BIOMATHEMATICS

Prof. Ranjith Padinhateeri
Department of Bioscience & Bioengineering,
IIT Bombay

Differential equations part 2

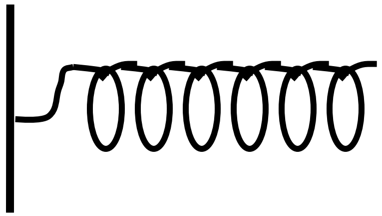
Relation between force (f) and Energy (E)

$$f = -\frac{dE}{dR}$$

Example 1: Spring

$$f \propto -R$$

$$f = -kR$$



Energy Calculation

$$\frac{dE}{dR} = -f$$

$$\frac{dE}{dR} = kR$$

$$dE = kR\,dR$$

$$\int dE = \int kR\,dR$$

$$E = k\,\frac{R^2}{2} + constant$$

Example 2: Charge system

$$f = \frac{+q^2}{4\pi\varepsilon_0\varepsilon_r R^2}$$

Coulomb's Law

Energy Calculation

$$\frac{dE}{dR} = -f$$

$$\frac{dE}{dR} = -\frac{q^2}{4\pi\varepsilon_0\varepsilon_r R^2}$$

$$dE = -\frac{q^2}{4\pi\varepsilon_0\varepsilon_r R^2}\, dR$$

$$\int dE = -\frac{q^2}{4\pi\varepsilon_0\varepsilon_r}\int \frac{1}{R^2}\, dR$$

$$\int dE = -\frac{q^2}{4\pi\varepsilon_0\varepsilon_r}\int R^{-2}\,dR$$

$$E = \frac{q^2}{4\pi\varepsilon_0\varepsilon_r}\frac{1}{R} + constant$$

Uniform Concentration

$$\frac{dC}{dX} = 0$$

$$C = constant$$

Concentration gradient

$$\frac{dC}{dX} = k$$

$$dC = k\,dX$$

$$C = kX + constant$$

Newton's equation

$$ma = f$$

$$\frac{d^2x}{dt^2} = f/m$$

Solving this differential equation one can get position as a function of time

Second order differential equation

$$\frac{d^2x}{dt^2} = f(x)$$

f=mg = a constant

$$\frac{d^2x}{dt^2} = f/m$$

$$\frac{d^2x}{dt^2} = g$$

Eg: An object falling under gravity

First integration: To get velocity

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = g$$

$$\frac{d}{dt}(v) = g \Rightarrow v(t) = gt + v(0)$$

Where, v=dx/dt
v(0)=velocity at t=0: a constant

Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay

Second Integration: To get position

$$v(t) = gt + v(0)$$

$$\frac{dx}{dt} = gt + v_0$$

$$\int dx = \int (gt + v_0)dt$$

$$\Rightarrow x(t) = g\frac{t^2}{2} + v_0 t + x_0$$

Two constants of integration

$$x(t) = g\frac{t^2}{2} + v_0 t + x_0$$

x_0 and v_0 are two constants of integration

To solve a second order differential equation you need to know two constants of integration

Summary

- First and second order differential equation
- Relation between energy and force
- Newton’s equation

Prof. Ranjith Padinhateeri, Biosciences and Bioengineering, IIT Bombay