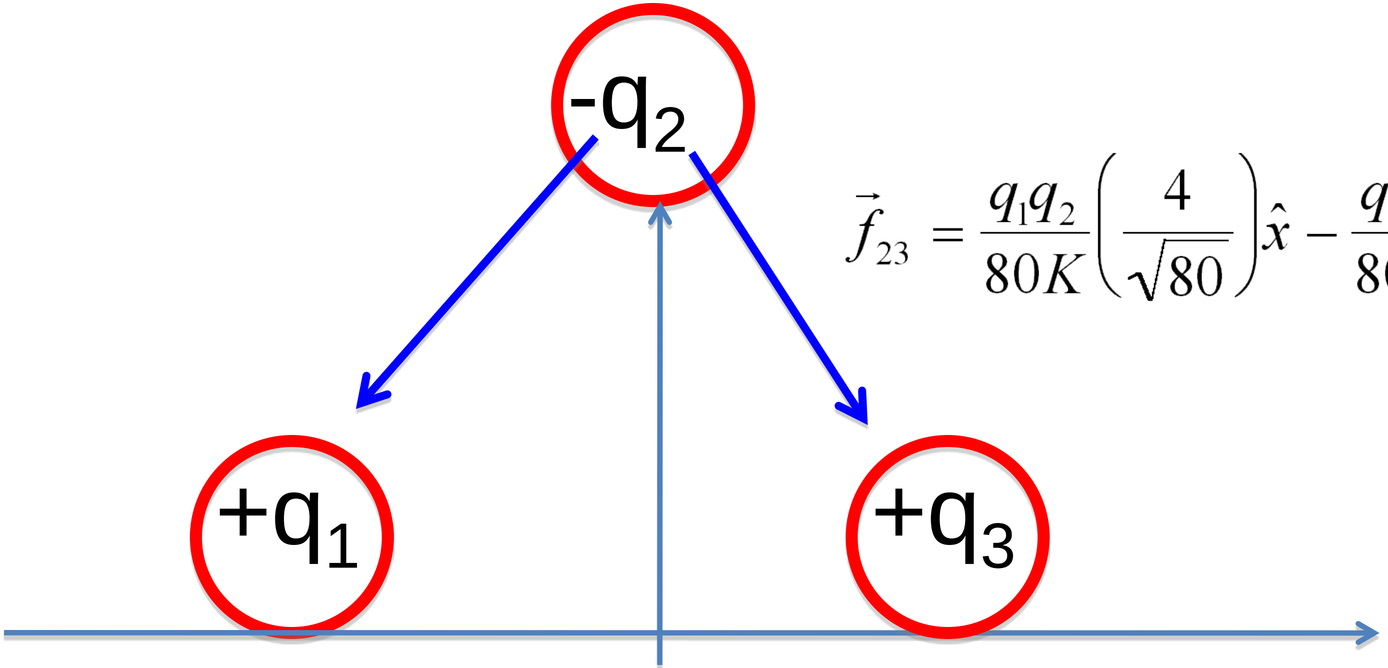
Force on charge q_2 due to q_3

$$\hat{r}_{23} = \frac{\vec{r}_{23}}{\left|\vec{r}_{23}\right|} = \frac{4\hat{x} - 8\hat{y}}{\sqrt{4^2 + 8^2}}$$



$$\vec{r}_{23} = 4\hat{x} - 8\hat{y}$$

$$\vec{f}_{23} = \frac{q_1 q_2}{80K}\left(\frac{4}{\sqrt{80}}\right)\hat{x} - \frac{q_1 q_2}{80K}\left(\frac{8}{\sqrt{80}}\right)\hat{y}$$



$$\vec{f}_{23} = \frac{q_1 q_2}{80K}\left(\frac{4}{\sqrt{80}}\right)\hat{x} - \frac{q_1 q_2}{80K}\left(\frac{8}{\sqrt{80}}\right)\hat{y}$$

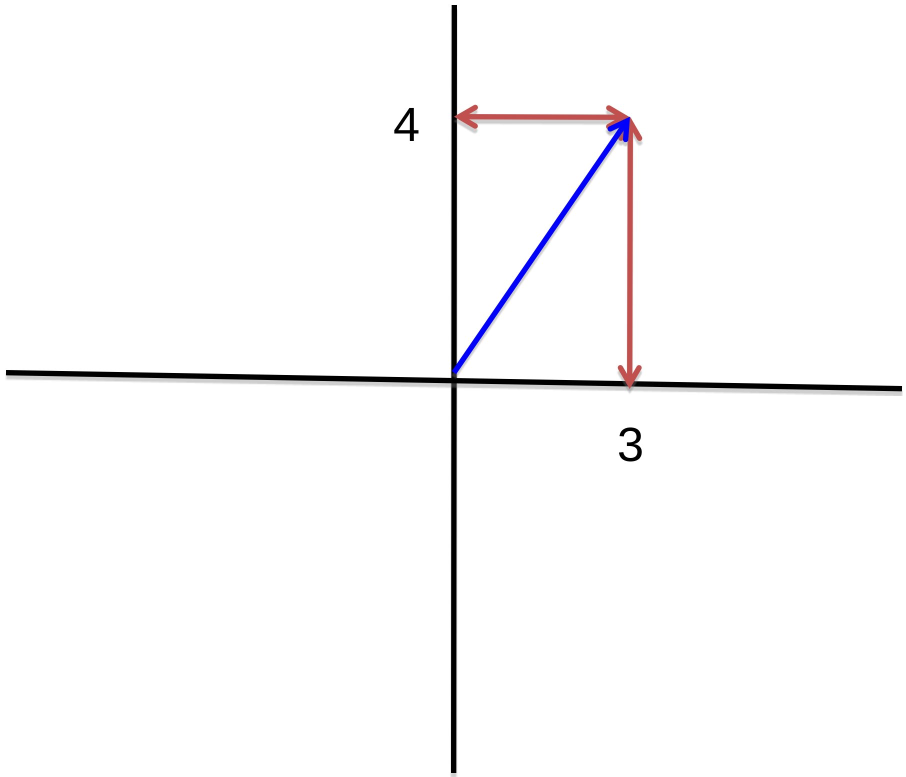
$$\vec{f}_{21} = -0.6\,\frac{q_1 q_2}{100K}\,\hat{x} - 0.8\,\frac{q_1 q_2}{100K}\,\hat{y}$$

$$\vec{f}_2 = \vec{f}_{21} + \vec{f}_{23}$$

Summary

- Finding out resultant force
- Unit vector
- Magnitude of a vector
- Direction of a vector

We saw that two numbers specify a vector in 2D e.g. position of a particle.

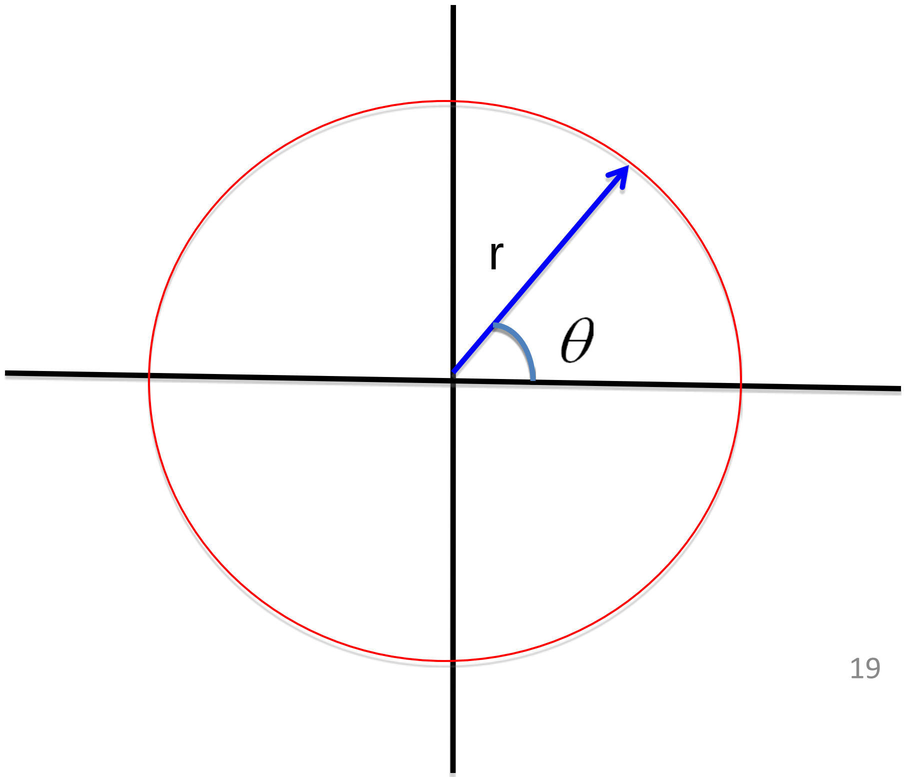


Plane polar co-ordinate

We can represent the same using a distance and an angle.

$$x = r\cos\theta$$

$$y = r\sin\theta$$



Addition of vectors

Vectors can be added by adding individual components. E.g.

$$\vec{A} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$$

$$\vec{B} = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = (a_1 + b_1)\hat{x} + (a_2 + b_2)\hat{y} + (a_3 + b_3)\hat{z}$$

Subtraction of vectors

Vectors can be subtracted by subtracting their individual components. E.g.

$$\vec{A} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$$

$$\vec{B} = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$$

$$\vec{C} = \vec{A} - \vec{B}$$

$$\vec{C} = (a_1 - b_1)\hat{x} + (a_2 - b_2)\hat{y} + (a_3 - b_3)\hat{z}$$

Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay

Dot product of vectors

$$\vec{f} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$$

$$\vec{x} = b_1\hat{x} + b_2\hat{y} + b_3\hat{z}$$

$$C = \vec{f} \bullet \vec{x}$$

$$C = \left|\vec{f}\right|\left|\vec{x}\right|\cos\theta$$

$$C = a_1b_1 + a_2b_2 + a_3b_3$$

Summary

- Addition of vectors
- Plan polar co-ordinates
- Dot product